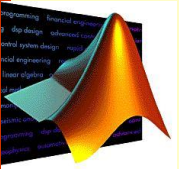


Chapter 03

LINEAR ALGEBRAIC EQUATIONS



Linear Algebraic Equations

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

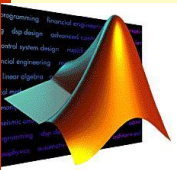
$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

.

.

.

$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n$$

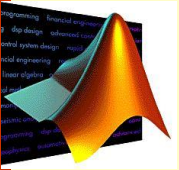


Example

$$3X_1 + 2X_2 + X_3 = 11$$

$$3X_1 + X_2 + 3X_3 = 16$$

$$X_1 + 2X_2 + X_3 = 7$$



Gaussian Elimination with Backward Substitution

$$\left(\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{array} \right)$$

Elimination process

$$\left(\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ & a'_{22} & a'_{23} & b'_2 \\ & & a''_{33} & b''_3 \end{array} \right)$$

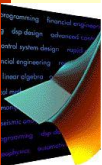


$$x_3 = \frac{b''_3}{a''_{33}},$$

Backward substitution

$$x_2 = \frac{(b'_2 - a'_{23}x_3)}{a'_{22}},$$

$$x_1 = \frac{(b_1 - a_{12}x_2 - a_{13}x_3)}{a_{11}}.$$



Original equations

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \quad \dots (2.1)$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \quad \dots (2.2)$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \quad \dots (2.3)$$

Devide eq (2.1) by a_{11}

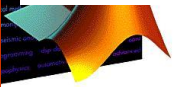
$$x_1 + \frac{1}{a_{11}} a_{12}x_2 + \frac{1}{a_{11}} a_{13}x_3 = \frac{1}{a_{11}} b_1 \quad \dots (2.4)$$

Eliminate a_{21} by

[Eq. (2.2) - a_{21} x Eq. (2.4)]

$$\left(a_{22} - \frac{a_{21}}{a_{11}} a_{12} \right) x_2 + \left(a_{23} - \frac{a_{21}}{a_{11}} a_{13} \right) x_3 = \left(b_2 - \frac{a_{21}}{a_{11}} b_1 \right)$$

$$a'_{22}x_2 + a'_{23}x_3 = b'_2 \quad \dots (2.5)$$



Eliminate a_{31} Eq. (2.3) by

[Eq. (2.3) - a_{31} x Eq. (2.4)]

$$\left(a_{32} - \frac{a_{31}}{a_{11}} a_{12} \right) x_2 + \left(a_{33} - \frac{a_{31}}{a_{11}} a_{13} \right) x_3 = \left(b_3 - \frac{a_{31}}{a_{11}} b_1 \right)$$

$$a'_{32} x_2 + a'_{33} x_3 = b'_3 \quad \dots (2.6)$$

Divide eq (2.5) by a'_{22}

$$x_2 + \frac{1}{a'_{22}} a'_{23} x_3 = \frac{1}{a'_{22}} b'_2 \quad \dots (2.7)$$

Eliminate a'_{31} Eq. (2.6) by

[Eq. (2.6) - a'_{32} x Eq. (2.7)]

$$\left(a'_{33} - \frac{a'_{32}}{a'_{22}} a'_{23} \right) x_3 = \left(b'_3 - \frac{a'_{32}}{a'_{22}} b'_2 \right)$$

$$a''_{33} x_3 = b''_3 \quad \dots (2.8)$$

Backward substitution

Solution of x_3 form eq (2.8)

