Chapter 03
LINEAR ALGEBRAIC EQUATIONS

Linear Algebraic Equations

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$$

$$.$$

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$$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n$$

Example

$$
\begin{aligned}
3X_1 &+ 2X_2 + X_3 = 11 \\
3X_1 &+ X_2 + 3X_3 = 16 \\
X_1 &+ 2X_2 + X_3 = 7
\end{aligned}
$$

Gaussian Elimination with Backward Substitution

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \mid b_1 \\ a_{21} & a_{22} & a_{23} \mid b_2 \\ a_{31} & a_{32} & a_{33} \mid b_3 \end{pmatrix}$$

Elimination process

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \mid b_1 \\ & a'_{22} & a'_{23} \mid b'_2 \\ & & a''_{33} \mid b''_3 \end{pmatrix}$$

Backward substitution

$$x_3 = \frac{b''_3}{a''_{33}},$$

$$x_2 = \frac{\left(b'_2 - a'_{23}x_3\right)}{a'_{22}},$$

$$x_1 = \frac{\left(b_1 - a_{12}x_2 - a_{13}x_3\right)}{a_{11}}.$$

Original equations

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \qquad \text{.... (2.1)}$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \qquad \text{.... (2.2)}$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \qquad \text{.... (2.3)}$$

Devide eq (2.1) by a_{11}

$$x_1 + \frac{1}{a_{11}} a_{12}x_2 + \frac{1}{a_{11}} a_{13}x_3 = \frac{1}{a_{11}} b_1 \qquad \text{.... (2.4)}$$

Eliminate a_{21} by

[Eq. (2.2) – a_{21} x Eq. (2.4)]

$$\left(a_{22} - \frac{a_{21}}{a_{11}} a_{12}\right)x_2 + \left(a_{23} - \frac{a_{21}}{a_{11}} a_{13}\right)x_3 = \left(b_2 - \frac{a_{21}}{a_{11}} b_1\right)$$

$$a'_{22}x_2 + a'_{23}x_3 = b'_2 \qquad \text{.... (2.5)}$$

Eliminate a_{31} Eq. (2.3) by

[Eq. (2.3) – a_{31} x Eq. (2.4)]

$$\left(a_{32} - \frac{a_{31}}{a_{11}} a_{12}\right) x_2 + \left(a_{33} - \frac{a_{31}}{a_{11}} a_{13}\right) x_3 = \left(b_3 - \frac{a_{31}}{a_{11}} b_1\right)$$

$$a'_{32} x_2 + a'_{33} x_3 = b'_3 \qquad \text{.... (2.6)}$$

Devide eq (2.5) by a'$_{22}$

$$x_2 + \frac{1}{a'_{22}} a'_{23} x_3 = \frac{1}{a'_{22}} b'_2 \qquad \text{.... (2.7)}$$

Eliminate a'$_{31}$ Eq. (2.6) by

[Eq. (2.6) – a'$_{32}$ x Eq. (2.7)]

$$\left(a'_{33} - \frac{a'_{32}}{a'_{22}} a'_{23}\right) x_3 = \left(b'_3 - \frac{a'_{32}}{a'_{22}} b'_2\right)$$

$$a''_{33} x_3 = b''_3 \qquad \text{.... (2.8)}$$

Backward substitution

Solution of x_3 form eq (2.8)

$$x_3 = \frac{b_3''}{a_{33}''}$$

Solution of x_2 form eq (2.5)

$$x_2 = \frac{\left(b_2' - a_{23}' x_3\right)}{a_{22}'}$$

Solution of x_1 form eq (2.1)

$$x_1 = \frac{\left(b_1 - a_{12} x_2 - a_{13} x_3\right)}{a_{11}}$$

Example

$$3X_1 + 2X_2 + 3X_3 = 10$$

$$3X_1 + X_2 + 2X_3 = 9$$

$$X_1 + X_2 + 2X_3 = 5$$

```
function [x] = elimgauss(A,B,n)
%       This program to solve Linear Algebraic Equations
%       by Gaussian elimination
%       n is matrix orde
%       A is matrix n x n
%       B is matrix n x 1
%       x is matrix output n x 1 for solution

%       Nama File : elimgauss.m
%       Surakarta, March 2017
%       Chemical Engineering Departement
%       Sebelas Maret University
%   ----------------------------------------------------------------

%   Gaussian elimination
for k=1:n
    for i=k+1:n
        m(i,k)=A(i,k)/A(k,k);
        for j=k:n
            A(i,j)=A(i,j)-m(i,k)*A(k,j);
        end
        B(i)=B(i)-m(i,k)*B(k);
    end
end

%   Backward substitution
x(n)=B(n)/A(n,n);
for i=n-1:-1:1
    sum=B(i);
    for k=i+1:n
        sum=sum-A(i,k)*x(k);
    end
    x(i)=sum/A(i,i);
end
```

Matrix Inversion

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ . & & & & \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ . \\ b_n \end{bmatrix}$$

$$\mathbf{A} \qquad \mathbf{x} = \mathbf{B}$$

Solution

$$A\,x = B$$

$$A^{-1}A\,x = A^{-1}B$$

$$I\,x = A^{-1}B$$

$$x = A^{-1}B$$

Matlab

```
x = inv(A)*B
```

or

```
x = A\B
```

Example

$$x_1 + 2x_2 + 3x_3 = 366$$
$$4x_1 + 5x_2 + 6x_3 = 804$$
$$7x_1 + 8x_2 = 351$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \begin{matrix} = \\ = \\ = \end{matrix} \begin{bmatrix} 366 \\ 804 \\ 351 \end{bmatrix}$$

A x = B

```
x = inv(A)*B
```

Solution

```
>> A=[1 2 3 ; 4 5 6 ; 7 8 0]
 A =
     1     2     3
     4     5     6
     7     8     0
>> det(A)
ans =
    27
```

Program

```
%Program for solution linear algebraic equations
A=[1 2 3 ; 4 5 6 ; 7 8 0 ]
b=[366 ; 804 ; 351]
disp('x=inv(A)*b')
x=inv(A)*b
disp('atau')
disp('x=A\b')
x=A\b
```

Ex 01

$$
\begin{aligned}
10x_1 - x_2 + 2x_3 &= 6 \\
-x_1 + 11x_2 - x_3 + 3x_4 &= 25 \\
2x_1 - x_2 + 10x_3 - x_4 &= -11 \\
3x_2 - x_3 + 8x_4 &= 15
\end{aligned}
$$