

## Proyek 13-15.

$$1. a) f(x) = x^{2/3} (3-x), [-2, 2]$$

$$f(x) = 3x^{2/3} - x^{5/3}$$

$$f'(x) = 3 \cdot \frac{2}{3} x^{-1/3} - \frac{5}{3} x^{2/3}$$

$$f'(x) = 2x^{-1/3} - \frac{5}{3} x^{2/3}$$

$$f'(x) = x^{-1/3} \left(2 - \frac{5}{3}x\right) = \frac{2 - \frac{5}{3}x}{\sqrt[3]{x}}$$

Titik kritis:

• Titik ujung selang:  $[-2, 2]$ • Titik stasioner (saat  $f'(x) = 0$ )

$$f'(x) = 0$$

$$x^{-1/3} \left(2 - \frac{5}{3}x\right) = 0$$

$$2 - \frac{5}{3}x = 0$$

$$\frac{5}{3}x = 2$$

$$x = \frac{6}{5}$$

• Titik singular (saat  $f'(x)$  tidak terdefinisi) $f'(x)$  tidak terdefinisi

$$x^{-1/3} \left(2 - \frac{5}{3}x\right) = 0$$

$$x^{-1/3} = 0$$

$$x = 0$$

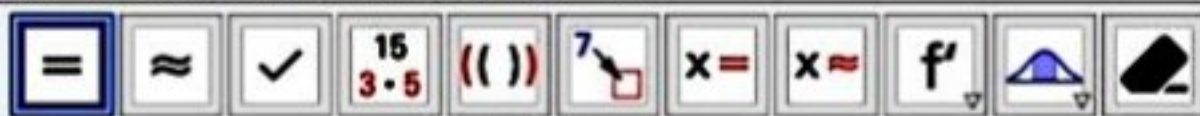
Mencari nilai maksimum, minimum:

$$f(-2) = (-2)^{2/3} (3 - (-2)) = 7,9301 \quad (\text{maksimum})$$

$$f(0) = (0)^{2/3} (3 - 0) = 0 \cdot 3 = 0 \quad (\text{minimum})$$

$$f(6/5) = (6/5)^{2/3} (3 - 6/5) = 2,03269$$

$$f(2) = (2)^{2/3} (3 - 2) = 1,5874$$



Algebra

Function

- $f(x) = \sqrt[3]{x^2}(3-x)$

Implicit Curve

- stasioner:  $-\text{cbrt}(x)^2 + 2/3 \text{cbrt}(x)$

Point

- A = (1.2, 2.03)
- B = (0, 0)
- C = (2, 1.59)
- D = (-2, 7.94)
- E = (0, 0)

CAS

Solve(stasioner)

- $x = \frac{6}{5}$

f(-2)

- $5\sqrt[3]{2^2}$

D := (-2.5cbrt(2^2))

- $D := (-2, 5\sqrt[3]{2^2})$

A := Extremum(f, -2, 2)

- $A := (1.2, 2.03)$

f(2)

