

Pemodelan Sistem

Identifikasi Sistem

- ▶ White Box → Analisis Matematis dari parameter system.
- ▶ Grey Box → Kombinasi.
- ▶ Black Box → Menggunakan analisis dari input-output system.

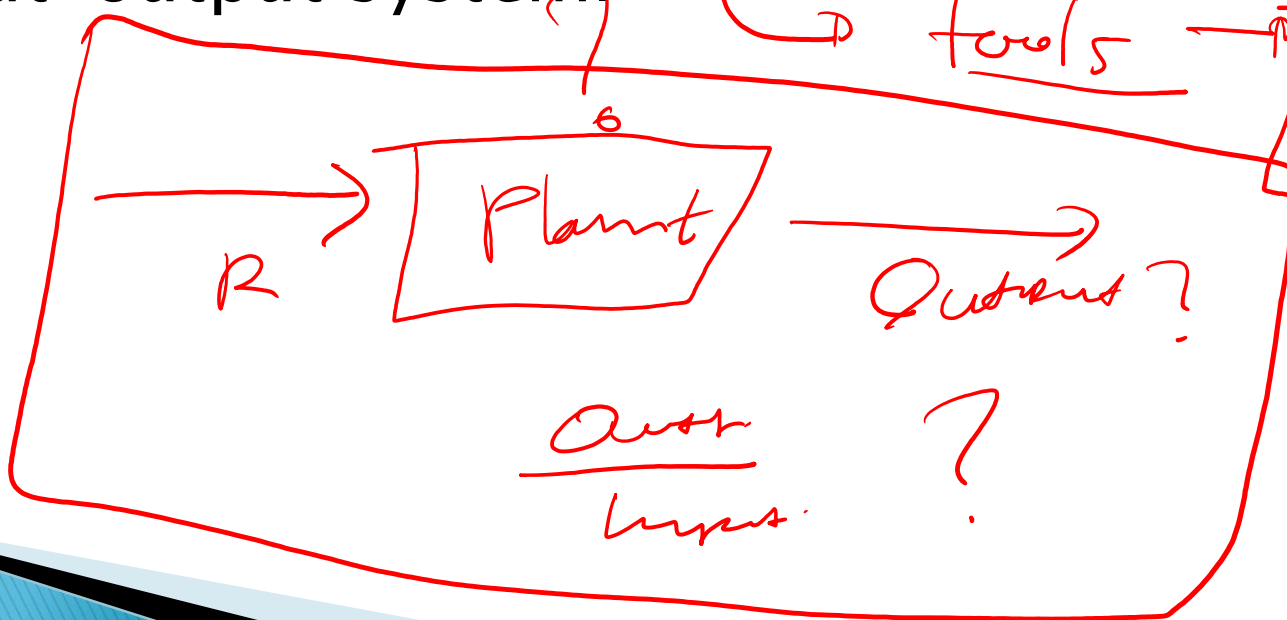
elektronik
+ mekanik

Superns:

↳ k
↳ d

tools

↳ Master
Sistem



Model

Review Matematika

- ▶ *Transformasi Laplace*

Problem dalam sistem kontrol adalah problem dinamik yg biasanya dideskripsikan dalam persamaan diferensial. Dengan transformasi Laplace, solusi persamaan diferensial lebih sederhana dan mudah

- ▶ *Partial Fraction Expansion*

karena berhadapan dengan pecahan simbolik (fungsi rasional) maka perlu metode ini untuk menyederhanakan persamaan

Transformasi Laplace

Jika terdapat fungsi $f(t)$ maka Transformasi Laplace dari $f(t)$ adalah $F(s)$

$$\mathcal{L}[f(t)] = F(s) = \int_{0-}^{\infty} f(t)e^{-st} dt$$

$$\mathcal{L}^{-1}[F(s)] = \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} F(s)e^{st} ds = f(t)u(t)$$



Jika terdapat fungsi $F(s)$ maka inverse Transformasi Laplace kembali ke $f(t)$ adalah

$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

Transformasi Laplace

PROBLEM: Find the Laplace transform of $f(t) = Ae^{-at}u(t)$.

SOLUTION: Since the time function does not contain an impulse function, we can replace the lower limit of Eq. (2.1) with 0. Hence,

$$\begin{aligned} F(s) &= \int_0^{\infty} f(t)e^{-st} dt = \int_0^{\infty} Ae^{-at}e^{-st} dt = A \int_0^{\infty} e^{-(s+a)t} dt \\ &= -\frac{A}{s+a} e^{-(s+a)t} \Big|_{t=0}^{\infty} = \frac{A}{s+a} \quad (2.3) \end{aligned}$$

TABLE 2.1 Laplace transform table

Item no.	$f(t)$	$F(s)$
1.	$\delta(t)$	1
2.	$u(t)$	$\frac{1}{s}$
3.	$tu(t)$	$\frac{1}{s^2}$
4.	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5.	$e^{-at}u(t)$	$\frac{1}{s+a}$
6.	$\sin \omega t u(t)$	$\frac{\omega}{s^2 + \omega^2}$
7.	$\cos \omega t u(t)$	$\frac{s}{s^2 + \omega^2}$

PROBLEM: Find the inverse Laplace transform of $F_1(s) = 1/(s+3)^2$.

SOLUTION: For this example we make use of the frequency shift theorem, Item 4 of Table 2.2, and the Laplace transform of $f(t) = tu(t)$, Item 3 of Table 2.1. If the inverse transform of $F(s) = 1/s^2$ is $tu(t)$, the inverse transform of $F(s+a) = 1/(s+a)^2$ is $e^{-at}tu(t)$. Hence, $f_1(t) = e^{-3t}tu(t)$.

Sifat-sifat Transformasi Laplace

Item no.	Theorem	Name
1.	$\mathcal{L}\{f(t)\} = F(s) = \int_{0^-}^{\infty} f(t)e^{-st} dt$	Definition
2.	$\mathcal{L}\{kf(t)\} = kF(s)$	Linearity theorem
3.	$\mathcal{L}\{f_1(t) + f_2(t)\} = F_1(s) + F_2(s)$	Linearity theorem
4.	$\mathcal{L}\{e^{-at}f(t)\} = F(s + a)$	Frequency shift theorem
5.	$\mathcal{L}\{f(t - T)\} = e^{-sT}F(s)$	Time shift theorem
6.	$\mathcal{L}\{f(at)\} = \frac{1}{a}F\left(\frac{s}{a}\right)$	Scaling theorem
7.	$\mathcal{L}\left[\frac{df}{dt}\right] = sF(s) - f(0^-)$	Differentiation theorem
8.	$\mathcal{L}\left[\frac{d^2f}{dt^2}\right] = s^2F(s) - sf(0^-) - f'(0^-)$	Differentiation theorem
9.	$\mathcal{L}\left[\frac{d^nf}{dt^n}\right] = s^nF(s) - \sum_{k=1}^n s^{n-k}f^{k-1}(0^-)$	Differentiation theorem
10.	$\mathcal{L}\left[\int_{0^-}^t f(\tau)d\tau\right] = \frac{F(s)}{s}$	Integration theorem
11.	$f(\infty) = \lim_{s \rightarrow 0} sF(s)$	Final value theorem ¹
12.	$f(0^+) = \lim_{s \rightarrow \infty} sF(s)$	Initial value theorem ²

