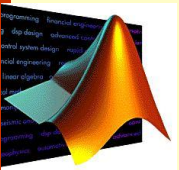
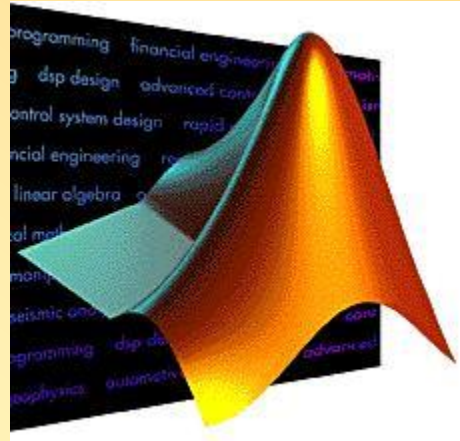


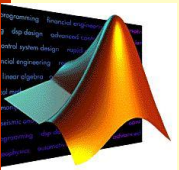
Regresi dan Interpolasi

05 Curve Fitting

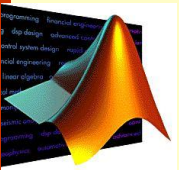


Introduction

- Data are often given for discrete values along a continuum. However, we may require estimates at points between the discrete values.
- This Chapter describes techniques to fit curves to such data to obtain intermediate estimates.

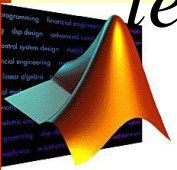


- There are two general approaches for curve fitting that are distinguished from each other on the basis of the amount of error associated with the data.
 - Regression
 - Interpolation



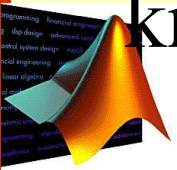
Regression

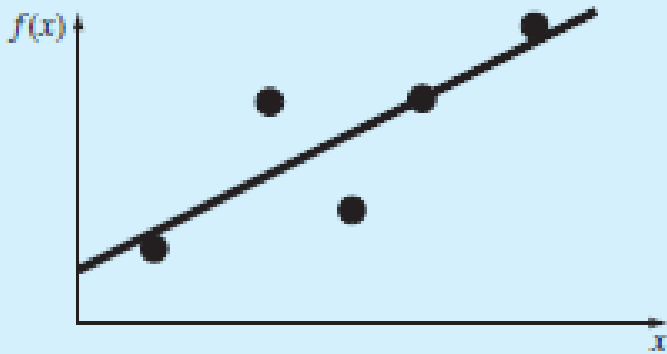
- The data exhibit a significant degree of error, the strategy is to derive a single curve that represents the general trend of the data.
- Because any individual data point may be incorrect, we make no effort to intersect every point. Rather, the curve is designed to follow the pattern of the points taken as a group. One approach of this nature is called *least-squares regression*



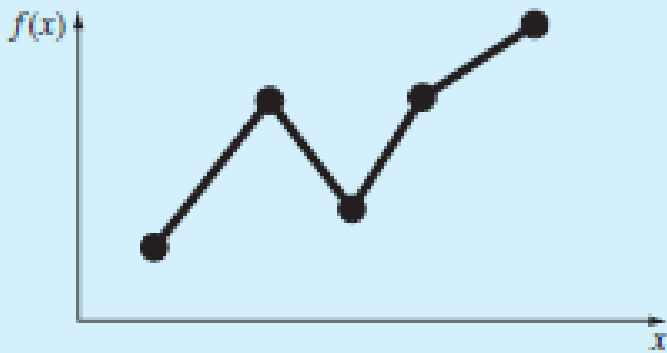
Interpolation

- The data are known to be very precise, the basic approach is to fit a curve or a series of curves that pass directly through each of the points.
- Such data usually originate from tables. Examples are values for the density of water or for the heat capacity of gases as a function of temperature.
- The estimation of values between well-known discrete points is called *interpolation*

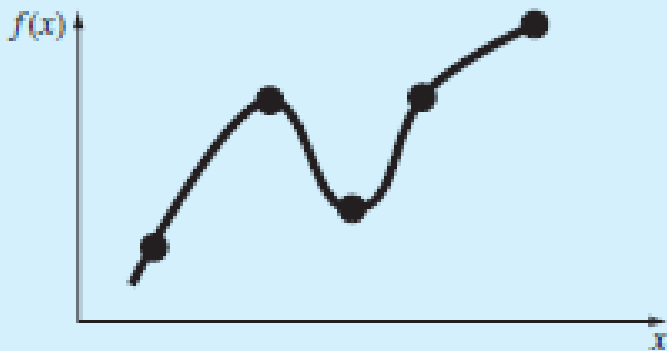




(a)

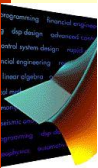


(b)



(c)

Three attempts to fit a “best” curve through five data points:
(a) least-squares regression,
(b) linear interpolation,
(c) curvilinear interpolation.



