TUGAS DISKUSI KALKULUS PEUBAH BANYAK

TAHAP 13-15

Disusun oleh Kelompok 1

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PROGRAM STUDI PENDIDIKAN MATEMATIKA

FAKULTAS KEGURUAN DAN ILMU PENDIDIKAN

UNIVERSITAS SEBELAS MARET SURAKARTA

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1. Susunlah suatu integral lipat (tidak usah dihitung) untuk menentukan luas daerah yang terletak di dalam kardioid $r = 1 - \sin\theta$ dan di luar lingkaran $r = 3\sin\theta$

   Perhatikan

   $r = 1 - \sin\theta$

   Saat $\theta = 0 \rightarrow r = 1$

   Saat $\theta = \frac{\pi}{2} \rightarrow r = 0$

   Saat $\theta = \pi \rightarrow r = 1$

   Saat $\theta = \frac{3}{2}\pi \rightarrow r = 2$

   Kemudian perhatikan

   $r = 3\sin\theta$

   $r^2 = 3r\sin\theta$

   $x^2 + y^2 = 3y$

   $x^2 + \left(y - \frac{3}{2}\right)^2 = \frac{9}{4}$

   Maka lingkaran $r = 3\sin\theta$ memiliki jari-jari $\frac{3}{2}$ dan berpusat di $(0, \frac{3}{2})$

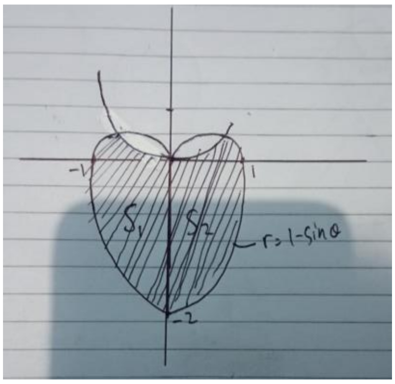
   Akan dicari perpotongan antara kardioid dan lingkaran

   $1 - \sin\theta \ = 3\sin\theta$

   $4\sin\theta = 1$

   $\sin\theta = \frac{1}{4}$

   $\theta = \sin^{-1}\frac{1}{4}$

   Dari Ilustrasi diperoleh

   $S_2 = \left\{(r,\theta)\middle|0 \leq r \leq 1 - \sin\theta\,;\ -\frac{\pi}{2} \leq \theta \leq \sin^{-1}\frac{1}{4}\right\}$ dan $\{(r,\theta)|0 \leq r \leq 3\sin\theta\,; 0 \leq \theta \leq \sin^{-1}\frac{1}{4}\}$

   Sehingga diperoleh luas daerahnya adalah

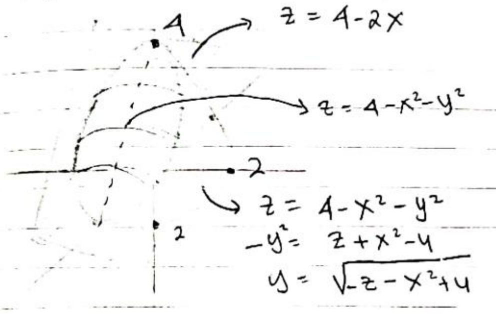
   $$\iint_S dA = \iint_{S1} dA_1 + \iint_{S2} dA_2$$

   Karena S1 dan S2 simetris sehingga

$$A_s = 2(\int_{-\frac{\pi}{2}}^{\sin^{-1}\frac{1}{4}} \int_0^1 r\,dr\,d\theta - \int_0^{\sin^{-1}\frac{1}{4}} \int_0^{3-\sin\theta} r\,dr\,d\theta$$

Diskusi Tahap 13-15

2) Susun integral lipat (tidak usah dihitung) untuk mencari volume dari benda pejal yang dibatasi oleh permukaan-permukaan yang gambarnya diberikan.



$z = 4 - x^2 - y^2$

$-y^2 = z + x^2 - 4$

$y = \sqrt{-z - x^2 + 4}$

Jawab :

- Mencari jejak bidang $z = 4 - 2x$
  1) Jejak di bidang yz ($x = 0$)
     $z = 4 - 2.0 \Rightarrow z = 4$
     → Berupa garis $z = 4$ //
  2) Jejak di bidang xz ($y = 0$)
     $z = 4 - 2x$ → berupa garis //
  3) Jejak di bidang xy ($z = 0$)
     $0 = 4 - 2x \Rightarrow x = 2$ (garis)

- Kemudian mencari jejak bidang $z = 4 - x^2 - y^2$
  1) Jejak di bidang yz ($x = 0$)
     $z = 4 - 0^2 - y^2$
     $z = 4 - y^2$ (berupa parabola)
  2) Jejak di bidang xz ($y = 0$)
     $z = 4 - x^2 - 0^2$
     $z = 4 - x^2$ (berupa parabola)
  3) Jejak di bidang xy ($z = 0$)
     $0 = 4 - x^2 - y^2$
     $x^2 + y^2 = 4$ (berupa lingkaran)