Partial Fraction Expansion

Misalnya terdapat fungsi dalam s dari hasil transformasi Laplace

$$\longrightarrow G(s) = \frac{Q(s)}{P(s)}$$

di mana $Q(s)$ dan $P(s)$ adalah polinom dalam s

Akar-akar (yang membuat persamaan menjadi nol) dari $Q(s)$ disebut *zero* dari $G(s)$

Akar-akar (yang membuat persamaan menjadi nol) dari $P(s)$ disebut *pole* dari $G(s)$

Jika diasumsikan orde (pangkat tertinggi) dari $P(s)$ lebih besar dari $Q(s)$ maka

$$P(s) = s^n + a_{n-1}s^{n-1} + \dots + a_1s + a_0$$

Dimana a_1, a_0 dst adalah koefisien real

maka terdapat beberapa jenis penyederhanaan

Partial Fraction Expansion

1. Jika pole bilangan real dan berbeda

$$G(s) = \frac{Q(s)}{P(s)} = \frac{Q(s)}{(s + s_1)(s + s_2) \cdots (s + s_n)}$$

$$s_1 \neq s_2 \neq \cdots \neq s_n.$$

maka

$$G(s) = \frac{K_{s1}}{s + s_1} + \frac{K_{s2}}{s + s_2} + \cdots + \frac{K_{sn}}{s + s_n}$$

$$K_{s1} = \left[(s + s_1) \frac{Q(s)}{P(s)} \right] \Big|_{s=-s_1} = \frac{Q(-s_1)}{(s_2 - s_1)(s_3 - s_1) \cdots (s_n - s_1)}$$

K_{s2} , K_{s3} dst diperoleh dengan cara yang sama

Ilustrasi

$$G(s) = \frac{5s + 3}{(s + 1)(s + 2)(s + 3)} = \frac{5s + 3}{s^3 + 6s^2 + 11s + 6}$$

Dapat dituliskan

$$G(s) = \frac{K_{-1}}{s + 1} + \frac{K_{-2}}{s + 2} + \frac{K_{-3}}{s + 3}$$

$$K_{-1} = [(s + 1)G(s)] \Big|_{s=-1} = \frac{5(-1) + 3}{(2 - 1)(3 - 1)} = -1$$

$$K_{-2} = [(s + 2)G(s)] \Big|_{s=-2} = \frac{5(-2) + 3}{(1 - 2)(3 - 2)} = 7$$

$$K_{-3} = [(s + 3)G(s)] \Big|_{s=-3} = \frac{5(-3) + 3}{(1 - 3)(2 - 3)} = -6$$

$$G(s) = \frac{-1}{s + 1} + \frac{7}{s + 2} - \frac{6}{s + 3}$$

Bentuk yg mudah untuk diubah lagi ke t dengan Inverse transformasi Laplace

Partial Fraction Expansion

2. Jika terdapat akar yang berulang

$$G(s) = \frac{Q(s)}{P(s)} = \frac{Q(s)}{(s + s_1)(s + s_2) \cdots (s + s_{n-r})(s + s_l)^r}$$

$$G(s) = \frac{K_{s1}}{s + s_1} + \frac{K_{s2}}{s + s_2} + \cdots + \frac{K_{s(n-r)}}{s + s_{n-r}} \\ | \leftarrow n - r \text{ terms of simple poles} \rightarrow | \\ + \frac{A_1}{s + s_l} + \frac{A_2}{(s + s_l)^2} + \cdots + \frac{A_r}{(s + s_l)^r} \\ | \leftarrow r \text{ terms of repeated poles} \rightarrow |$$

$$A_r = [(s + s_l)^r G(s)] \Big|_{s=-s_l}$$

$$A_{r-1} = \frac{d}{ds} [(s + s_l)^r G(s)] \Big|_{s=-s_l}$$

$$A_{r-2} = \frac{1}{2!} \frac{d^2}{ds^2} [(s + s_l)^r G(s)] \Big|_{s=-s_l}$$

⋮

$$A_1 = \frac{1}{(r-1)!} \frac{d^{r-1}}{ds^{r-1}} [(s + s_l)^r G(s)] \Big|_{s=-s_l}$$

Ilustrasi

$$G(s) = \frac{1}{s(s+1)^3(s+2)} = \frac{1}{s^5 + 5s^4 + 9s^3 + 7s^2 + 2s}$$

$$\longrightarrow G(s) = \frac{K_0}{s} + \frac{K_{-2}}{s+2} + \frac{A_1}{s+1} + \frac{A_2}{(s+1)^2} + \frac{A_3}{(s+1)^3}$$

$$K_0 = [sG(s)] \Big|_{s=0} = \frac{1}{2}$$

$$K_{-2} = [(s+2)G(s)] \Big|_{s=-2} = \frac{1}{2}$$

$$A_3 = [(s+1)^3G(s)] \Big|_{s=-1} = -1$$

$$A_2 = \frac{d}{ds} [(s+1)^3G(s)] \Big|_{s=-1} = \frac{d}{ds} \left[\frac{1}{s(s+2)} \right] \Big|_{s=-1} = 0$$

$$A_1 = \frac{1}{2!} \frac{d^2}{ds^2} [(s+1)^3G(s)] \Big|_{s=-1} = \frac{1}{2} \frac{d^2}{ds^2} \left[\frac{1}{s(s+2)} \right] \Big|_{s=-1} = -1$$

maka hasilnya

$$G(s) = \frac{1}{2s} + \frac{1}{2(s+2)} - \frac{1}{s+1} - \frac{1}{(s+1)^3}$$

Partial Fraction Expansion

3. Jika polanya adalah pasangan bilangan kompleks

$$s = -\sigma + j\omega \quad \text{and} \quad s = -\sigma - j\omega$$

Koefisien bisa dicari dengan →

$$K_{-\sigma+j\omega} = (s + \sigma - j\omega)G(s) \Big|_{s=-\sigma+j\omega}$$

$$K_{-\sigma-j\omega} = (s + \sigma + j\omega)G(s) \Big|_{s=-\sigma-j\omega}$$

Ilustrasi

$$F(s) = \frac{3}{s(s^2 + 2s + 5)} = \frac{3}{s(s + 1 + j2)(s + 1 - j2)}$$
$$= \frac{K_1}{s} + \frac{K_2}{s + 1 + j2} + \frac{K_3}{s + 1 - j2}$$

$$K_2 = \frac{3}{s(s + 1 - j2)} \Big|_{s \rightarrow -1 - j2} = -\frac{3}{20}(2 + j1)$$

K_3 adalah pasangan bilangan kompleks dari K_2
 K_1 bisa dicari dari cara sebelumnya

$$F(s) = \frac{3/5}{s} - \frac{3}{20} \left(\frac{2 + j1}{s + 1 + j2} + \frac{2 - j1}{s + 1 - j2} \right)$$

Dengan inverse transformasi Laplace \longrightarrow

$$f(t) = \frac{3}{5} - \frac{3}{20} \left[(2 + j1)e^{-(1+j2)t} + (2 - j1)e^{-(1-j2)t} \right]$$
$$= \frac{3}{5} - \frac{3}{20} e^{-t} \left[4 \left(\frac{e^{j2t} + e^{-j2t}}{2} \right) + 2 \left(\frac{e^{j2t} + e^{-j2t}}{2j} \right) \right]$$

Latihan

PROBLEM: Find the Laplace transform of $f(t) = te^{-5t}$.