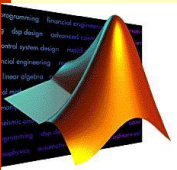
$$x_3 = \frac{b_3''}{a_{33}}$$

Solution of x_2 form eq (2.5)

$$x_2 = \frac{(b_2' - a_{23}'x_3)}{a_{22}'}$$

Solution of x_1 form eq (2.1)

$$x_1 = \frac{(b_1 - a_{12}x_2 - a_{13}x_3)}{a_{11}}$$

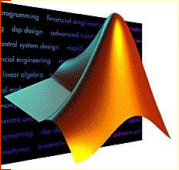


Example

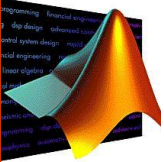
$$3X_1 + 2X_2 + 3X_3 = 10$$

$$3X_1 + X_2 + 2X_3 = 9$$

$$X_1 + X_2 + 2X_3 = 5$$



03 Linear Algebraic Equations



```
function [x] = elimgauss(A,B,n)
%       This program to solve Linear Algebraic Equations
%       by Gaussian elimination
%       n is matrix orde
%       A is matrix n x n
%       B is matrix n x 1
%       x is matrix output n x 1 for solution

%       Nama File : elimgauss.m
%       Surakarta, March 2017
%       Chemical Engineering Departement
%       Sebelas Maret University
%       -----

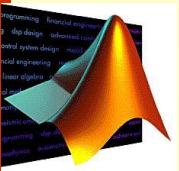
%       Gaussian elimination
for k=1:n
    for i=k+1:n
        m(i,k)=A(i,k)/A(k,k);
        for j=k:n
            A(i,j)=A(i,j)-m(i,k)*A(k,j);
        end
        B(i)=B(i)-m(i,k)*B(k);
    end
end

%       Backward substitution
x(n)=B(n)/A(n,n);
for i=n-1:-1:1
    sum=B(i);
    for k=i+1:n
        sum=sum-A(i,k)*x(k);
    end
    x(i)=sum/A(i,i);
end
```

Matrix Inversion

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \cdot & & & & \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \cdot \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \cdot \\ b_n \end{bmatrix}$$

$$\mathbf{A} \quad \mathbf{x} \quad = \quad \mathbf{B}$$



Solution

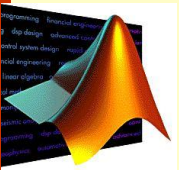
$$\begin{aligned} \mathbf{A} \mathbf{x} &= \mathbf{B} \\ \mathbf{A}^{-1} \mathbf{A} \mathbf{x} &= \mathbf{A}^{-1} \mathbf{B} \\ \mathbf{I} \mathbf{x} &= \mathbf{A}^{-1} \mathbf{B} \\ \mathbf{x} &= \mathbf{A}^{-1} \mathbf{B} \end{aligned}$$

Matlab

$$\mathbf{x} = \text{inv}(\mathbf{A}) * \mathbf{B}$$

or

$$\mathbf{x} = \mathbf{A} \setminus \mathbf{B}$$



Example

$$x_1 + 2x_2 + 3x_3 = 366$$

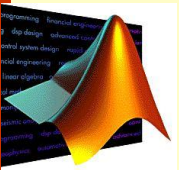
$$4x_1 + 5x_2 + 6x_3 = 804$$

$$7x_1 + 8x_2 = 351$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 366 \\ 804 \\ 351 \end{bmatrix}$$

$$\mathbf{A} \mathbf{x} = \mathbf{B}$$

$$\mathbf{x} = \text{inv}(\mathbf{A}) * \mathbf{B}$$



Solution

```
>> A=[1 2 3 ; 4 5 6 ; 7 8 0]
```

```
A =
```

```
1    2    3
```

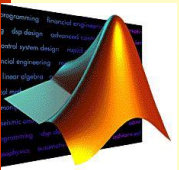
```
4    5    6
```

```
7    8    0
```

```
>> det(A)
```

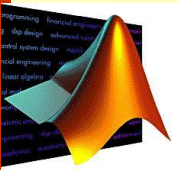
```
ans =
```

```
27
```



Program

```
%Program for solution linear algebraic equations  
A=[1 2 3 ; 4 5 6 ; 7 8 0 ]  
b=[366 ; 804 ; 351]  
disp('x=inv(A)*b')  
x=inv(A)*b  
disp('atau')  
disp('x=A\b')  
x=A\b
```



Ex 01

$$10x_1 - x_2 + 2x_3 = 6$$

$$-x_1 + 11x_2 - x_3 + 3x_4 = 25$$

$$2x_1 - x_2 + 10x_3 - x_4 = -11$$

$$3x_2 - x_3 + 8x_4 = 15$$