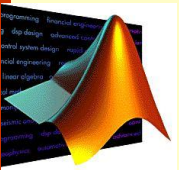
5.1. Regression

a. Linear Regression

- The primary objective of this method is to how least-squares regression can be used to fit a straight line to measured data.

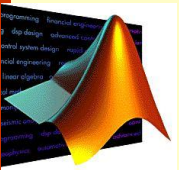
The mathematical expression for the straight line is,

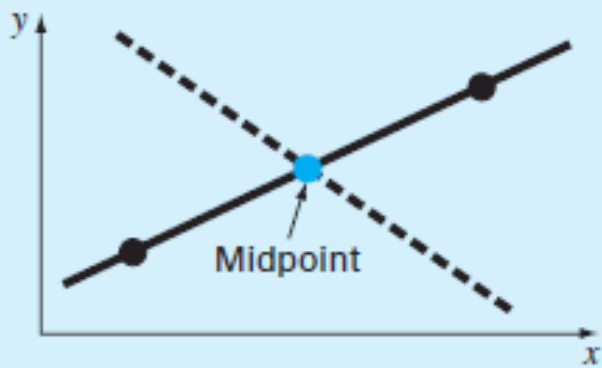
$$y = a_0 + a_1x$$



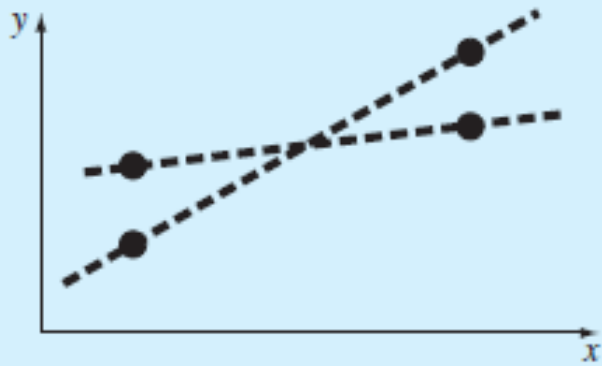
- One strategy for fitting a “best” line through the data would be to minimize the sum of the residual errors for all the available data, as in

$$\sum_{i=1}^n e_i = \sum_{i=1}^n (y_i - a_0 - a_1 x_i)$$

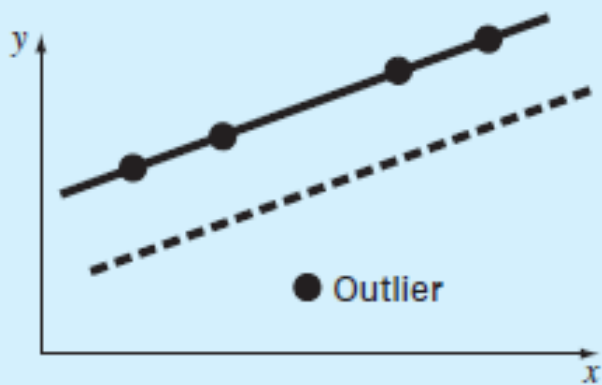




(a)



(b)



(c)

Examples of some criteria for “best fit” that are inadequate for regression:

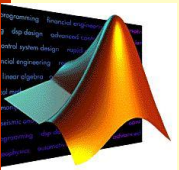
(a) minimizes the sum of the residuals,

(b) minimizes the sum of the absolute values of the residuals, and

(c) minimizes the maximum error of any individual point.

- A strategy that overcomes the shortcomings of the aforementioned approaches is to minimize the sum of the squares of the residuals:

$$S_r = \sum_{i=1}^n e_i^2 = \sum_{i=1}^n (y_i - a_0 - a_1 x_i)^2$$



- To determine values for a_0 and a_1 , Eq. above is differentiated with respect to each unknown coefficient:

$$\frac{\partial S_r}{\partial a_0} = -2 \sum (y_i - a_0 - a_1 x_i)$$

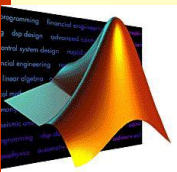
$$\frac{\partial S_r}{\partial a_1} = -2 \sum [(y_i - a_0 - a_1 x_i) x_i]$$

$$0 = \sum y_i - \sum a_0 - \sum a_1 x_i$$

$$0 = \sum x_i y_i - \sum a_0 x_i - \sum a_1 x_i^2$$

$$n a_0 + \left(\sum x_i \right) a_1 = \sum y_i$$

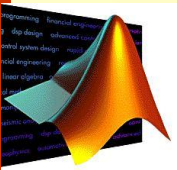
$$\left(\sum x_i \right) a_0 + \left(\sum x_i^2 \right) a_1 = \sum x_i y_i$$