PROBLEM: Find the inverse Laplace transform of $F(s) = 10/[s(s + 2)(s + 3)^2]$.

Fungsi Transfer



$$\frac{C(s)}{R(s)} = G(s) = \frac{(b_m s^m + b_{m-1} s^{m-1} + \dots + b_0)}{(a_n s^n + a_{n-1} s^{n-1} + \dots + a_0)}$$

$G(s)$ adalah fungsi transfer

Output dapat dicari dengan

$$C(s) = R(s)G(s)$$

Ilustrasi

Tentukan fungsi transfer dari

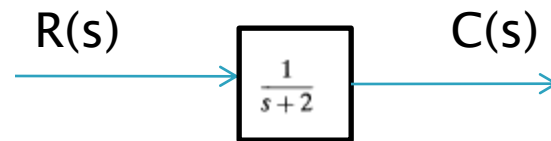
$$\frac{dc(t)}{dt} + 2c(t) = r(t)$$

Dengan transformasi Laplace



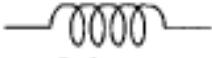
$$sC(s) + 2C(s) = R(s)$$

Fungsi transfer

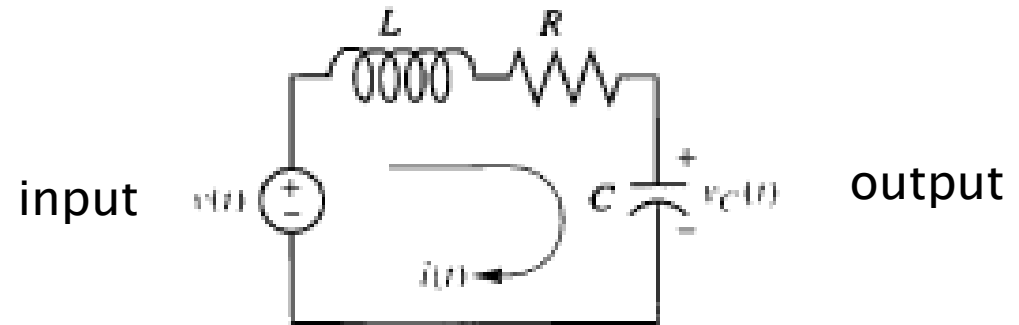
$$G(s) = \frac{C(s)}{R(s)} = \frac{1}{s+2}$$



Pemodelan Sistem Elektrik

Component	Voltage-current	Current-voltage	Voltage-charge	Impedance $Z(s) = V(s)/I(s)$	Admittance $Y(s) = I(s)/V(s)$
 Capacitor	$v(t) = \frac{1}{C} \int_0^1 i(\tau) d\tau$	$i(t) = C \frac{dv(t)}{dt}$	$v(t) = \frac{1}{C} q(t)$	$\frac{1}{Cs}$	Cs
 Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R} v(t)$	$v(t) = R \frac{dq(t)}{dt}$	R	$\frac{1}{R} = G$
 Inductor	$v(t) = L \frac{di(t)}{dt}$	$i(t) = \frac{1}{L} \int_0^1 v(\tau) d\tau$	$v(t) = L \frac{d^2 q(t)}{dt^2}$	Ls	$\frac{1}{Ls}$

Ilustrasi



Dari hukum Kirchoff (loop) tegangan

$$L \frac{di(t)}{dt} + Ri(t) + \frac{1}{C} \int_0^t i(\tau) d\tau = v(t)$$

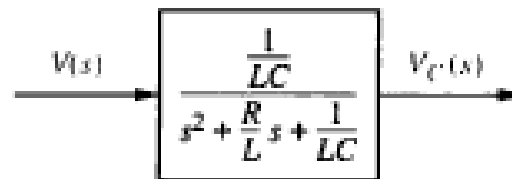
$$i(t) = dq(t)/dt$$

$$L \frac{d^2q(t)}{dt^2} + R \frac{dq(t)}{dt} + \frac{1}{C} q(t) = v(t)$$

Dengan transformasi Laplace

$$q(t) = Cv_C(t)$$

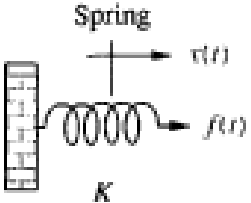
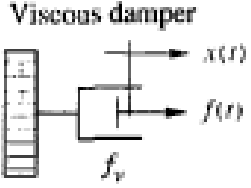
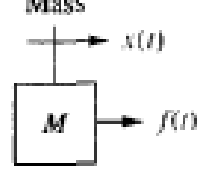
$$LC \frac{d^2v_C(t)}{dt^2} + RC \frac{dv_C(t)}{dt} + v_C(t) = v(t) \rightarrow (LCs^2 + RCs + 1)V_C(s) = V(s)$$



$$\frac{V_C(s)}{V(s)} = \frac{1/LC}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Pemodelan Sistem Mekanik

Sistem Translasi

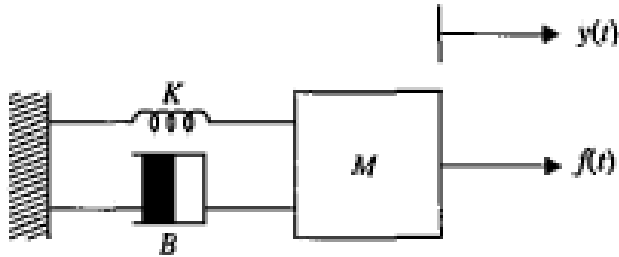
Component	Force-velocity	Force-displacement	Impedance $Z_M(s) = F(s)/X(s)$
<p>Spring</p> 	$f(t) = K \int_0^t v(\tau) d\tau$	$f(t) = Kx(t)$	K
<p>Viscous damper</p> 	$f(t) = f_v v(t)$	$f(t) = f_v \frac{dx(t)}{dt}$	$f_v s$
<p>Mass</p> 	$f(t) = M \frac{dv(t)}{dt}$	$f(t) = M \frac{d^2 x(t)}{dt^2}$	$M s^2$

Pemodelan Sistem Mekanik

Sistem Rotasi

Component	Torque-angular velocity	Torque-angular displacement	Impedance $Z_M(s) = T(s)/\theta(s)$
<p>Spring K</p>	$T(t) = K \int_0^t \omega(\tau) d\tau$	$T(t) = K\theta(t)$	K
<p>Viscous damper D</p>	$T(t) = D\omega(t)$	$T(t) = D \frac{d\theta(t)}{dt}$	Ds
<p>Inertia J</p>	$T(t) = J \frac{d\omega(t)}{dt}$	$T(t) = J \frac{d^2\theta(t)}{dt^2}$	Js^2