- Lalu cari batas daerah integrasi dan S
  $S = \{ (x, z) \mid 4 - 2x \le z \le 4 - x^2 ,\ 0 \le x \le 2 \}$
  Sehingga volumenya :

$$V = 2.\left(\int_0^2 \int_{4-2x}^{4-x^2} \sqrt{-z - x^2 + 4} \quad dz\,dx \right)$$

$$= \int_0^2 \sqrt{-(4 - x^2) - x^2 + 4} - \sqrt{-4 + 2x - x^2 + 4} \quad dx$$

$$= \int_0^2 \frac{2}{3} \sqrt{-4 + x^2 - x^2 + 4} - \sqrt{-\cancel{x}^2 + 2x} \quad dx$$

$$= \int_0^2 \frac{2}{3} \sqrt{((2 - x)(x))}^3 \ dx$$

$$= \frac{2}{3} \int_0^2 (2x (\sqrt{(2 - x)x} - x^2 \sqrt{(2x)x}) \ dx$$

$$= -\frac{2}{3} \int_0^2 x^2 \sqrt{2x - x^2}\ dx + \frac{4}{3} \int_0^2 x \sqrt{2x - x^2} \quad dx$$

$= \frac{2}{3}\int_0^2 \sqrt{-(x-1)^2+1}\;\; x^2\,dx + \frac{4}{3}\int_0^2 \sqrt{2x-x^2}\;\; x\,dx$

$u = x-1$

saat $x = 0$, $u = -1$ } maka batasnya berubah

saat $x = 2$, $u = 1$

$= \frac{2}{3}\int_{-1}^{1} (u+1)^2\sqrt{1-u^2}\,du + \frac{4}{3}\int_0^2 x\sqrt{2x-x^2}\,dx$

$\Rightarrow$ misal $u = \sin x$, $du = \cos dx$

$= \frac{2}{3}\int_{-\pi/2}^{\pi/2} (\sin x+1)^2(1-\sin^2 x))\,dx + \frac{4}{3}\int_0^2 x\sqrt{-x^2+2x}\,dx$

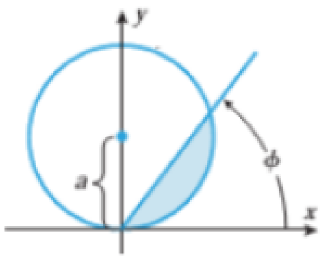
$= \left(-\frac{1}{6}\sin^3 x\cos x\right)\Big]_{-\pi/2}^{\pi/2} + \frac{1}{2}\int_{-\pi/2}^{\pi/2}\sin^2 x\,dx + \frac{4}{3}\int_{-\pi/2}^{\pi/2}\sin^3 x\,dx - \frac{2}{3}\times\int_{-\pi/2}^{\pi/2}\sin x\,dx$

$-\frac{2}{3}\times\int_{-\pi/2}^{\pi/2} dx + \frac{4}{3}\int_0^2 x\sqrt{-x^2+2x}\,dx$

$= -\frac{5\pi}{12} + \frac{4}{3}\int_{-\pi/2}^{\pi/2}\sin^3 x\,dx - \frac{4}{3}\int_{-\pi/2}^{\pi/2}\sin x\,dx + \frac{4}{3}\int_0^2 x\sqrt{-x^2+2x}\,dx$

$= \frac{\pi}{4} + \frac{4}{3}\int_{-\pi/2}^{\pi/2} \Rightarrow \frac{\pi}{4}$ //

3. Buktikan bahwa luas daerah yang diarsir pada gambar di bawah ini adalah $a^2\emptyset - \frac{1}{2}a^2 sin2\emptyset$



Jawab:

Daerah yang diarsir merupakan tembereng, maka $L_{tembereng} = L_{juring} - L_{segitiga}$

Luas segitiga yang diketahui sisi-sudut-sisi, yaitu:

$$L = \frac{1}{2}a^2 sin\emptyset$$

Dan luas juringnya

$$L = \frac{\emptyset}{360°} \times \pi \times a^2$$

Maka

$L_{tembereng} = L_{juring} - L_{segitiga}$

$L_{tembereng} = \frac{\emptyset}{360°} \times \pi \times a^2 - \frac{1}{2}a^2 sin\emptyset$

$L_{tembereng} = \frac{\emptyset}{2}a^2 - \frac{1}{2}a^2 sin\emptyset \mid \times 2$

$L_{tembereng} = \emptyset a^2 - \frac{1}{2}a^2 sin\emptyset$

$L_{tembereng} = a^2\emptyset \frac{1}{2} a^2 sin\emptyset$

Jadi, terbukti bahwa luas daerah yang diarsir pada gambar diatas adalah $a^2\emptyset - \frac{1}{2}a^2 sin2\emptyset$

(4.) Buat sketsa benda pejal yang dibatasi oleh silinder $x^2 + z^2 = 9$, bidang-bidang $x = 0$, $y = 0$, $z = 0$ dan $x + 2y = 2$, kemudian hitung volumenya.

