DISKUSI 13-15

Kelompok 3

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1. Susunlah suatu integral lipat (tidak usah dihitung) untuk menentukan luas daerah yang terletak di dalam kardioid $r = 1 - \sin\theta$ dan di luar lingkaran $r = 3\sin\theta$

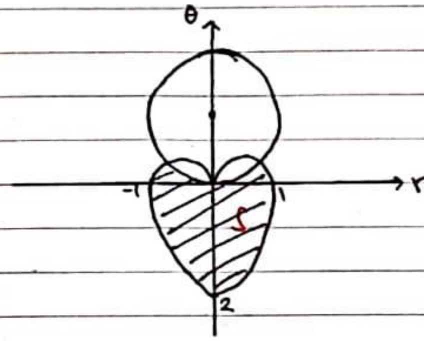
Jawab:

Diketahui, $r = 1 - \sin\theta$ (kardioid)

$r = 3\sin\theta$ (lingkaran)

·) $r = 1 - \sin\theta$

θ	r
0	1
$\frac{\pi}{2}$	0
π	1
$\frac{3\pi}{2}$	2



·) $r = 3\sin\theta$

$r^2 = 3r\sin\theta$

$x^2 + y^2 = 3y$

$x^2 + \left(y - \frac{3}{2}\right)^2 = \frac{9}{4}$

O pada pusat $(0, \frac{3}{2})$; $r = \frac{3}{2}$

·) mencari titik potong $r = 1 - \sin\theta$ dan $r = 3\sin\theta$

$1 - \sin\theta = 3\sin\theta$

$4\sin\theta = 1$

$\sin\theta = \frac{1}{4}$

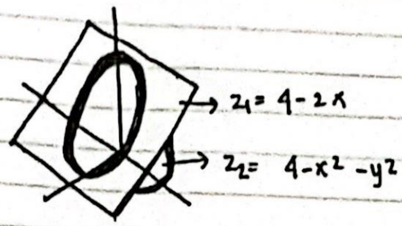
$\theta = \sin^{-1}\left(\frac{1}{4}\right)$

·) $A(S_1) = \int_{-\frac{\pi}{2}}^{0} \int_{0}^{1-\sin\theta} r \, dr \, d\theta + \int_{0}^{\sin^{-1}(\frac{1}{4})} \int_{3\sin\theta}^{1-\sin\theta} r \, dr \, d\theta$

$A(S) = 2A(S_1)$

$S_1 = \{(r,\theta) \mid -\frac{\pi}{2} \le \theta \le 0, 0 \le r \le 1 - \sin\theta\} \cup \{(r,\theta) \mid 0 \le \theta \le \sin^{-1}(\frac{1}{4}), 3\sin\theta \le r \le 1 - \sin\theta\}$

2. Susun integral lipat (tdk dihitung) untuk mencari volume benda pejal yang dibatasi : permukaan



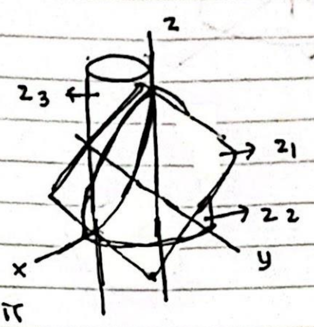
Jawab:

$z = 4 - x^2 - y^2$

$z = 4 - 2x$

$\Rightarrow \quad 4-2x = 4-x^2-y^2$

$\quad x^2+y^2-2x = 0 \ \ldots z_3$



Menggunakan koordinat silindris

- $z = 4-x^2-y^2$ (paraboloida)
  $z = 4 - r^2$ //
- $x^2+y^2-2x = 0$ (tabung)
  $r = 2\cos\theta$ //

Variabel z berjalan dari bidang xy ke atas paraboloida, yaitu mulai dari 0 ke $4-r^2$. nilai θ tetap. r berjalan mulai dari 0 ke $\cos\theta$ dan θ berjalan dari 0 ke $\frac{\pi}{2}$. Jadi

$$V = \iiint_S dv$$

$$= \int_0^{\frac{\pi}{2}} \int_0^{\cos\theta} \int_0^{4-r^2} r \ dz \, dr \, d\theta$$