Ilustrasi



Sistem massa pegas peredam

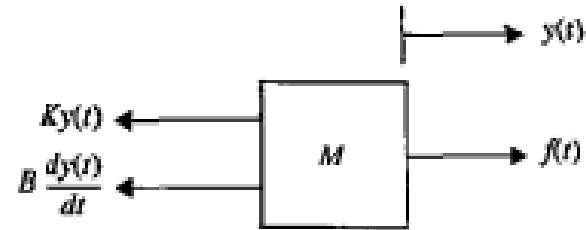


Diagram Benda Bebas

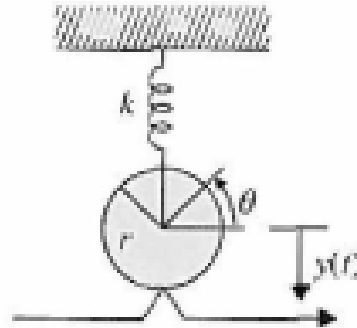
Persamaan kesetimbangan

$$f(t) - B \frac{dy(t)}{dt} - Ky(t) = M \frac{d^2y(t)}{dt^2}$$

Dengan transformasi Laplace dengan asumsi kondisi awal nol

$$\longrightarrow \frac{Y(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K}$$

Latihan

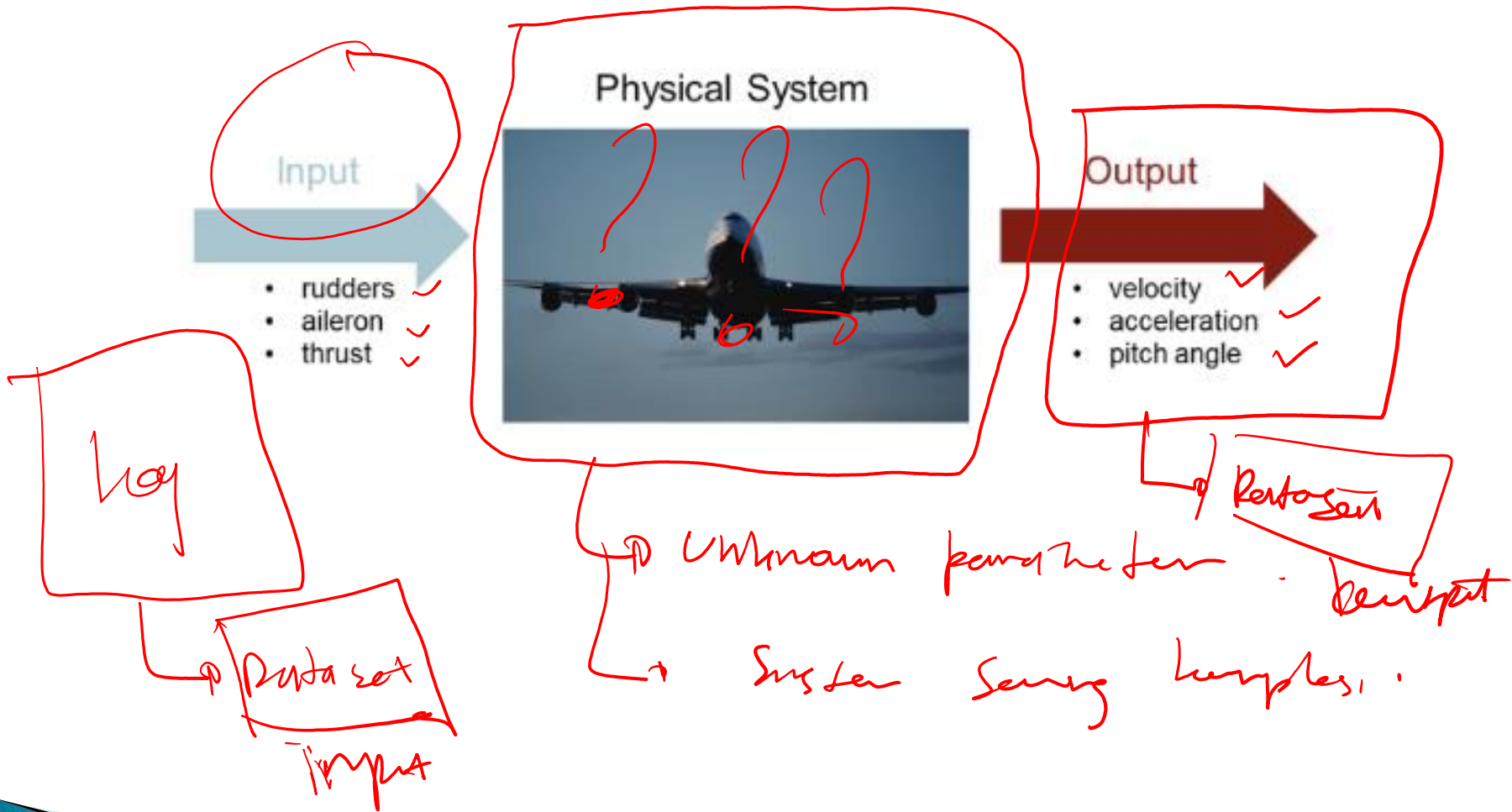


Sistem suspensi kendaraan yang melewati jalan yang bergelombang,
Tentukan fungsi transfer sistem tersebut (output $y(t)$)

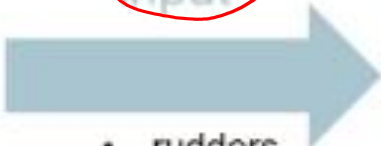
Kesimpulan

- ▶ Transformasi Laplace digunakan untuk mencari solusi persamaan diferensial dengan menjadikannya menjadi persamaan aljabar yang dapat dimanipulasi dengan mudah
- ▶ Partial Fraction Expansion digunakan untuk memecahkan fungsi rasional ke dalam komponen-komponen akar-akarnya
- ▶ Pemodelan sistem elektrik : Hukum Kirchoff dan Hukum Ohm
- ▶ Pemodelan sistem mekanik : Hukum Newton

Black-Box Methode



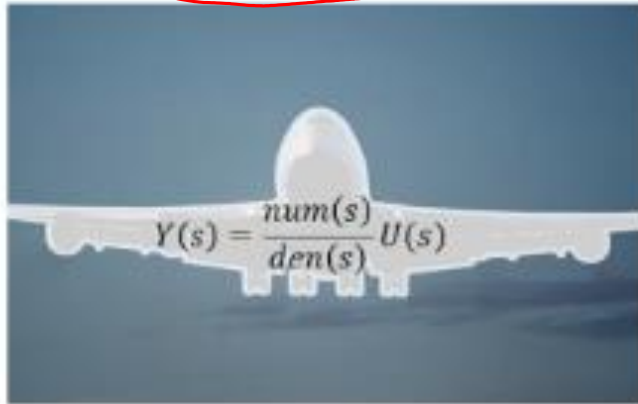
u
Input



- rudders
- aileron
- thrust



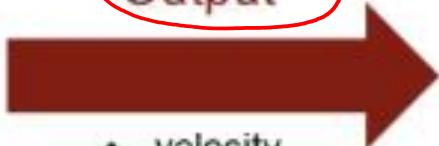
Linear Model



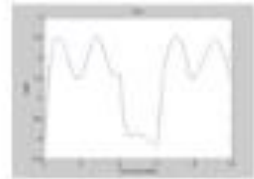
u, y: measured time or frequency domain signals