- These are called the *normal equations*. They can be solved simultaneously for

$$a_1 = \frac{n \sum x_i y_i - \sum x_i \sum y_i}{n \sum x_i^2 - (\sum x_i)^2}$$

$$a_0 = \bar{y} - a_1 \bar{x}$$



Koefisien Determinasi (R-square)

$$R^2 = 1 - \frac{SS\ Error}{SS\ Total} = 1 - \frac{\sum(y_i - \hat{y}_i)^2}{\sum(y_i - \bar{y})^2}$$

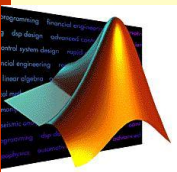
Keterangan :

y_i = observasi respon ke - i

\bar{y} = rata - rata

\hat{y}_i = ramalan respon ke - i

$0 < R^2 < 1$; makin mendekati 1 berarti model persamaan makin baik

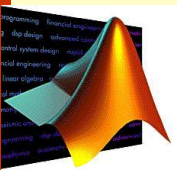


Problem

- Fit a straight line to the values in Table 14.1.

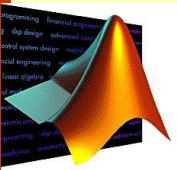
TABLE 14.1 Experimental data for force (N) and velocity (m/s) from a wind tunnel experiment.

$v, \text{ m/s}$	10	20	30	40	50	60	70	80
$F, \text{ N}$	25	70	380	550	610	1220	830	1450

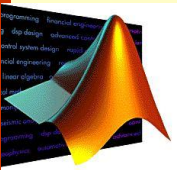


LINEARIZATION OF NONLINEAR RELATIONSHIPS

- Linear regression provides a powerful technique for fitting a best line to data. However, it is predicated on the fact that the relationship between the dependent and independent variables is linear.
- This is not always the case, and the first step in any regression analysis should be to plot and visually inspect the data to ascertain whether a linear model applies.



- In some cases, techniques such as polynomial regression, which is described in next sub Chap., are appropriate. For others, transformations can be used to express the data in a form that is compatible with linear regression.



Contoh linierisasi hubungan yang tidak linier

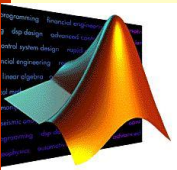
1. $y = a x^b$

2. $y = a e^{bx}$

3. $y = a + \frac{b}{x}$

4. $y = \frac{x}{a + bx}$

5. $y = \frac{ax}{b + x}$



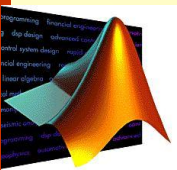
b. Regresi Polinomial

Jika pada regresi linier, data-data didekati dengan persamaan garis $y = a_1x + a_0$, maka pada regresi polinomial, data-data didekati dengan persamaan polinomial, yaitu :

$$y = a_0 + a_1x + a_2x^2 + \dots + a_mx^m$$

sehingga

$$SSE = \sum_{n=1}^n (y_{\text{data}} - y_{\text{pers}})^2$$



05 Curve Fitting

$$a_0n + a_1\sum x_i + a_2\sum x_i^2 + \dots + a_m\sum x_i^m = \sum y_i$$

$$a_0\sum x_i + a_1\sum x_i^2 + a_2\sum x_i^3 + \dots + a_m\sum x_i^{m+1} = \sum x_i y_i$$

$$a_0\sum x_i^2 + a_1\sum x_i^3 + a_2\sum x_i^4 + \dots + a_m\sum x_i^{m+2} = \sum x_i^2 y_i$$

.

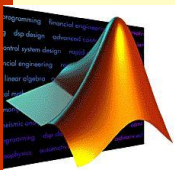
.