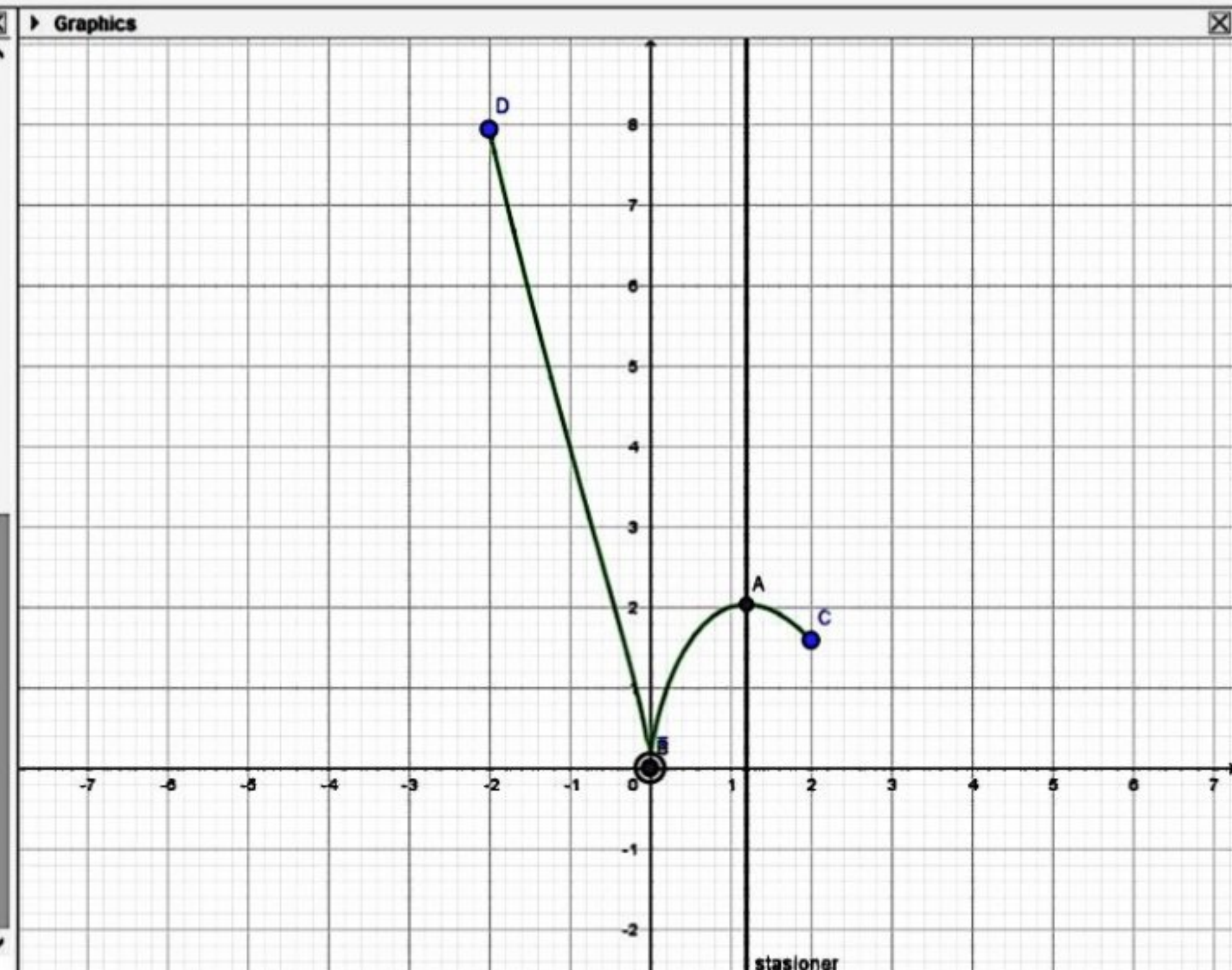
- $\sqrt[3]{2^2}$

C := (2, cbrt(2^2))

- $C := (2, \sqrt[3]{2^2})$

E := (0, 0)

- $E := (0, 0)$



$$b. f(x) = \sqrt{x} + \cos x, \quad [0, 2\pi]$$

$$f'(x) = \frac{1}{2} x^{-1/2} - \sin x = \frac{1}{2\sqrt{x}} - \sin x$$

Titik kritis:

• Titik ujung selang:  $[0, 2\pi]$

• Titik stasioner (saat  $f'(x) = 0$ )

$$f'(x) = 0$$

$$\frac{1}{2\sqrt{x}} - \sin x = 0$$

$$\frac{1}{2\sqrt{x}} = \sin x$$

$$\frac{1}{2} = \sin x \cdot \sqrt{x}$$

$$\sin \frac{\pi}{3} = \sin x \cdot \sqrt{x}$$

$$\sin \frac{\pi}{3} = \sin x \cdot x^{3/2}$$

GeoGebra Classic 5

File Edit View Options Tools Window Help

Algebra CAS Graphics

1  $f(x) := x^{1/2} + \cos(x), 0 \leq x \leq 2\pi$   
 $\rightarrow f(x) := \text{If}(0 \leq x \leq 2\pi, \sqrt{x} + \cos(x))$