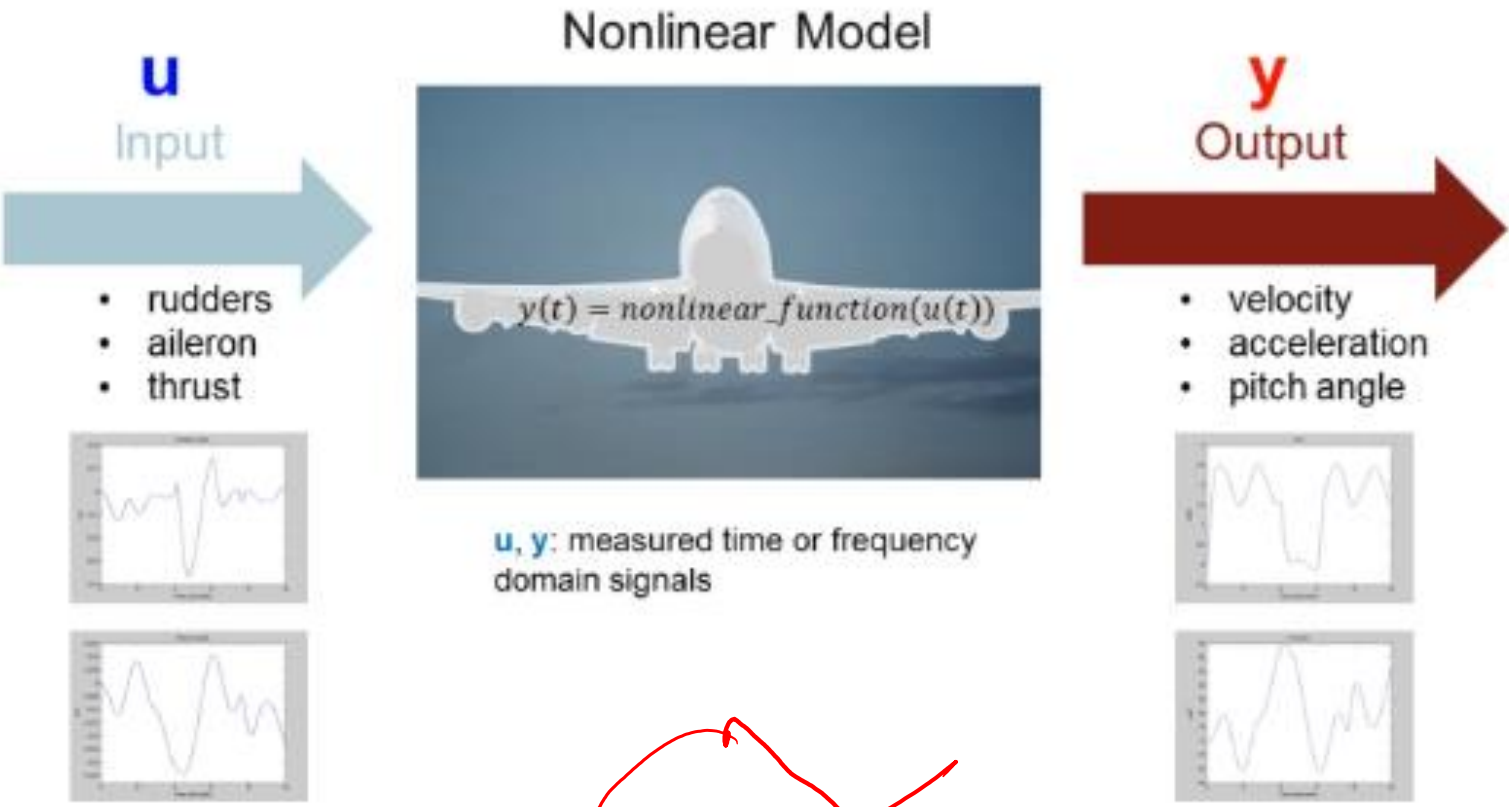
(t)
 s

y
Output

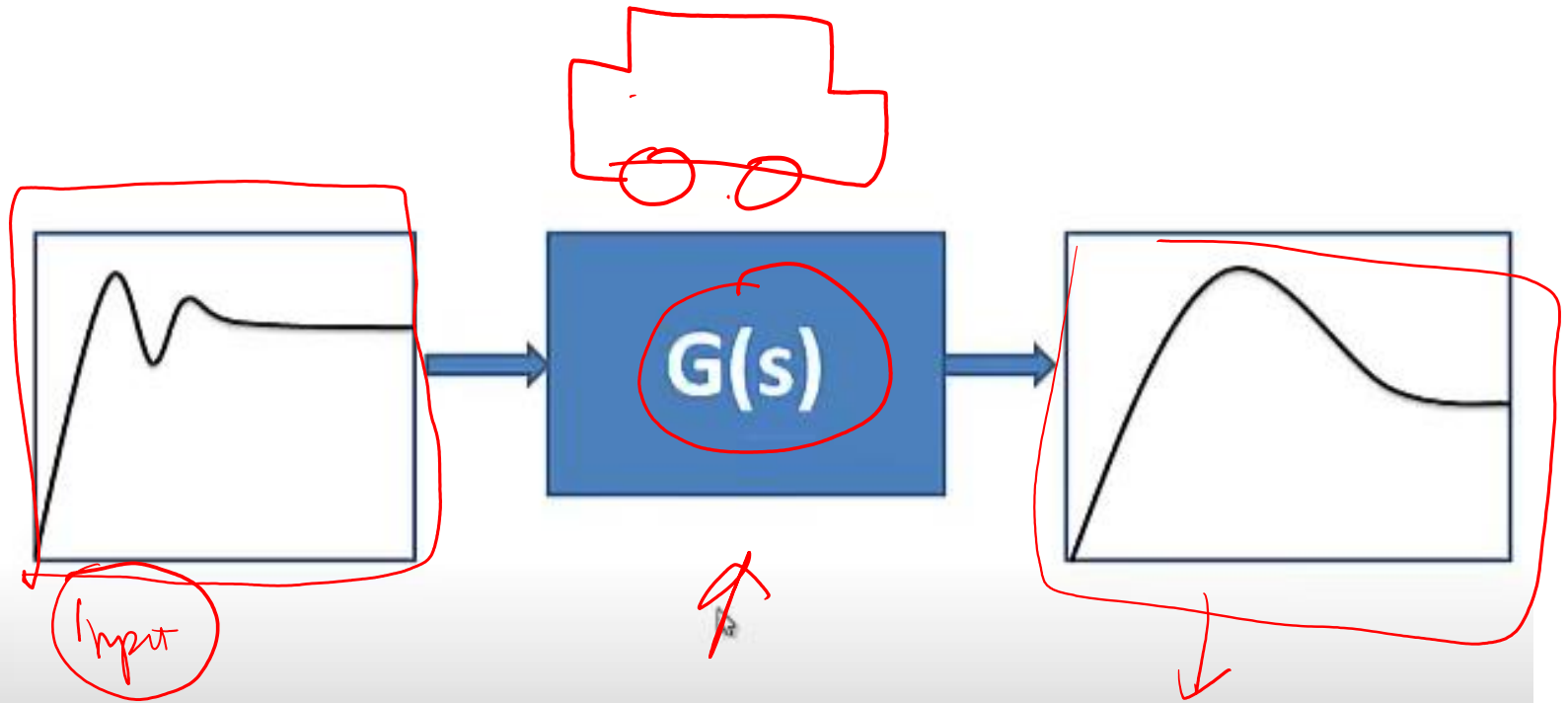


- velocity
- acceleration
- pitch angle





Handwritten red scribble: A red circle with a square inside it, and some lines extending from the bottom left and right, possibly representing a transfer function or a specific signal.



Input

Disturbance input

Physical system
(plant).
Disturbance output

• DC MOTOR SPEED MODELING

By transfer function

$$T = K_t i$$

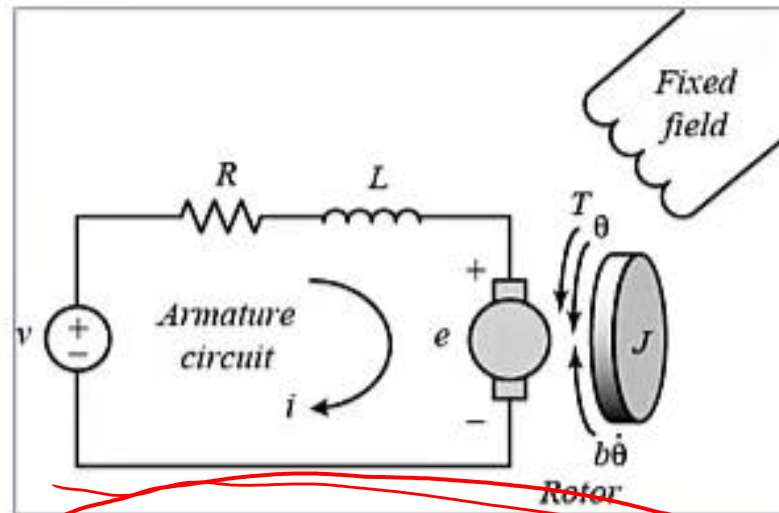
$$e = K_e \dot{\Theta}$$

$$J\ddot{\Theta} + b\dot{\Theta} = Ki$$

$$L \frac{di}{dt} + Ri = V - K \dot{\Theta}$$

$$s(Js + b)\Theta(s) = KI(s)$$

$$(Ls + R)I(s) = V(s) - Ks\Theta(s)$$