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.

$$a_0\sum x_i^m + a_1\sum x_i^{m+1} + a_2\sum x_i^{m+2} + \dots + a_m\sum x_i^{2m} = \sum x_i^m y_i$$

$$\begin{bmatrix} n & \sum x_i & \sum x_i^2 & \dots & \sum x_i^m \\ \sum x_i & \sum x_i^2 & \sum x_i^3 & \dots & \sum x_i^{m+1} \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 & \dots & \sum x_i^{m+2} \\ \dots & \dots & \dots & \dots & \dots \\ \sum x_i^m & \sum x_i^{m+1} & \sum x_i^{m+2} & \dots & \sum x_i^{2m} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \dots \\ a_m \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum x_i y_i \\ \sum x_i^2 y_i \\ \dots \\ \sum x_i^m y_i \end{bmatrix}$$



Regresi dg Matlab

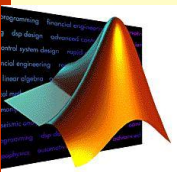
Fungsi Polyfit

Dalam Matlab, fungsi `polyfit` menyelesaikan masalah pencocokan kurva untuk kuadrat terkecil.

Penggunaan fungsi `polyfit` akan menghasilkan suatu persamaan polinomial yang paling mendekati data.

Jika derajat fungsi `polyfit` dipilih $n = 1$, maka akan dihasilkan persamaan garis lurus yaitu regresi linier.

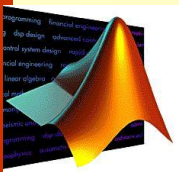
```
kons = polyfit(x, y, 1)
```



Soal 1

Bennett & Meyers memberikan data-data konduktivitas termal aseton. Hubungan $\ln(k)$ dan $\ln(T)$ mendekati persamaan garis lurus, dengan T dalam R ($R = ^\circ F + 460$). Tentukan nilai k pada $300 ^\circ F$!

k [BTU/jam.ft. $^\circ F$]	T ($^\circ F$)
0,0057	32
0,0074	115
0,0099	212
0,0147	363



Soal 2

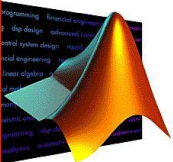
Tekanan uap toluena pada persamaan Antoine

Data tekanan uap toluena dalam $^{\circ}\text{C}$ dan Torr adalah

T ($^{\circ}\text{C}$)	-26,7	-4,4	6,4	18,4	31,8	40,3
p (torr)	1	5	10	20	40	60

Tentukan konstanta-konstanta persamaan Antoine yang menyatakan hubungan T dan p sebagai berikut

$$p = \exp\left(a + \frac{b}{T + 273,2}\right)$$



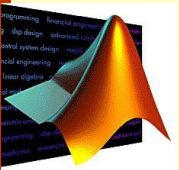
lanjutan

$$p = \exp\left(a + \frac{b}{T + 273,2}\right)$$

$$\ln(p) = a + \frac{b}{T + 273,2}$$

$$y = a + b x$$

$$y = \ln(p) \text{ dan } x = \frac{1}{T + 273,2}$$

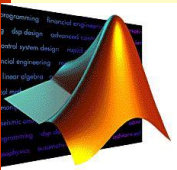


lanjutan

```
% Data - data
p = [ 1 5 10 20 40 60]; % C
T = [-26.7 -4.4 6.4 18.4 31.8 40.3]; % Torr

% Linierisasi
y = log(p);
x = 1./(T+273.15);

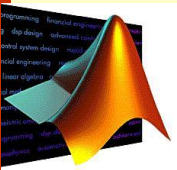
kons = polyfit(x,y,1);
a = kons(2)
b = kons(1)
```



Regresi Polinomial

data-data didekati dengan persamaan polinomial, yaitu :

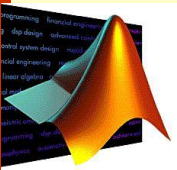
$$y = a_0 + a_1x + a_2x^2 + \dots + a_mx^m$$



lanjutan

Fungsi `polyfit` dari Matlab dapat digunakan untuk derajat $n = 2$, yang disebut **polinomial kuadratis**. Hasil fungsi `polyfit` adalah vektor baris yang berisi koefisien-koefisien polinomial.

Selain fungsi `polyfit`, Matlab juga mempunyai fungsi `polyval` untuk mengevaluasi polinomial pada tiap titik yang ingin diketahui nilainya.



Soal 2

Persamaan Fit untuk Data Tekanan Uap Benzena

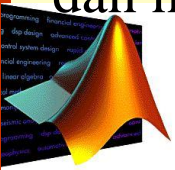
Data tekanan uap murni benzena pada berbagai temperatur ditunjukkan pada tabel.

Temperatur (°C)	Tekanan (mmHg)	Temperatur (°C)	Tekanan (mmHg)
-36,7	1	15,4	60
-19,6	5	26,1	100
-11,5	10	42,2	200
-2,6	20	60,6	400
7,6	40	80,1	760

Hubungan tekanan sebagai fungsi temperatur dapat dinyatakan sebagai persamaan empiris polinomial sederhana sebagai berikut :

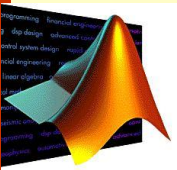
$$P = a_0 + a_1T + a_2T^2 + a_3T^3 + \dots + a_nT^n$$

dengan a_0, a_1, \dots, a_n adalah parameter yang ditentukan dengan regresi dan n adalah orde polinomial.