2 Derivative(f)  
 $\rightarrow \text{If}(0 \leq x \leq 2\pi, \frac{1}{2}(\sqrt{x})^{-1} - \sin(x))$

3  $\text{stasioner} := \text{If}(0 \leq x \leq 2\pi, \frac{1}{2}(\sqrt{x})^{-1} - \sin(x)) = 0$   
 $\rightarrow \text{stasioner} : \text{If}(0 \leq x \leq 2\pi, \frac{1}{2}(\sqrt{x})^{-1} - \sin(x)) = 0$

4 Solve(stasioner)  
 $\rightarrow ?$

5  $C := \text{Extremum}(f, 0, 2\pi)$   
 $\rightarrow C := (0.66, 1.6)$

6

7

Algebra view:

- $C = (0.66, 1.6)$
- $f(x) = \sqrt{x} + \cos(x), (0 \leq x \leq 2\pi)$
- stasioner:  $\text{If}(0 \leq x \leq 2\pi, \frac{1}{2}(\sqrt{x})^{-1} - \sin(x)) = 0$
- stasioner<sub>1</sub>:  $\text{If}(0 \leq x \leq 2\pi, \frac{1}{2}(\sqrt{x})^{-1} - \sin(x)) = 0$
- $E = (2.84, 0.73)$

Graphics view:

stasioner<sub>1</sub> Activate Windows  
 Go to Settings to activate Windows.

Input:

$$a. y = \frac{x}{\sqrt{4-x^2}} = \frac{x}{(4-x^2)^{1/2}}$$

$$D_f = \{x \in \mathbb{R} \mid \frac{x}{4-x^2} \text{ terdefinisi}\}$$

$$D_f = \{x \in \mathbb{R} \mid 4-x^2 > 0\}$$

$$D_f = \{x \in \mathbb{R} \mid -2 < x < 2\}$$

$$y' = \frac{(1)(4-x^2)^{1/2} - \left(\frac{1}{2}(4-x^2)^{-1/2}(-2x)\right)x}{(4-x^2)^{1/2}^2}$$

$$= \frac{(4-x^2)^{1/2} + x^2(4-x^2)^{-1/2}}{4-x^2} = \frac{(4-x^2)^{1/2}((4-x^2) + x^2)}{4-x^2}$$

$$= \frac{4(4-x^2)^{-1/2}}{4-x^2} = \frac{4}{(4-x^2)\sqrt{4-x^2}}$$

• Titik kritis.

• Stasioner =  $f'(x) = 0$  (tidak ada)

Karena agar bernilai 0 maka pembilang harus 0 padahal pembilang sudah pasti 4.

• Singular:  $f'(x)$  tidak terdefinisi.

$$\frac{4}{(4-x^2)\sqrt{4-x^2}} \text{ tidak terdefinisi}$$

Tidak terdefinisi apabila penyebut 0.

$(4-x^2)\sqrt{4-x^2}$  bernilai 0, apabila:

$$\bullet 4-x^2 = 0$$

$$x^2 = 4$$

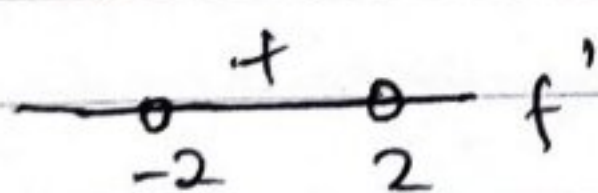
$$x = \pm 2$$

$$\bullet \sqrt{4-x^2} = 0$$

$$4-x^2 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$



$f$  naik di  $(-2, 2)$

tidak pernah turun.

Tidak terdeteksi nilai maksimum dan minimumnya.