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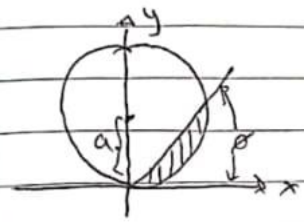
3. Buktikan bahwa luar daerah yang diarsir di bawah ini adalah $a^2\phi - \frac{1}{2}a^2 \sin 2\phi$



yang diarsir adalah temberêng

rumus luar temberêng : luar juring - luar segitiga

luar segitiga jika diketahui sisi -sudut - sisi maka,

$L = \frac{1}{2} a^2 \sin \theta$

luar juringnya adalah

$L = \frac{\theta}{360^\circ} \cdot \pi \cdot a^2$

~~Rumus~~ luar temberêng : luar juring - luar segitiga

$\frac{\theta}{360} \cdot \pi \cdot a^2 - \frac{1}{2} a^2 \sin \theta$

$\frac{\theta}{2} \cdot a^2 - \frac{1}{2} a^2 \sin \theta \quad \Big| \times 2$

$a^2 \theta - \frac{1}{2} a^2 \sin 2\theta$

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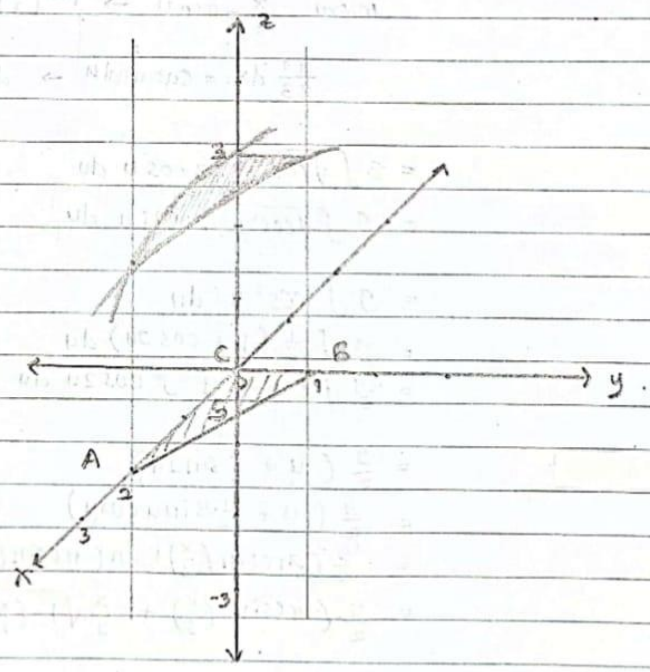
(4) Buat sketsa benda pejal yang dibatasi oleh silinder $x^2 + z^2 = 9$, bidang-bidang $x = 0$, $y = 0$, $z = 0$ dan $x + 2y = 2$, kmd hit. volumenya.

Jawab:

- $x^2 + z^2 = 9$
  - Jejak dibidang xy $(z = 0) \to x = \pm 3$
  - Jejak dibidang xz $(y = 0) \to x^2 + z^2 = 9$
  - Jejak dibidang yz $(x = 0) \to z = \pm 3$
- $x + 2y = 2$
  - Jejak dibidang xy $(z = 0) \to x + 2y = 2 \Leftrightarrow x = 2 - 2y$
  - Jejak di bidang xz $(y = 0) \to x = 2$
  - Jejak di bidang yz $(x = 0) \to y = 1$
- Sketsa :



- Titik potong $x + 2y = 2$ dan $y = 0$ adalah $A(2, 0)$

  Titik potong $x + 2y = 2$ dan $x = 0$ adalah $B(0, 1)$
- $S = \{(x, y) \mid 0 \le x \le 2 \; ; \; 0 \le y \le \frac{2-x}{2}\}$ $\to$ himp. y sederhana

  Akan dicari vol. benda pejal di bawah permukaan $x^2 + z^2 = 9$, di atas S

$$V = \iint_S \sqrt{9 - x^2} \; dA$$

$$= \int_0^2 \int_0^{\frac{2-x}{2}} \sqrt{9 - x^2} \; dy \, dx$$

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$$= \int_0^2 \left(\frac{2-x}{2}\right)\sqrt{9-x^2}\,dx$$

$$= \int_0^2 \left(1-\frac{x}{2}\right)\sqrt{9-x^2}\,dx$$

$$= \int_0^2 \left(\sqrt{9-x^2}\right)dx - \int_0^2 \frac{x}{2}\sqrt{9-x^2}\,dx$$

$$= \int_0^2 \sqrt{9-x^2}\,dx - \frac{1}{2}\int_0^2 x\sqrt{9-x^2}\,dx$$

(*) $\int \sqrt{9-x^2}\,dx = \int \sqrt{9\left(1-\frac{x^2}{9}\right)}\,dx$

$$= \int 3\sqrt{1-\left(\frac{x}{3}\right)^2}\,dx$$

misal $\frac{x}{3} = \sin u \rightarrow 1-\left(\frac{x}{3}\right)^2 \Leftrightarrow 1-\sin^2 u = \cos^2 u$