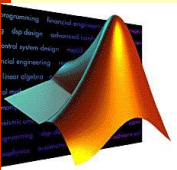
lanjutan

```
vp = [ 1 5 10 20 40 60 100 200 400 760];  
T = [-36.7 -19.6 -11.5 -2.6 7.6 15.4 26.1  
42.2 60.6 80.1];  
% Penggunaan fungsi polyfit  
m = 3; %orde polinomial  
k = polyfit(T, vp, m);  
% Evaluasi persamaan polinomial  
z = polyval(k, 55)
```

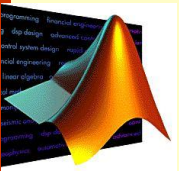


lanjutan

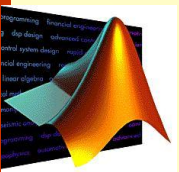
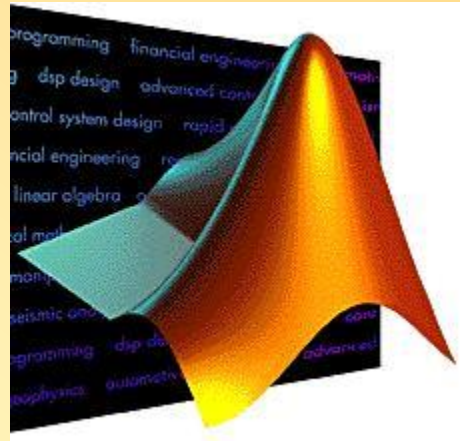
```
vp = [ 1 5 10 20 40 60 100 200 400 760];  
T = [-36.7 -19.6 -11.5 -2.6 7.6 15.4 26.1  
42.2 60.6 80.1];  
% Penggunaan fungsi polyfit  
m = 3; %orde polinomial  
k = polyfit(T, vp, m);  
% Evaluasi persamaan polinomial  
z = polyval(k, T);  
% Cetak hasil  
plot(T, z, 'k-', T, vp, 'ko', 'linewidth', 2)  
xlabel('T(C)')  
ylabel('vp(mmHg)')
```



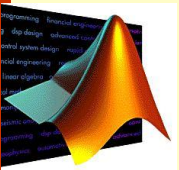
Next meeting



05 Curve Fitting

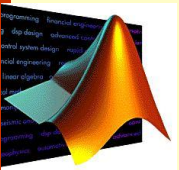


- There are two general approaches for curve fitting that are distinguished from each other on the basis of the amount of error associated with the data.
 - Regression
 - Interpolation



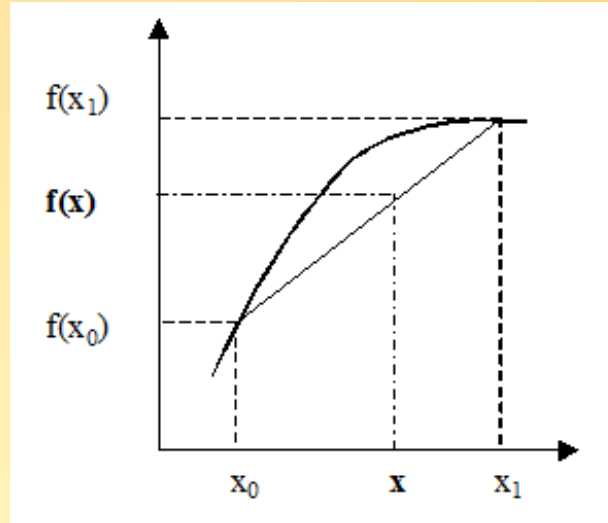
6.2. Interpolation

- Interpolation is used to estimate intermediate values between precise data points.



a. Linear Interpolation

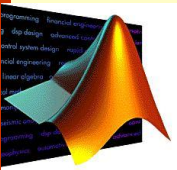
- The simplest form of interpolation is to connect two data points with a straight line. This technique, called *linear interpolation*, is depicted graphically in Fig.



$$\frac{f(x) - f(x_0)}{x - x_0} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$f(x) = f(x_0) + \frac{f(x_1) - f(x_0)}{(x_1 - x_0)} (x - x_0)$$

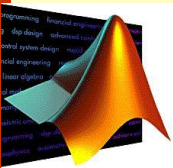
Newton linear-interpolation formula.



