TUGAS KALKULUS

MATA KULIAH KALKULUS PEUBAH BANYAK

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Pendidikan Matematika

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Nomor 1

1.) Susunlah suatu integral lipat (tidak usah dihitung) untuk menentukan luas daerah yang terletak di dalam kardioid $r = 1 - \sin\theta$ dan di luar lingkaran $r = 3\sin\theta$

Jawab

Akan disusun suatu integral lipat untuk menentukan luas daerah yang terletak di dalam kardioid $r = 1 - \sin\theta$ dan di luar lingkaran $r = 3\sin\theta$

Titik potong $r = 1 - \sin\theta$ dan $r = 3\sin\theta$

$$1 - \sin\theta = 3\sin\theta$$
$$4\sin\theta = 1$$
$$\sin\theta = \frac{1}{4}$$
$$\theta = \sin^{-1}\left(\frac{1}{4}\right)$$

·) $r = 1 - \sin\theta$ (kardioid)

θ	r
0	1
$\frac{\pi}{2}$	0
π	1
$3\frac{\pi}{2}$	2

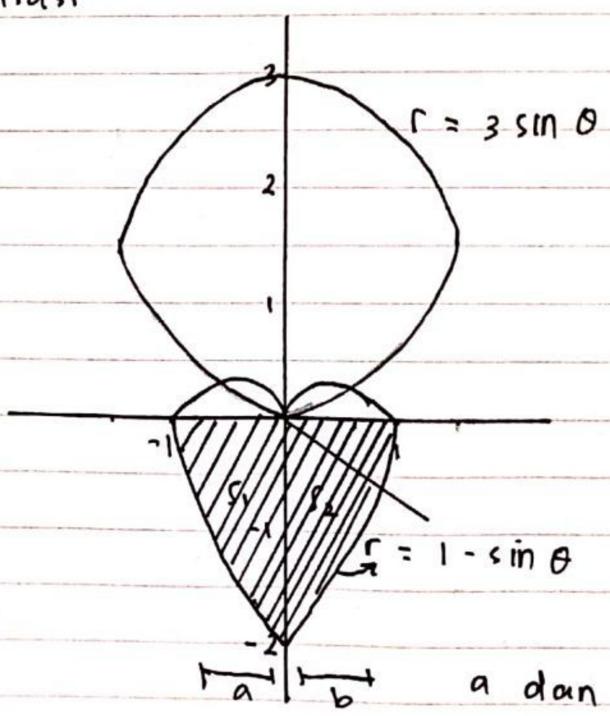
·) $r = 3\sin\theta$

$$r^2 = 3r\sin\theta$$
$$x^2 + y^2 = 3y$$
$$x^2 + \left(y - \frac{3}{2}\right)^2 = \frac{9}{4}$$

⊙ dengan pusat $\left(0, \frac{3}{2}\right)$ dan

jari-jari $r = \frac{3}{2}$

Ilustrasi



a dan b simetris

Dari ilustrasi di atas diperoleh,

$S_1 = \{(r,\theta) \mid -\frac{\pi}{2} \leq \theta \leq 0,\ 0 \leq r \leq 1 - \sin\theta\}$

$= \{(r,\theta) \mid 0 \leq \theta \leq \sin^{-1}\left(\frac{1}{4}\right),\ 3\sin\theta \leq r \leq 1 - \sin\theta\}$

Sehingga,

$$A(S_1) = \int_{-\frac{\pi}{2}}^{0} \int_{0}^{1-\sin\theta} r\, dr\, d\theta + \int_{0}^{\sin^{-1}\left(\frac{1}{4}\right)} \int_{3\sin\theta}^{1-\sin\theta} r\, dr\, d\theta$$