$$\frac{\dot{\Theta}(s)}{V(s)} = \frac{K}{(Js + b)(Ls + R) + K^2}$$

physical parameters are:

(J)	moment of inertia of the rotor	0.01 kg.m ²	✓
(b)	motor viscous friction constant	0.1 N.m.s	✓
(Ke)	electromotive force constant	0.01 V/rad/sec	✓
(Kt)	motor torque constant	0.01 N.m/Amp	✓
(R)	electric resistance	1 Ohm	✓
(L)	electric inductance	0.5 H	✓

The screenshot shows the MATLAB R2015b software interface. The main window displays the script file 'model.m' with the following code:

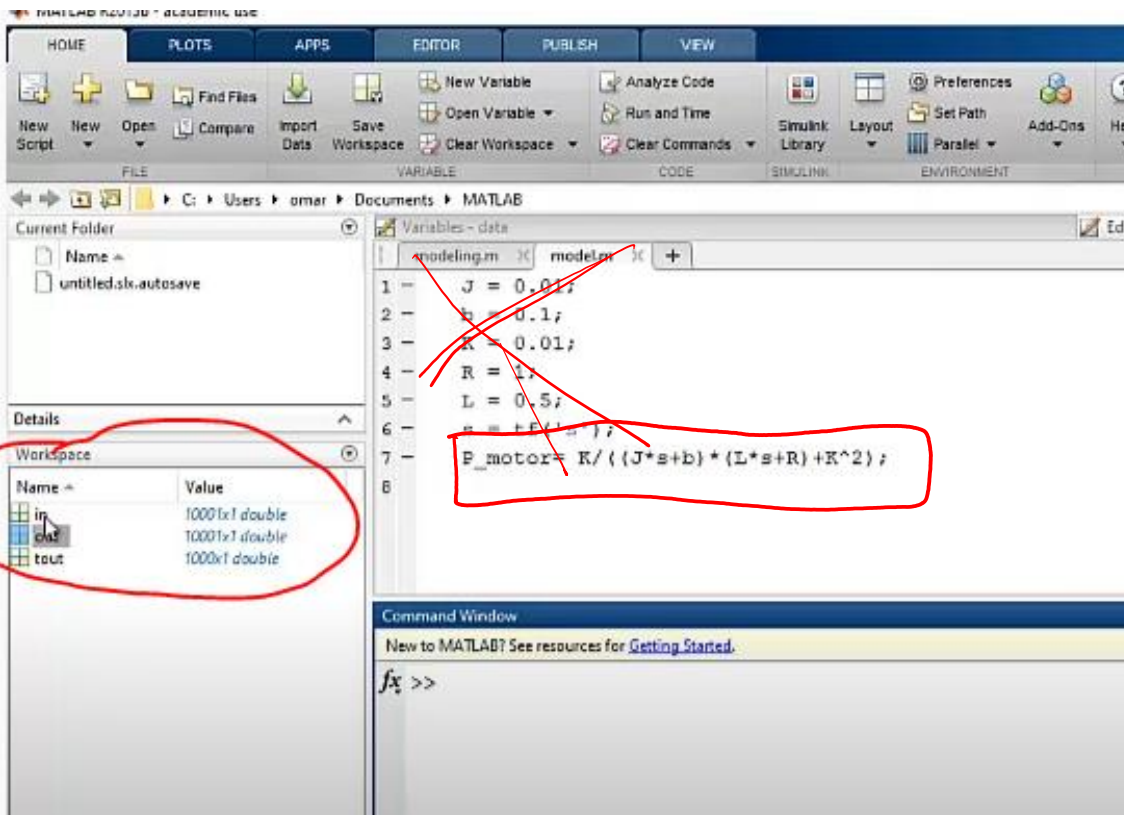
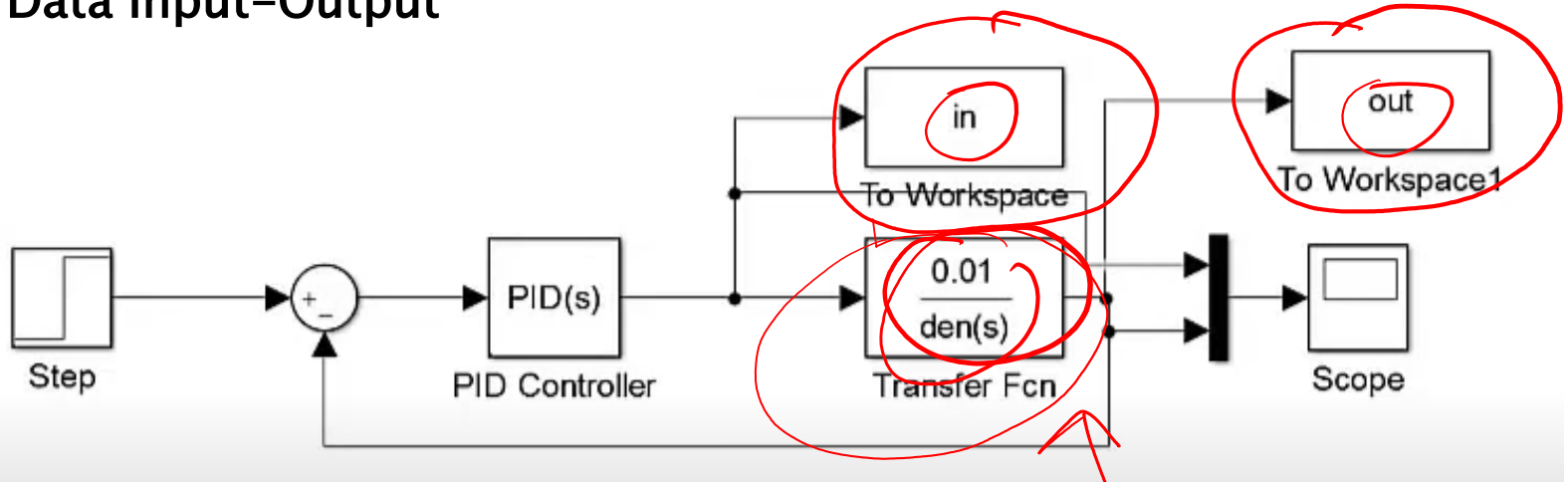
```
1 J = 0.01;  
2 b = 0.1;  
3 K = 0.01;  
4 R = 1;  
5 L = 0.5;  
6 s = tf('s');  
7 P_motor = K / ((J*s+b) * (L*s+R) + K^2);  
8
```