- Find ρ , μ , and ν at $T = 27\text{ }^\circ\text{C}$

$T, \text{ }^\circ\text{C}$	$\rho, \text{ kg/m}^3$	$\mu, \text{ N} \cdot \text{s/m}^2$	$\nu, \text{ m}^2/\text{s}$
-40	1.52	1.51×10^{-5}	0.99×10^{-5}
0	1.29	1.71×10^{-5}	1.33×10^{-5}
20	1.20	1.80×10^{-5}	1.50×10^{-5}
50	1.09	1.95×10^{-5}	1.79×10^{-5}
100	0.946	2.17×10^{-5}	2.30×10^{-5}
150	0.835	2.38×10^{-5}	2.85×10^{-5}
200	0.746	2.57×10^{-5}	3.45×10^{-5}
250	0.675	2.75×10^{-5}	4.08×10^{-5}
300	0.616	2.93×10^{-5}	4.75×10^{-5}
400	0.525	3.25×10^{-5}	6.20×10^{-5}
500	0.457	3.55×10^{-5}	7.77×10^{-5}

$$f(x) = f(x_0) + \frac{f(x_1) - f(x_0)}{(x_1 - x_0)} (x - x_0)$$



b. Quadratic Interpolation

- A strategy for improving the estimate is to introduce some curvature into the line connecting the points. If three data points are available, this can be accomplished with a second-order polynomial (also called a quadratic polynomial or a parabola).

Bentuk umum

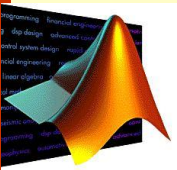
$$f(x) = b_0 + b_1(x-x_0) + b_2(x-x_0)(x-x_1)$$

dengan

$$b_0 = f(x_0)$$

$$b_1 = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}$$

$$b_2 = \frac{\frac{f(x_2) - f(x_1)}{(x_2 - x_1)} - \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}}{(x_2 - x_0)}$$



- Find ρ , μ , and ν at $T = 27\text{ }^\circ\text{C}$

$T, \text{ }^\circ\text{C}$	$\rho, \text{ kg/m}^3$	$\mu, \text{ N} \cdot \text{s/m}^2$	$\nu, \text{ m}^2/\text{s}$
-40	1.52	1.51×10^{-5}	0.99×10^{-5}
0	1.29	1.71×10^{-5}	1.33×10^{-5}
20	1.20	1.80×10^{-5}	1.50×10^{-5}
50	1.09	1.95×10^{-5}	1.79×10^{-5}
100	0.946	2.17×10^{-5}	2.30×10^{-5}
150	0.835	2.38×10^{-5}	2.85×10^{-5}
200	0.746	2.57×10^{-5}	3.45×10^{-5}
250	0.675	2.75×10^{-5}	4.08×10^{-5}
300	0.616	2.93×10^{-5}	4.75×10^{-5}
400	0.525	3.25×10^{-5}	6.20×10^{-5}
500	0.457	3.55×10^{-5}	7.77×10^{-5}

Bentuk umum

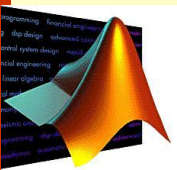
$$f(x) = b_0 + b_1(x-x_0) + b_2(x-x_0)(x-x_1)$$

dengan

$$b_0 = f(x_0)$$

$$b_1 = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}$$

$$b_2 = \frac{\frac{f(x_2) - f(x_1)}{(x_2 - x_1)} - \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}}{(x_2 - x_0)}$$



General Form of Newton's Interpolating Polynomials

Bentuk umum interpolasi polinomial Newton

$$f(x) = b_0 + b_1(x-x_0) + b_2(x-x_0)(x-x_1) + \dots + b_n(x-x_0)(x-x_1)(x-x_2)\dots(x-x_n)$$

dengan

$$b_0 = f(x_0)$$

$$b_1 = f[x_1, x_0]$$

$$b_2 = f[x_2, x_1, x_0]$$

.

.

.