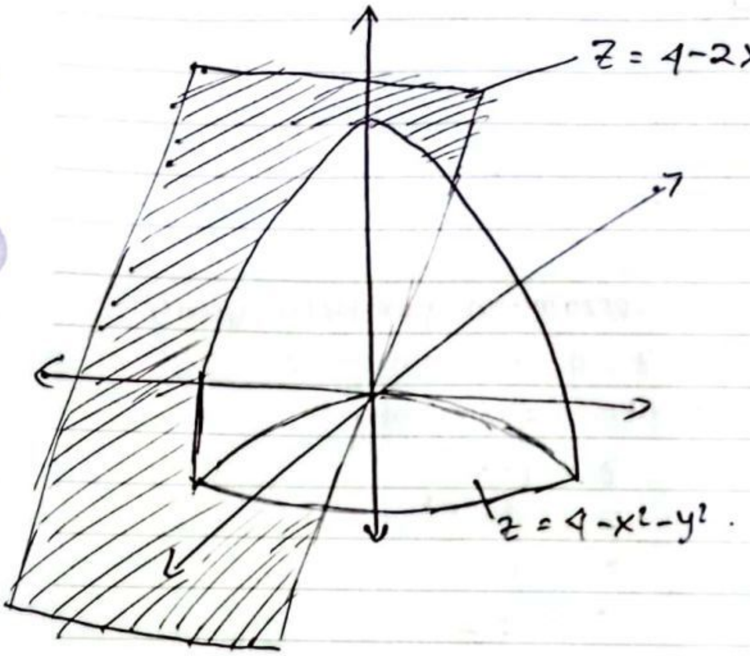
$$A(S) = 2\, A(S_1)$$

$$= 2\left(\int_{-\frac{\pi}{2}}^{0} \int_{0}^{1-\sin\theta} r\, dr\, d\theta + \int_{0}^{\sin^{-1}\left(\frac{1}{4}\right)} \int_{3\sin\theta}^{1-\sin\theta} r\, dr\, d\theta\right)$$

Nomor 2

DISKUSI 13 - 15

(2) Susun integral lipat (tidak usah dihitung) untuk mencari volume dari benda pejal yang dibatasi oleh permukaan - permukaan yang gambarnya diberikan.



Jawab :

•) $z = 4-2x$

Jejak dibidang xy ($z=0$)

$0 = 4-2x$

$2x = 4$

$x = 2$ (garis

Jejak dibidang xz ($y=0$)

$z = 4-2x$ (garis)

Jejak dibidang yz ($x=0$)

$z = 4-2(0)$

$z = 4$ (garis)

•) $z = 4-x^2-y^2$

Jejak dibidang xy ($z=0$)

$0 = 4-x^2-y^2$

$x^2+y^2 = 4$ (lingkaran)

pusat $(0,0)$, $r = \cancel{4}\ 2$

Jejak dibidang xz ($y=0$)

$z = 4-x^2-0^2$

$z = 4-x^2$ (Parabola)

Jejak dibidang yz ($x=0$)

$z = 4-0^2-y^2$.

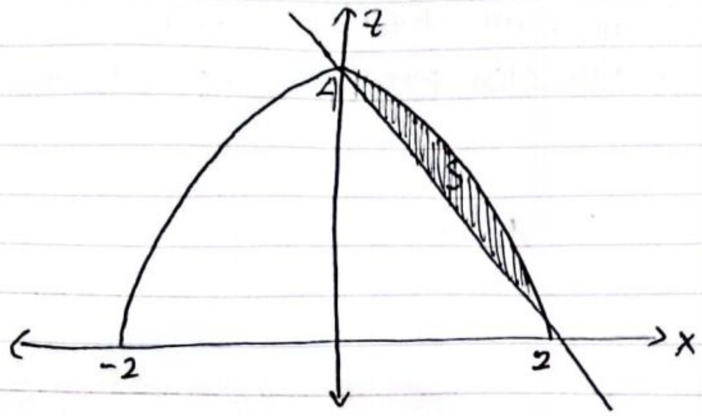
$z = 4-y^2$ (Parabola)

$z = 4-x^2-y^2$

$y = \sqrt{4-x^2-z}$

(2)

Daerah integrasi pada bidang-xz :



(-2,0) dan (2,0) diperoleh dari persamaan parabola yang memiliki nilai ketika $x=0$ dan $z=0$ dibidang -xz.

- ketika $x=0$, maka:

$$z = 4-x^2$$
$$z = 4$$

- ketika $z=0$, maka :

$$z = 4-x^2$$
$$0 = 4-x^2$$
$$x^2 = 4$$
$$x = \pm 2$$

Akan dicari,
Volume benda pejal yang dibatasi oleh permukaan $z = 4-x^2-y^2$ dan $z = 4-2x$ dapat dipandang sebagai 2 kalinya volume benda pejal dibawah permukaan $y = \sqrt{4-x^2-z^2}$ diatas S dengan

$$S = \{(x,z) \mid 4-2x \leq z \leq 4-x^2,\ 0 \leq x \leq 2\}.$$

Maka,

$$V = 2\left(\int_0^2 \int_{4-2x}^{4-x^2} \sqrt{4-x^2-z}\ dz\ dx\right)$$

$$= \int_0^2 \frac{2}{3}\left(-(x-2)x\right)^{3/2} dx$$

$$= \frac{2}{3} \int_0^2 \left(-(x-2)\,x\right)^{3/2} dx$$

$$= \frac{2}{3} \int_0^2 \left((2-x)x\right)^{3/2} dx$$

$$= \frac{2}{3} \int_0^2 \left(2x\sqrt{2x-x^2} - x^2\sqrt{2x-x^2}\right) dx$$

$$= -\frac{2}{3} \int_0^2 x^2\sqrt{2x-x^2}\ dx + \frac{4}{3}\int_0^2 x\sqrt{2x-x^2}\ dx$$