• Kecekungan (turunan kedua  $f$ )

$$f'(x) = \frac{4}{(4-x^2)(\sqrt{4-x^2})} = \frac{4}{(4-x^2)^{3/2}}$$

$$f''(x) = \frac{0(4-x^2)(\sqrt{4-x^2}) - \frac{3}{2}(4-x^2)^{1/2}(-2x)(4)}{((4-x^2)^{3/2})^2}$$

$$f''(x) = \frac{-12x(4-x^2)^{1/2}}{(4-x^2)^3} = \frac{12x}{(4-x^2)^{5/2}}$$

• Titik kritis

• Stasioner: (saat  $f''(x) = 0$ )

$$f''(x) = 0$$
$$\frac{12x}{(4-x^2)^{5/2}} = 0$$

Akan bernilai 0 saat pembilang nol.

$$12x = 0$$
$$x = 0$$

• Singular: (saat  $f''(x)$  tidak terdefinisi)

$f''(x)$  tidak terdefinisi

$$\frac{12x}{(4-x^2)^{5/2}} \text{ tidak terdefinisi}$$

Akan tak terdefinisi apabila penyebut = 0

$$(4-x^2)^{5/2} = 0$$

$$4-x^2 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$



$f$  akan cekung ke bawah pada  $(-2, 0)$  atau  $(0, 2)$

Titik belok  $(0, f(0)) = (0, 0)$ .

$$f(0) = \frac{0}{\sqrt{4-0}} = \frac{0}{2} = 0$$

Mencari asimtot

Uji asimtot tegak

Perhatikan

$$\lim_{x \rightarrow -2} \frac{x}{\sqrt{4-x^2}}$$

$$\lim_{x \rightarrow -2} x = -2 < 0$$

$$\lim_{x \rightarrow -2} \sqrt{4-x^2} = 0$$

Jika  $x \rightarrow -2^-$  maka  $\sqrt{4-x^2} \rightarrow 0^-$

$$\lim_{x \rightarrow -2^-} \frac{x}{\sqrt{4-x^2}} = +\infty$$

$$\lim_{x \rightarrow -2^+} \frac{x}{\sqrt{4-x^2}} = -\infty$$

$\rightarrow$  jika  $x \rightarrow -2^+$  maka  $\sqrt{4-x^2} \rightarrow 0^+$

Uji asimtot miring Tidak punya, berdasarkan grafik.

$$\frac{x}{\sqrt{4-x^2}}$$

Uji asimtot datar

$$\text{Perhatikan} = \lim_{x \rightarrow \infty} \frac{x}{\sqrt{4-x^2}} = \text{tidak terdefinisi}$$

$$\lim_{x \rightarrow -\infty} \frac{x}{\sqrt{4-x^2}} = \text{tidak terdefinisi}$$

Jadi  $f(x) = \frac{x}{\sqrt{4-x^2}}$  tidak punya asimtot datar

2. (b)  $f(x) = 2\sqrt{x} + \frac{2}{\sqrt{x}} - 3$       (ambil)  $\rightarrow D_f = \{x \in \mathbb{R} \mid \text{ly terdefinisi}\}$   
 $= \{x \in \mathbb{R} \mid x > 0\}$

$$f'(x) = \frac{d}{dx} \left( 2\sqrt{x} + \frac{2}{\sqrt{x}} - 3 \right)$$

$$= \frac{d}{dx} 2\sqrt{x} + \frac{d}{dx} \left( \frac{2}{\sqrt{x}} \right) - \frac{d}{dx} 3$$

$$= 2 \times \frac{1}{2\sqrt{x}} - 2 \times \frac{1}{\sqrt{x}^2} - 0$$

$$f'(x) = \frac{x-1}{x\sqrt{x}}$$

Titik kritis

• Stasioner (saat  $f'(x) = 0$ )