The code is highlighted with a red box. The Command Window at the bottom shows the prompt 'fx >>'.

The Workspace window on the left shows the following variables and their values:

Name	Value
a	[1,2.0121e+03,2.4120...
ans	4x4 double
b	[0,-9.9800e-04,0.0040...
den	[0.0050,0.0600,0.1001]
in	1x1 double timeseries
num	0.0100
out	1x1 double timeseries
ss1	1x1 idss
sys	1x1 tf
sysa	1x1 tf
test1d	501x1 double
t1	1x1 idtf
t4	1x1 idtf
tout	1000x1 double
yin	500x1 double
yout	500x1 double

Retrive Data Input-Output



T(s)?



Using "ident" in MATLAB

The image shows the MATLAB System Identification tool interface. The main window is titled "System Identification - Untitled" and contains several sections:

- Import data:** A grid of buttons for importing data. Red handwritten numbers 1, 2, 3, and 4 are written on the first four buttons. A red arrow points from the number 4 to the "To Workspace" button.
- Operations:** Includes a "Preprocess" dropdown and an "Estimate" dropdown.
- Model Views:** A grid of buttons for selecting model views. A red arrow points from the "Model output" checkbox to the "ident" command in the Command Window.
- Data Views:** Includes checkboxes for "Time plot", "Data spectra", and "Frequency function".
- Workspace Variable:** A section with "Input" and "Output" fields. The "Input" field contains "in" and the "Output" field contains "out", both circled in red.
- Data Information:** Includes fields for "Data name" (containing "test", circled in red), "Starting time" (0), and "Sample time" (0.001).

The Command Window at the bottom shows the following text:

```
New to MATLAB? See resources for Getting Started.  
>> ident  
fx >>
```

Red annotations include circles around the "ident" command and the "Data name" field, and arrows pointing from the "To Workspace" button and the "Model output" checkbox to the Command Window.

Training Data to Get Transfer Function

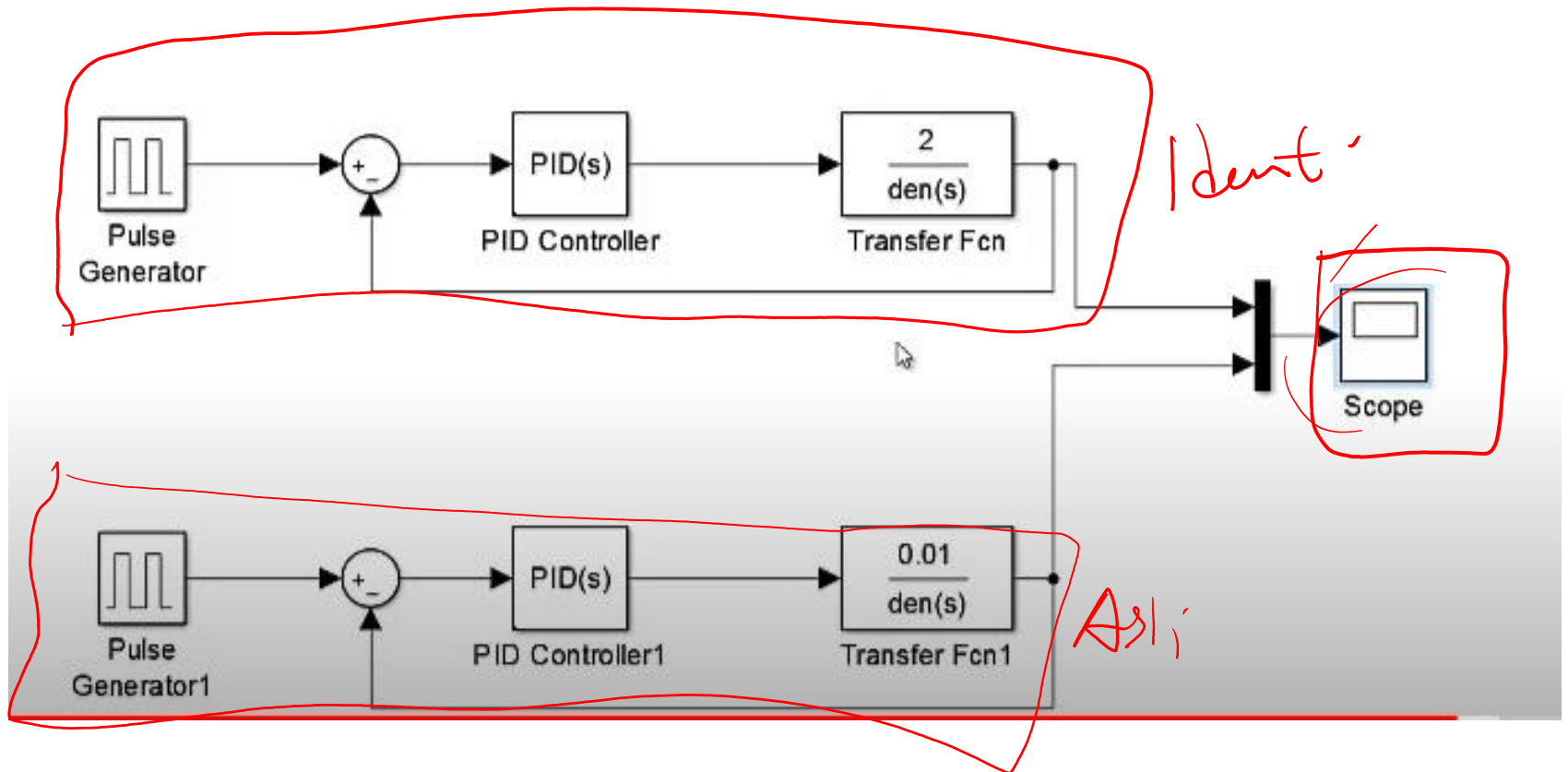
The image displays the MATLAB System Identification software interface. The main window, titled "Plant Identification Progress", shows the results of a transfer function identification process. The "Estimation Progress" table lists 14 iterations with various numerical values. The "Result" section indicates that the estimation was successful, with a fit to estimation data of 100% and a Final Prediction Error (FPE) of 1.66906e-18. A red checkmark is placed next to the "Result" label.