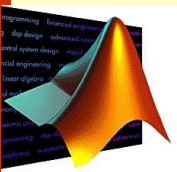
$$b_n = f[x_n, x_{n-1}, \dots, x_1, x_0]$$

Differensiasi terbagi pertama

$$f[x_i, x_j] = \frac{f(x_i) - f(x_j)}{(x_i - x_j)}$$

Differensiasi terbagi kedua

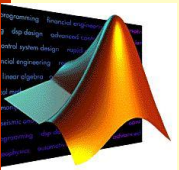
$$f[x_i, x_j, x_k] = \frac{f[x_i, x_j] - f[x_j, x_k]}{x_i - x_k}$$



05 Curve Fitting

Interpolasi polinomial order 3

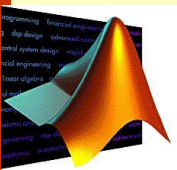
i	x_i	$f(x_i)$	I	II	III
0	x_0	$f(x_0)$	$f[x_1, x_0]$		
1	x_1	$f(x_1)$	$f[x_2, x_1]$	$f[x_2, x_1, x_0]$	$f[x_3, x_2, x_1, x_0]$
2	x_2	$f(x_2)$	$f[x_3, x_2]$	$f[x_3, x_2, x_1]$	
3	x_3	$f(x_3)$			



From data of acetic acid density at 25 °C, find the density of acetic acid at 65 % !

Data densitas asam asetat pada 25 °C

Konsentrasi (%)	Densitas (g/cm ³)
0	0,997
10	1,011
20	1,024
30	1,035
40	1,045
50	1,053
60	1,06
70	1,064
80	1,065
90	1,061
100	1,044



Interpolation with Matlab

Function

`interp1` for linier interpolation.

`YI = interp1 (X,Y,XI,'the methode')`

For find Y YI if $X = X1$, with data Y vs X

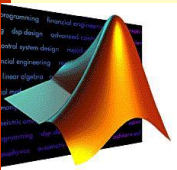
The methode

`'nearest'` -

`'linier'` -

`'spline'` -

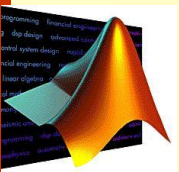
`'cubic'` -



Konduktivitas termal aseton

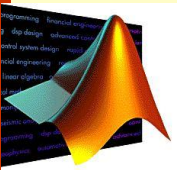
Bennett & Meyers memberikan data-data konduktivitas termal aseton. Hubungan $\ln(k)$ dan $\ln(T)$ mendekati persamaan garis lurus, dengan T dalam R ($R = ^\circ F + 460$). Tentukan nilai k pada $300\ ^\circ F$!

k [BTU/jam.ft.$^\circ F$]	T ($^\circ F$)
0,0057	32
0,0074	115
0,0099	212
0,0147	363



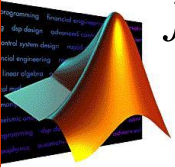
lanjutan

```
k = [0.0057 0.0074 0.0099 0.0147];  
T = [32 115 212 363]+460;  
    % Hubungan k dan T linier  
ln_k = log(k);  
ln_T = log(T);  
    % Interpolasi Linier  
T300 = log(300+460);  
k300 = interp1(ln_T,ln_k,T300,'linier');  
    % Hasil  
k_pd_300 = exp(k300)
```

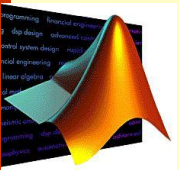


b. Splines Interpolation

- n^{th} -order polynomials were used to interpolate between $(n+1)$ data points.
- For example, for eight points, we can derive a perfect seventh-order polynomial. This curve would capture all the meanderings (at least up to and including seventh derivatives) suggested by the points.
- However, there are cases where these functions can lead to erroneous results because of round-off error and oscillations.
- An alternative approach is to apply lower-order polynomials in a piecewise fashion to subsets of data points. Such connecting polynomials are called *spline functions*.



- For example, third-order curves employed to connect each pair of data points are called *cubic splines*.
- These functions can be constructed so that the connections between adjacent cubic equations are visually smooth.



Larutan metil dietanol amin (MDEA) banyak digunakan untuk absorpsi gas seperti H_2S dan CO_2 . Dalam suatu menara absorpsi, perlu diketahui viskositas 43 % larutan metil dietanol amin (MDEA) dalam air pada $45^\circ C$. Untuk itu, ingin dipergunakan persamaan berikut:

$$\ln \mu = B_1 + \frac{B_2}{T} + B_3 T$$

Dengan μ dalam poise, dan T dalam K. B_i adalah parameter dengan ketentuan sebagai berikut:

Fraksi berat MDEA	B_1	$B_2 \cdot 10^{-3}$	$B_3 \cdot 10^2$
0	-19,52	3,913	2,112
0,1	-22,14	4,475	2,470
0,2	-25,16	5,157	2,859
0,3	-28,38	5,908	3,255
0,4	-31,52	6,678	3,634
0,5	-34,51	7,417	3,972