$$f'(x) = 0$$

$$\frac{x-1}{x\sqrt{x}} = 0$$

$$x-1 = 0$$

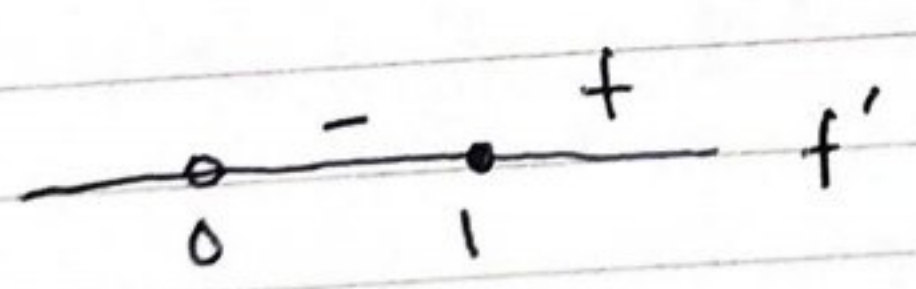
$$x = 1$$

• Singular (saat  $f'(x)$  tidak terdefinisi)

$$\frac{x-1}{x\sqrt{x}}$$

Tidak terdefinisi saat  $x=0$  tetapi  $0 \notin D_f$

Berarti tidak memiliki titik singular



f turun di  $(0, 1)$   
naik di  $(1, \infty)$

$$f(1) = 2\sqrt{1} + \frac{2}{\sqrt{1}} - 3$$

$$= 1$$



• Kecekungan (turunan kedua)

$$f'(x) = \frac{x-1}{x\sqrt{x}}$$

$$f''(x) = \frac{-x+3}{2x^2\sqrt{x}}$$

• Titik kritis

Stasioner saat  $f'(x) = 0$

$$\frac{-x+3}{2x^2\sqrt{x}} = 0$$

$$-x+3 = 0$$

$$x = 3$$

(Mempunyai titik stasioner)

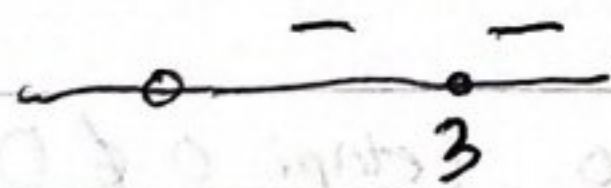
• Singular (saat  $f''(x)$  tidak terdefinisi)

$$\frac{-x+3}{2x^2\sqrt{x}} \text{ tidak terdefinisi}$$

$$\text{tdk terdefinisi saat } 2x^2\sqrt{x} = 0$$

$$x = 0, \text{ tetapi } x \notin Df$$

Jd tdk mempunyai titik singular



graf f selalu cekung ke bawah

b.

Uji asimtot tegak

Perhatikan :  $\lim_{x \rightarrow 0} 2\sqrt{x} + \frac{2}{\sqrt{x}} - 3 = \lim_{x \rightarrow 0} \frac{2x + 2 - 3\sqrt{x}}{\sqrt{x}}$

$$\lim_{x \rightarrow 0} 2x + 2 - 3\sqrt{x} = 2 > 0$$

$$\lim_{x \rightarrow 0} \sqrt{x} = 0$$

$$x \rightarrow 0$$

dika  $x \rightarrow 0^-$  maka  $\sqrt{x} \rightarrow 0^-$

$$\lim_{x \rightarrow 0^-} 2\sqrt{x} + \frac{2}{\sqrt{x}} - 3 = -\infty$$

$$\lim_{x \rightarrow 0^+} 2\sqrt{x} + \frac{2}{\sqrt{x}} - 3 = -\infty \rightarrow \text{dika } x \rightarrow 0^+ \text{ maka } \sqrt{x} \rightarrow 0^+$$

Uji asimtot miring = Tidak punya asimtot miring berdasarkan grafik

Uji asimtot datar

Perhatikan

$$\lim_{x \rightarrow \infty} 2\sqrt{x} + \frac{2}{\sqrt{x}} - 3 = +\infty$$

$$\lim_{x \rightarrow -\infty} 2\sqrt{x} + \frac{2}{\sqrt{x}} - 3 = -\infty$$

Tidak punya asimtot datar

GeoGebra

- Algebra
- Tools
- Table

- $f: y = \frac{x}{\sqrt{4-x^2}}$
- $g: y = 2\sqrt{x} + \frac{2}{\sqrt{x}} - 3$
- Input...

GeoGebra Graphing Calculator