JAWAB :

$x^2 + z^2 = 9$

Jejak di bidang xy $(z = 0)$

$x^2 - 9 = 0$

$(x+3)(x-3) = 0$

$x = -3 \lor x = 3$

Jejak di bidang yz $(x = 0)$

$z^2 - 9 = 0$

$(z-3)(z+3) = 0$

$z = 3 \lor z = -3$

Jejak di bidang xz $(y = 0)$

$x^2 + z^2 = 9$ (ling. $r = 3$, pusat $(0,0)$)

$x + 2y = 2$

Jejak di bidang xy $(z = 0)$

$x + 2y = 2$ (bidang)

Jejak di bidang yz $(x = 0)$

$2y = 2$

$y = 1$

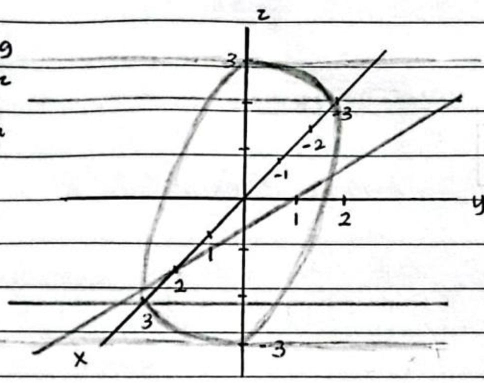
Jejak di bidang xz $(y = 0)$

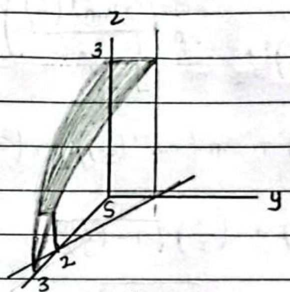
$x = 2$

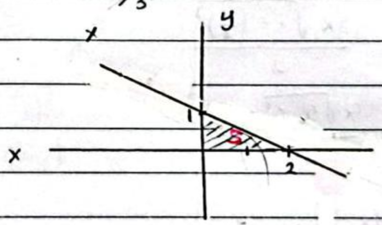
$x^2 + z^2 = 9$

$z^2 = 9 - x^2$

$z = \sqrt{9 - x^2}$







$V = \iint_S \sqrt{9 - x^2}\, dA$

Akan dicari volume benda pejal di bawah permukaan $z = \sqrt{9 - x^2}$ di atas S dengan $S = \{(x,y) \mid 0 \le x \le 2 \; ; \; 0 \le y \le \frac{1}{2}(2-x)\}$

$$V = \int_0^2 \int_0^{\frac{1}{2}(2-x)} \sqrt{9 - x^2}\, dy\, dx$$

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$$= \int_0^2 \left(1-\frac{x}{2}\right)\sqrt{9-x^2}\,dx$$

$$= \int_0^2 \sqrt{9-x^2}\,dx - \int_0^2 \frac{x}{2}\sqrt{9-x^2}\,dx$$

$$= \int_0^2 \sqrt{9-x^2}\,dx - \frac{1}{2}\int_0^2 x\sqrt{9-x^2}\,dx$$

✱ Mencari $\int_0^2 \sqrt{9-x^2}\,dx$

Permisalan

$x = 3\sin u \longrightarrow u = \sin^{-1}\left(\frac{x}{3}\right)$

$dx = 3\cos u\,du$

Saat $x = 0 \longrightarrow u = \sin^{-1}(0)$

$x = 2 \longrightarrow u = \sin^{-1}\left(\frac{2}{3}\right)$

$$\int_0^2 \sqrt{9-x^2}\,dx = \int_{\sin^{-1}0}^{\sin^{-1}\left(\frac{2}{3}\right)} \sqrt{9-9\sin^2 u}\;3\cos u\,du$$

$$= \int_{\sin^{-1}0}^{\sin^{-1}\left(\frac{2}{3}\right)} 3\cos u\;3\cos u\,du$$

$$= \int_{\sin^{-1}0}^{\sin^{-1}\left(\frac{2}{3}\right)} 9\cos^2 u\,du$$

$$= 9\int_{\sin^{-1}0}^{\sin^{-1}\left(\frac{2}{3}\right)} \frac{1+\cos 2u}{2}\,du$$