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(2)

$$= -\frac{2}{3}\int_0^2 \sqrt{1-(x-1)^2}\, x^2\, dx + \frac{4}{3}\int_0^2 x\sqrt{2x-x^2}\, dx$$

$u = x-1$; $\to u = 0-1 = -1$; $\to u = 2-1 = 1$; $\} \sqrt{1-(x-1)^2}\, x^2$

$du = dx$

$$= -\frac{2}{3}\int_{-1}^{1} (u+1)^2\sqrt{1-u^2}\, du + \frac{4}{3}\int_0^2 x\sqrt{2x-x^2}\, dx$$

$u = \sin(x)$ $\quad du = \cos x\, dx$.

$$= -\frac{2}{3}\int_{-\pi/2}^{\pi/2} (\sin(x)+1)^2 \cos(x)\sqrt{\cos^2(x)}\, dx +$$

$$\frac{4}{3}\int_0^2 x\sqrt{2x-x^2}\, dx$$

$$= -\frac{2}{3}\int_{-\pi/2}^{\pi/2} (\sin(x)+1)^2 (1-\sin^2(x))\, dx + \frac{4}{3}\int_0^2 x\sqrt{2x-x^2}\, dx$$

$$= \frac{2}{3}\int_{-\pi/2}^{\pi/2} \sin^4(x)\, dx + \frac{4}{3}\int_{-\pi/2}^{\pi/2} \sin^3(x)\, dx - \frac{4}{3}\int_{-\pi/2}^{\pi/2} \sin(x)\, dx$$

$$- \frac{2}{3}\times\int_{-\pi/2}^{\pi/2} 1\, dx + \frac{4}{3}\int_0^2 x\sqrt{2x-x^2}\, dx$$

$$= \left(-\frac{1}{6}\sin^3(x)\cos(x)\right)\Big|_{-\pi/2}^{\pi/2} + \frac{1}{2}\int_{-\pi/2}^{\pi/2} \sin^2(x)\, dx + \frac{4}{3}\int_{-\pi/2}^{\pi/2} \sin^3(x)\, dx$$

$$- \frac{4}{3}\int_{-\pi/2}^{\pi/2} \sin(x)\, dx - \frac{2}{3}\times\int_{-\pi/2}^{\pi/2} 1\, dx + \frac{4}{3}\int_0^2 x\sqrt{2x-x^2}\, dx$$

$$= -\frac{1}{8}\int_{-\pi}^{\pi} \cos p\, dp - \frac{5}{12}\times\int_{-\pi/2}^{\pi/2} 1\, dx + \frac{4}{3}\int_{-\pi/2}^{\pi/2} \sin^3(x)\, dx$$

$$- \frac{4}{3}\int_{-\pi/2}^{\pi/2} \sin(\cancel{x})\, dx + \frac{4}{3}\int_0^2 x\sqrt{2x-x^2}\, dx$$

$$= -\frac{5\pi}{12} + \frac{4}{3}\int_{-\pi/2}^{\pi/2} \sin^3(x)\, dx - \frac{4}{3}\int_{-\pi/2}^{\pi/2} \sin(x)\, dx + \frac{4}{3}\int_0^2 x\sqrt{2x-x^2}\, dx$$

$$= \frac{\pi}{4} + \frac{4}{3}\int_{-\pi/2}^{\pi/2} \sin v\, dv$$

$$= \frac{\pi}{4} + \left(-\frac{4\cos(v)}{3}\right)\Big|_{-\pi/2}^{\pi/2}$$

$$= \frac{\pi}{4} + 0 = \frac{\pi}{4} \approx 0{,}78540$$

(2)