Overlaid on the main window is the "Data/model Info: tf1" dialog box. This dialog shows the model name as "tf1" and the color as "0.75 0.75 0.75". The transfer function is displayed as $s^2 + 12.05 s + 20.02$. A red circle highlights the text "from input 'u' to output 'y1':" and the transfer function equation. Below the dialog is the "Diary and Notes" window, which contains the following text:

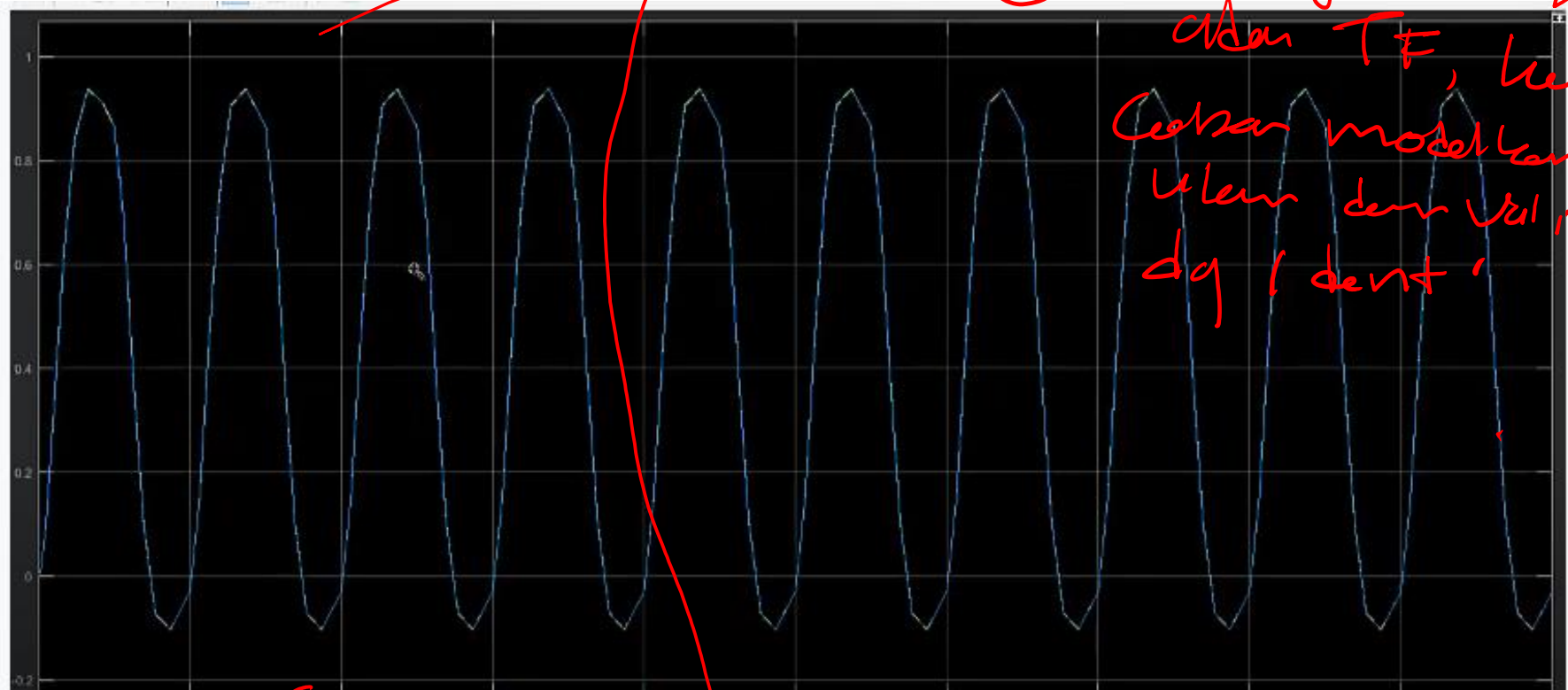
```
% Import test
% Transfer function estimation
Options = tfestOptions;
Options.Display = 'on';
% Est = tfest(test, 2, 0, Options);
```

At the bottom of the main window, the name of the identified model is shown as "Name: tf1".

3. Validasi



Validasi



Sumber

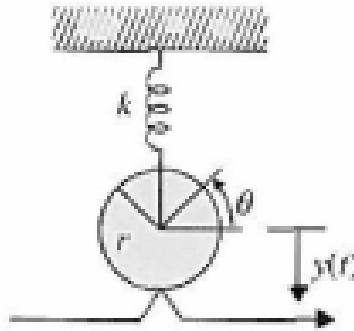
- ①. Simulasikan ulang tutorial
- ②. Coba jurnal/TA atau TF, kembangkan modelkan ulang dan validasi dg identifikasi

Fungsinya

Tutorial :

1. <https://www.mathworks.com/videos/system-identification-toolbox-overview-64920.html>
2. https://www.youtube.com/watch?v=Zez7kcbRKSo&ab_channel=MATLAB

Tugas :



1. Sistem suspensi kendaraan yang melewati jalan yang bergelombang, Tentukan fungsi transfer sistem tersebut (output $y(t)$)

2. Lakukan pemodelan Motor DC dengan System Identification Toolbox : https://www.youtube.com/watch?v=Zez7kcbRKSo&ab_channel=MATLAB