$$= \frac{9}{2}\left[\int_{\sin^{-1}0}^{\sin^{-1}\left(\frac{2}{3}\right)} 1+\cos 2u\,du\right] \qquad \sin^{-1} 0 = 0$$

$$= \frac{9}{2}\left[u + \frac{\sin 2u}{2}\right]_0^{\sin^{-1}\left(\frac{2}{3}\right)}$$

$$= \frac{9}{2}\left[\left(\sin^{-1}\left(\frac{2}{3}\right) + \frac{\sin\left(2\sin^{-1}\left(\frac{2}{3}\right)\right)}{2}\right)\right] \qquad \sqrt{1-\frac{4}{9}} = \sqrt{\frac{5}{9}}$$

$$= \frac{9}{2}\sin^{-1}\left(\frac{2}{3}\right) + \frac{9}{4}\sin\left(2\sin^{-1}\left(\frac{2}{3}\right)\right) \qquad = \frac{\sqrt{5}}{3}$$

$$= \frac{9}{2}\sin^{-1}\left(\frac{2}{3}\right) + \frac{9}{\cancel{4}_2}\cdot\cancel{2}\sin\left(\sin^{-1}\left(\frac{2}{3}\right)\right)\underbrace{\cos\left(\sin^{-1}\left(\frac{2}{3}\right)\right)}$$

$$= \frac{9}{2}\sin^{-1}\left(\frac{2}{3}\right) + \frac{\cancel{9}}{\cancel{2}}\cdot\frac{\cancel{2}}{\cancel{3}}\cdot\frac{\sqrt{5}}{\cancel{3}}$$

$$= \frac{9}{2}\sin^{-1}\left(\frac{2}{3}\right) + \sqrt{5}$$

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* Mencari $\frac{1}{2}\int_0^2 x\sqrt{9-x^2}\,dx$

Permisalan

$t = 9-x^2$

$dt = -2x\,dx$

$-\frac{1}{2}dt = x\,dx$

Saat $x=0 \to t=9$

$x=2 \to t=5$

$\frac{1}{2}\int_0^2 x\sqrt{9-x^2}\,dx = -\frac{1}{4}\int_9^5 \sqrt{t}\,dt$

$= \frac{1}{4}\left[\frac{2}{3}t^{3/2}\right]_5^9$

$= \frac{1}{6}\left[27-5\sqrt{5}\right]$

$= \frac{9}{2}-\frac{5}{6}\sqrt{5}$

Sehingga

$V = \int_0^2 \sqrt{9-x^2}\,dx - \frac{1}{2}\int_0^2 x\sqrt{9-x^2}\,dx$

$= \frac{9}{2}\sin^{-1}\left(\frac{2}{3}\right)+\sqrt{5}-\frac{9}{2}+\frac{5}{6}\sqrt{5}$

$= \frac{9}{2}\sin^{-1}\left(\frac{2}{3}\right)+\frac{11}{6}\sqrt{5}-\frac{9}{2}$ //

5. Dengan mengggunakan koordinat polare , tentukan volume benda pejal di atas bidang-vy yang dibatasi oleh permukaan

$2x^2+2y^2+z^2=18$ dan $x^2+y^2=4$

Jawab :

Volume benda pejal dimisalkan V

$V = \iint_S \sqrt{18-\ 2x^2+2y^2}\ dA$

$V = \iint_S \sqrt{18-\ 2(x^2+y^2)}dA$ dengan

$S = \{(x,y)|x^2 + y^2 \leq 4 = \{(r,\theta)|0 \leq r \leq 2; 0 \leq \theta \leq 2\pi\}$

$V = \int_0^{2\pi} \int_0^2 \sqrt{18 - 2r^2} \cdot rdrd\theta = \sqrt{2} \int_0^{2\pi} \int_0^2 \sqrt{9 - r^2} rdrd\theta$

Misal : $p = 9 - r^2 \rightarrow -2rdr = dp \rightarrow rdr = -\frac{dp}{2}$

$$r = 0 \rightarrow p = 9, r = 2 \rightarrow p = 5$$

Jadi $V = \sqrt{2} \int_0^{2\pi} \int_9^5 \sqrt{p}(-\frac{dp}{2})d\theta = \frac{\sqrt{2}}{2} \int_5^9 \sqrt{pdp}\, d\theta = \frac{\sqrt{2}}{2} \int_0^{2\pi} \frac{2}{3}\left(27 - 5\sqrt{5}\right)d\theta = \frac{\sqrt{2}}{2}\frac{2}{3}\left(27 - 5\sqrt{5}\right) \int_0^{2\pi} d\theta = \frac{\sqrt{2}}{2} \cdot \frac{2}{3}\left(27 - 5\sqrt{5}\right)2\pi = \frac{\sqrt{2}}{3}\pi(27 - 5\sqrt{5})\sqrt{2}$