$\frac{1}{3}\,dx = \cos u\,du \rightarrow dx = 3\cos u\,du$

$$= 3\int \sqrt{\cos^2 u}\,.\,3\cos u\,du$$

$$= 9\int \cos u\,.\,\cos u\,du$$

$$= 9\int \cos^2 u\,du$$

$$= 9\int \frac{1}{2}(1+\cos 2u)\,du$$

$$= \frac{9}{2}\int 1\,du + \int \cos 2u\,du$$

$$= \frac{9}{2}\left(u + \frac{1}{2}\sin 2u\right)$$

$$= \frac{9}{2}\left(u + \frac{1}{2}2\sin u\cos u\right)$$

$$= \frac{9}{2}\left(\arcsin\left(\frac{x}{3}\right) + \sin\left(\arcsin\left(\frac{x}{3}\right)\right).\cos\left(\arcsin\left(\frac{x}{3}\right)\right)\right)$$

$$= \frac{9}{2}\left(\arcsin\left(\frac{x}{3}\right) + \frac{x}{3}\sqrt{1-\left(\frac{x}{3}\right)^2}\right)$$

$$= \left(\frac{9}{2}\left(\arcsin\left(\frac{x}{3}\right)\right)\right) + \left(\frac{9}{2}\cdot\frac{x}{3}\sqrt{1-\left(\frac{x}{3}\right)^2}\right)$$

$$= \frac{9}{2}\left(\arcsin\left(\frac{x}{3}\right)\right) + \frac{3x}{2}\sqrt{1-\left(\frac{x}{3}\right)^2}$$

$$= \frac{9}{2}\left(\arcsin\left(\frac{x}{3}\right)\right) + \frac{x}{2}\sqrt{9-x^2} + C$$

shg

$$\int_0^2 \sqrt{9-x^2}\,dx = \frac{9}{2}\left(\arcsin\left(\frac{x}{3}\right) + \frac{x}{2}\sqrt{9-x^2}\right)\Big|_0^2$$

$$= \frac{9}{2}\left(\arcsin\left(\frac{2}{3}\right) + \sqrt{5}\right)$$

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(*) $\int x\sqrt{9-x^2}\ dx$

misal $v = 9-x^2$

$dv = -2x\ dx$

$\frac{dv}{-2x} = dx$

shg $\int x\sqrt{9-x^2}\ dx = \int -\frac{1}{2}\sqrt{v}\ dv$

$= -\frac{1}{2}\int v^{1/2}\, dv$

$= -\frac{1}{2}\left(\frac{2}{3} v^{3/2}\right) + C$

$= -\frac{1}{3} v^{3/2} + C$

$= -\frac{1}{3}\sqrt{(9-x^2)^3} + C$

maka $-\frac{1}{2}\int_0^2 x\sqrt{9-x^2}\ dx = -\frac{1}{2}\cdot -\frac{1}{3}\left(\sqrt{(9-x^2)^3}\right)\Big|_0^2$

$= \frac{1}{6}(5\sqrt{5} - 27)$

$= \frac{9}{2}\left(\arcsin\left(\frac{2}{3}\right) + \sqrt{5}\right) + \frac{1}{6}(5\sqrt{5}-27)$

$= \frac{9}{2}\arcsin\left(\frac{2}{3}\right) + \frac{9\sqrt{5}}{2} + \frac{5\sqrt{5}}{6} - \frac{27}{6}$

$= \frac{9}{2}(0{,}72972) + \frac{27\sqrt{5}+5\sqrt{5}}{6} - \frac{27}{6}$

$= \frac{6{,}56748}{2} - \frac{27}{6} + \frac{16\sqrt{5}}{3}$

$= \frac{19{,}70244 - 27 + 71{,}55416}{6}$

$= \frac{64{,}2566}{6}$

$= 10{,}70943$

∴ Jadi, volume benda pejal yg dimaksud adalah 10,70943

5. Dengan menggunakan koordinat polar, tentukan volume benda pejal di atas bidang xy yang dibatasi oleh permukaan $2x^2+2y^2+z^2=18$ dan $x^2+y^2=4$

↳ Jawab :

1) Permukaan $2x^2+2y^2+z^2=18$

∴ Jejak di bidang xy $(z=0)$

$2x^2+2y^2=18$

$x^2+y^2=9$

(lingkaran dengan $P(0,0)$ dan $r=3$)

∴ Jejak di bidang xz $(y=0)$

$2x^2+z^2=18$

$\frac{x^2}{9}+\frac{z^2}{18}=1$

(elips)

∴ Jejak di bidang yz $(x=0)$

$2y^2+z^2=18$

$\frac{y^2}{9}+\frac{z^2}{18}=1$

(elips)