$V = (0{,}78540) \times 2 = \underline{\underline{1{,}5708}}$

maka didapatkan volume benda pejal yaitu $\underline{\underline{1{,}5708}}$

Nomor 3

No

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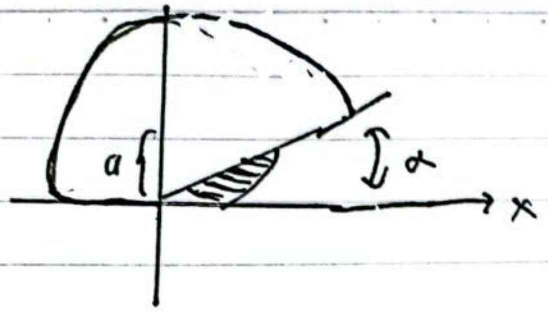
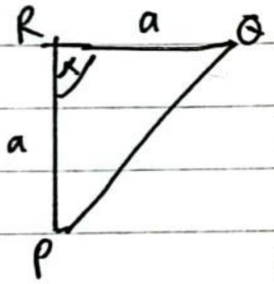


Bagian yang diarsir disebut tembereng

Ingat rumus Luas tembereng = Luas juring − Luas segitiga

Luas segitiga



$\sin \theta = \frac{PQ}{QR}$

$\sin \theta = \frac{PQ}{a}$

$PQ = a \sin \theta$

Luas segitiga $= \frac{1}{2} \cdot a \cdot t$

$= \frac{1}{2} PQ$

tinggi RO =

$\sin P = \frac{RO}{PR}$

$\sin P = \frac{RO}{a}$

$RO = \sin P \cdot a$

$RO = \sin \frac{1}{2}\theta \cdot a$

$\frac{\theta}{360} \cdot \pi \cdot a^2 - \frac{1}{2} a^2 \sin \theta$

$\frac{\theta}{2} \pi a^2 - \frac{1}{2} a^2 \sin \theta$

L segitiga $\frac{1}{2} a^2 \sin \theta$

L juring $\frac{\theta}{360} \pi a^2$

$\frac{\theta}{360} \pi a^2 - \frac{1}{2} a^2 \sin \theta \quad | \times 2$

$a^2 \theta - \frac{1}{2} a^2 \sin 2\theta$

Nomor 4

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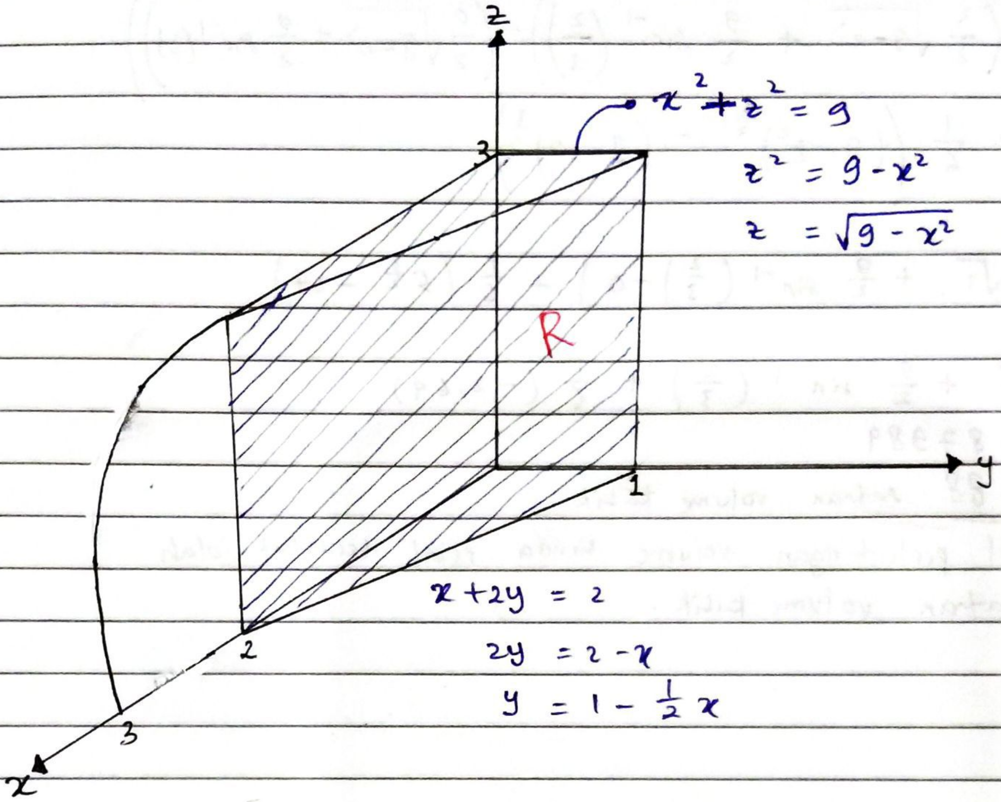
4. Sketsa benda pejal yang dibatasi oleh silinder $x^2 + z^2 = 9$, bidang-bidang $x = 0$, $y = 0$, $z = 0$, dan $x + 2y = 2$. Kemudian hitung volumenya!