Tentukan viskositas larutan pada 43 % larutan dan $45^\circ C$.



```
% Data parameter viskositas sebagai fungsi x
x = [0 0.1 0.2 0.3 0.4 0.5];
B1 = [-19.52 -22.14 -25.16 -28.38 -31.52 -34.51];
B2 = [3.913 4.475 5.157 5.908 6.678 7.417]*10^3;
B3 = [2.112 2.470 2.859 3.255 3.634 3.972]*10^-2;
```

```
% Data x dan T yang ingin dicari viskositasnya
```

```
x_i = 0.43;
```

```
% Konsentrasi
```

```
T_i = 45+273.15;
```

```
% Temperatur, K
```

```
% Interpolasi hubungan fraksi berat vs konstanta
```

```
% Konstanta B1, B2, dan B3 pada 0.43 ditentukan terlebih dahulu
```

```
% Interpolasi nilai B pada konsentrasi 0.43
```

```
B1_i = interp1(x,B1,x_i,'spline');
```

```
B2_i = interp1(x,B2,x_i,'spline');
```

```
B3_i = interp1(x,B3,x_i,'spline');
```

```
% Viskositas dihitung
```

```
viscA = exp(B1_i + B2_i/T_i + B3_i*T_i)
```

```
% Interpolasi hubungan fraksi berat vs viskositas
```

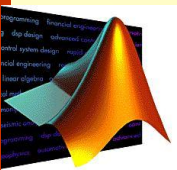
```
% Viskositas pada 45 C ditentukan terlebih dahulu
```

```
% Viskositas pada 45 C dihitung
```

```
visc = exp(B1 + B2./T_i + B3.*T_i)
```

```
% Interpolasi viskositas pada konsentrasi 0.43
```

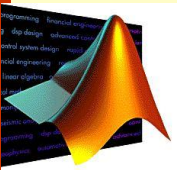
```
viscB = interp1(x,visc,x_i,'spline')
```



c. Two dimension Interpolation

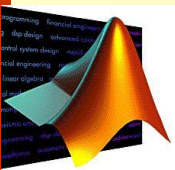
Interpolasi dua dimensi mempunyai prinsip yang sama dengan interpolasi satu dimensi. Perbedaannya adalah interpolasi dua dimensi menginterpolasikan fungsi dua variabel $z = f(x,y)$.

$ZI = \text{interp2}(X, Y, Z, XI, YI)$ adalah interpolasi untuk menentukan nilai ZI , dari fungsi Z pada titik XI dan YI dari matriks X dan Y . Metode yang ditampilkan pada interpolasi 1 dimensi dapat juga digunakan pada interpolasi 2 dimensi.



Tentukan berapakah ΔH (beda entalpi) dari uap yang didinginkan menjadi 480 °F dan 52 psia, jika kondisi mula-mula adalah 640 °F dan 92 psia !

Entalphi, H dalam Btu/lb				
p (psia)	T (°F)			
	400	500	600	700
50	1258,7	1282,6	-	-
55	1258,2	1282,2	-	-
90	-	-	1328,7	1378,1
95	-	-	1328,4	1377,8



% Kondisi awal

To = [600 700];

po = [90 95];

Ho = [1328.7 1378.1

1328.4 1377.8];

H1 = interp2(To,po,Ho,640,92)

% Temperatur, oF

% Tekanan, psia

% Entalphi, BTU/lb

% Interpolasi 2 dimensi

% Kondisi akhir

Tn = [400 500];

pn = [50 55];

Hn = [1258.7 1282.6

1258.2 1282.2];

H2 = interp2(Tn,pn,Hn,480,52)

% Temperatur, oF

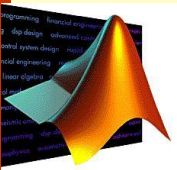
% Tekanan, psia

% Entalphi, BTU/lb

% Interpolasi 2 dimensi

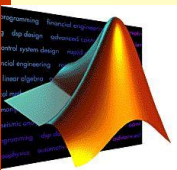
% Perhitungan beda entalpi

del_H = H2 - H1



Use the portion of the given steam table for superheated H_2O at 200 MPa to find the corresponding entropy s for a specific volume v of $0.108 \text{ m}^3/\text{kg}$ with linear interpolation

$v \text{ (m}^3/\text{kg)}$	0.10377	0.11144	0.1254
$s \text{ (kJ/kg}\cdot\text{K)}$	6.4147	6.5453	6.7664



The following data defines the sea-level concentration of dissolved oxygen for fresh water as a function of temperature:

$T, ^\circ\text{C}$	0	8	16	24	32	40
$o, \text{mg/L}$	14.621	11.843	9.870	8.418	7.305	6.413

Estimate $o(27)$ using (a) linear interpolation, (b) Newton's interpolating polynomial, and (c) cubic splines. Note that the exact result is 7.986 mg/L.