2) Permukaan $x^2+y^2=4$

∴ Jejak di bidang xy $(z=0)$

$x^2+y^2=4$

(lingkaran dengan $P(0,0)$ dan $r=2$)

∴ Jejak di bidang xz $(y=0)$

$x^2=4$

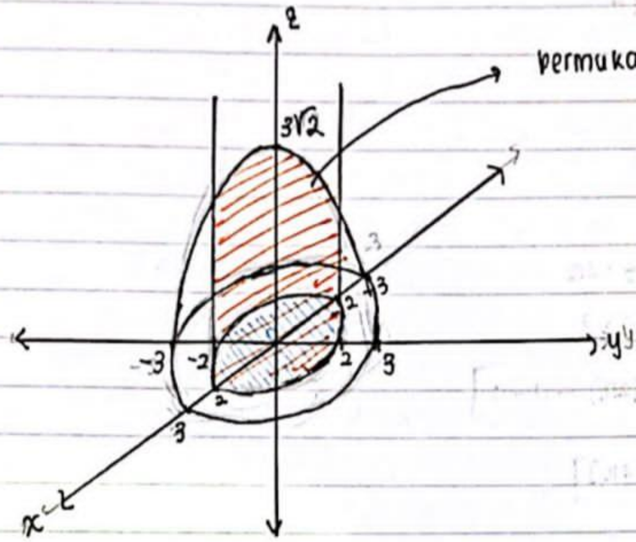
$(x+2)(x-2)=0$ (garis $x=2$ dan $x=-2$)

∴ Jejak di bidang yz $(x=0)$

$y^2=4$

$(y+2)(y-2)=0$ (garis $y=2$ dan $y=-2$)

sketsa :



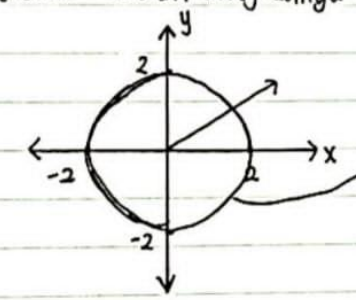
permukaan $2x^2+2y^2+z^2=18$

$$z^2=-2(x^2+y^2)+18$$

$$z^2=-2r^2+18$$

$$z=\sqrt{18-2r^2}$$

Diperoleh daerah integrasinya adalah



lingkaran dengan persamaan $x^2+y^2=4$

$$r^2=4$$

$$r=2 \vee r=-2$$

(TM)

$S=\{(r,\theta)\mid 0\le r\le 2\ ;\ 0\le\theta\le 2\pi\}$

Maka didapatkan

$$V=\int_0^{2\pi}\int_0^2 \sqrt{18-2r^2}\ r\ dr\ d\theta$$

misal $u=18-2r^2$ $\rightarrow$ saat $r=0\rightarrow u=18$

$du=-4r\ dr$ $\qquad r=2\rightarrow u=10$

$-\frac{du}{4}=r\,dr$

$$V=\int_0^{2\pi}\int_{18}^{10} u^{\frac{1}{2}}\cdot -\frac{du}{4}\ d\theta$$

$$=\int_0^{2\pi}\int_{10}^{18} \frac{1}{4}u^{\frac{1}{2}}\,du\ d\theta$$

$$=\int_0^{2\pi} \frac{1}{6}u^{\frac{3}{2}}\Big]_{u=10}^{u=18}\ d\theta$$

$$= \int_0^{2\pi} \frac{1}{6} 18\sqrt{18} - \frac{1}{6} \cdot 10\sqrt{10}\, d\theta$$

$$= \int_0^{2\pi} \frac{1}{6} (54\sqrt{2} - 10\sqrt{10})\, d\theta$$

$$= \frac{1}{6} \int_0^{2\pi} 54\sqrt{2} - 10\sqrt{10}\, d\theta$$

$$= \frac{1}{6} \left[54\sqrt{2}\,\theta - 10\sqrt{10}\,\theta \right]_{\theta=0}^{\theta=2\pi}$$

$$= \frac{1}{6} \left[54\sqrt{2}\,(2\pi) - 10\sqrt{10} \cdot (2\pi) - 0 - 0 \right]$$

$$= \frac{1}{6} \left[54\sqrt{2}\,(2\pi) - 10\sqrt{10}\,(2\pi) \right]$$

$$= 9\sqrt{2}\,(2\pi) - \frac{5\sqrt{10}}{3}\,(2\pi)$$

$$= \pi \left(18\sqrt{2} - \frac{10\sqrt{10}}{3} \right)$$

$$\approx 46{,}857.$$