Jawab :

1) Sketsa :



1) Menghitung volume

Berdasarkan sketsa tersebut, maka diperoleh daerah integrasi sebagai berikut.

$R = \{(x,y,z) \mid 0 \le x \le 2,\ 0 \le y \le 1 - \frac{1}{2}x,\ 0 \le z \le \sqrt{9 - x^2}\}$

Sehingga volume dapat dicari sebagai berikut:

$$V = \iiint_R dV$$

$$V = \int_{x=0}^{x=2} \int_{y=0}^{y=1-\frac{1}{2}x} \int_{z=0}^{z=\sqrt{9-x^2}} dz\, dy\, dx$$

$$= \int_0^2 \int_0^{1-\frac{1}{2}x} [z]^{\sqrt{9-x^2}} dy\, dx$$

$$= \int_0^2 \int_0^{1-\frac{1}{2}x} \left[\sqrt{9-x^2} - 0\right] dy\, dx$$

$$= \int_0^2 \int_0^{1-\frac{1}{2}x} \sqrt{9-x^2}\, dy\, dx$$

$$= \int_0^2 \sqrt{9-x^2}\left(1 - \frac{1}{2}x\right) dx$$

$$= \int_0^2 \left(\sqrt{9-x^2} - \frac{1}{2}x\sqrt{9-x^2}\right) dx$$

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$$V = \int_0^2 \sqrt{9-x^2}\,dx - \int_0^2 \tfrac{1}{2}x\sqrt{9-x^2}\,dx$$

$$= \left[\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)\right]_0^2$$

$$= \left(\left(\frac{2}{2}\sqrt{9-2^2} + \frac{9}{2}\sin^{-1}\left(\frac{2}{3}\right)\right) - \left(\frac{0}{2}\sqrt{9-0^2} + \frac{9}{2}\sin^{-1}(0)\right)\right)$$

$$+ \frac{1}{6}\left((9-2^2)^{\frac{3}{2}} - (9-0)^{\frac{3}{2}}\right)$$

$$= \left(\sqrt{\ } + \frac{9}{2}\sin^{-1}\left(\frac{2}{3}\right) - 0\right) + \frac{1}{6}\left(5^{\frac{3}{2}} - 27\right)$$

$$= \sqrt{5} + \frac{9}{2}\sin^{-1}\left(\frac{2}{3}\right) + \frac{1}{6}(-2{,}64)$$

$$= 2{,}87984$$

$\approx$ 2,88 satuan volume kubik

Jadi, hasil perhitungan volume benda pejal tersebut ialah 2,88 satuan volume kubik.

No 5. Nomor 5

Misal volume benda pejal tersebut adalah V maka

$V = \iint_S \sqrt{18 - 2(x^2 + y^2)}\, dA$ dengan

$S = \{(x,y) \mid x^2 + y^2 \leq 4\} = \{(r,\theta) \mid 0 \leq r \leq 2 ;\ 0 \leq \theta \leq 2\pi\}$

$V = \int_0^{2\pi} \int_0^2 \sqrt{18 - 2r^2} \cdot r\,dr\,d\theta = \sqrt{2} \int_0^{2\pi} \int_0^2 \sqrt{9 - r^2} \cdot r\,dr\,d\theta .$

Misal $9 - r^2 = p \rightarrow -2r\,dr = dp \rightarrow r\,dr = -\frac{dp}{2}$

saat $r = 0 \rightarrow p = 9$, saat $r = 2 \rightarrow p = 5$

Sehingga,

$V = \sqrt{2} \int_0^{2\pi} \int_9^5 \sqrt{p} \left(-\frac{dp}{2}\right) d\theta = \frac{\sqrt{2}}{2} \int_5^9 \sqrt{p}\,dp\, d\theta$

$= \frac{\sqrt{2}}{2} \int_0^{2\pi} \frac{2}{3}(27 - 5\sqrt{5})\, d\theta$

$= \frac{\sqrt{2}}{2} \cdot \frac{2}{3}(27 - 5\sqrt{5}) \int_0^{2\pi} d\theta = \frac{\sqrt{2}}{2} \cdot \frac{2}{3}(27 - 5\sqrt{5})\, 2\pi$

$= \frac{2}{3}\pi (27 - 5\sqrt{5})\sqrt{2} .$