Pemodelan Sistem

Identifikasi Sistem

- White Box → Analisis Matematis dari parameter system.
- Grey Box → Kombinasi.
- Black Box → Menggunakan analisis dari input-output system.

Review Matematika

- *Transformasi Laplace*

  Problem dalam sistem kontrol adalah problem dinamik yg biasanya dideskripsikan dalam persamaan diferensial. Dengan transformasi Laplace, solusi persamaan diferensial lebih sederhana dan mudah
- *Partial Fraction Expansion*

  karena berhadapan dengan pecahan simbolik (fungsi rasional) maka perlu metode ini untuk menyederhanakan persamaan

Transformasi Laplace

Jika terdapat fungsi f(t) maka Transformasi Laplace dari f(t) adalah F(s) →

$$\mathscr{L}[f(t)] = F(s) = \int_{0-}^{\infty} f(t)e^{-st}\,dt$$

$$\mathscr{L}^{-1}[F(s)] = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} F(s)e^{st}ds = f(t)u(t)$$

← Jika terdapat fungsi F(s) maka inverse Transformasi Laplace kembali ke f(t) adalah

$$\begin{aligned} u(t) &= 1 \quad & t > 0 \\ &= 0 \quad & t < 0 \end{aligned}$$

Transformasi Laplace

PROBLEM: Find the Laplace transform of $f(t) = Ae^{-at}u(t)$.

SOLUTION: Since the time function does not contain an impulse function, we can replace the lower limit of Eq. (2.1) with 0. Hence,

$$F(s) = \int_0^\infty f(t)e^{-st}\,dt = \int_0^\infty Ae^{-at}e^{-st}\,dt = A\int_0^\infty e^{-(s+a)t}\,dt$$

$$= -\frac{A}{s+a}e^{-(s+a)t}\Big|_{t=0}^{\infty} = \frac{A}{s+a} \qquad (2.3)$$

TABLE 2.1 Laplace transform table

Item no.	$f(t)$	$F(s)$
1.	$\delta(t)$	1
2.	$u(t)$	$\frac{1}{s}$
3.	$tu(t)$	$\frac{1}{s^2}$
4.	$t^n u(t)$	$\frac{n!}{s^n+1}$
5.	$e^{-at}u(t)$	$\frac{1}{s+a}$
6.	$\sin \omega t u(t)$	$\frac{\omega}{s^2+\omega^2}$
7.	$\cos \omega t u(t)$	$\frac{s}{s^2+\omega^2}$

PROBLEM: Find the inverse Laplace transform of $F_1(s) = 1/(s+3)^2$.

SOLUTION: For this example we make use of the frequency shift theorem, Item 4 of Table 2.2, and the Laplace transform of $f(t) = tu(t)$, Item 3 of Table 2.1. If the inverse transform of $F(s) = 1/s^2$ is $tu(t)$, the inverse transform of $F(s+a) = 1/(s+a)^2$ is $e^{-at}tu(t)$. Hence, $f_1(t) = e^{-3t}tu(t)$.

Sifat-sifat Transformasi Laplace

Item no.	Theorem		Name
1.	$\mathscr{L}[f(t)] = F(s)$	$= \int_{0-}^{\infty} f(t)e^{-st}dt$	Definition
2.	$\mathscr{L}[kf(t)]$	$= kF(s)$	Linearity theorem
3.	$\mathscr{L}[f_1(t) + f_2(t)]$	$= F_1(s) + F_2(s)$	Linearity theorem
4.	$\mathscr{L}[e^{-at}f(t)]$	$= F(s+a)$	Frequency shift theorem
5.	$\mathscr{L}[f(t-T)]$	$= e^{-sT}F(s)$	Time shift theorem
6.	$\mathscr{L}[f(at)]$	$= \frac{1}{a}F\left(\frac{s}{a}\right)$	Scaling theorem
7.	$\mathscr{L}\left[\frac{df}{dt}\right]$	$= sF(s) - f(0-)$	Differentiation theorem
8.	$\mathscr{L}\left[\frac{d^2f}{dt^2}\right]$	$= s^2F(s) - sf(0-) - f'(0-)$	Differentiation theorem
9.	$\mathscr{L}\left[\frac{d^nf}{dt^n}\right]$	$= s^nF(s) - \sum_{k=1}^{n} s^{n-k}f^{k-1}(0-)$	Differentiation theorem
10.	$\mathscr{L}\left[\int_{0-}^{t} f(\tau)d\tau\right]$	$= \frac{F(s)}{s}$	Integration theorem
11.	$f(\infty)$	$= \lim_{s\to 0} sF(s)$	Final value theorem[1]
12.	$f(0+)$	$= \lim_{s\to\infty} sF(s)$	Initial value theorem[2]

Partial Fraction Expansion

Misalnya terdapat fungsi dalam s dari hasil transformasi Laplace → $G(s) = \frac{Q(s)}{P(s)}$

di mana Q(s) dan P(s) adalah polinom dalam s

Akar-akar (yang membuat persamaan menjadi nol) dari Q(s) disebut *zero* dari G(s)

Akar-akar (yang membuat persamaan menjadi nol) dari P(s) disebut *pole* dari G(s)

Jika diasumsikan orde (pangkat tertinggi) dari P(s) lebih besar dari Q(s) maka

$$P(s) = s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0$$

Dimana a_1, a_0 dst adalah koefisien real

maka terdapat beberapa jenis penyederhanaan

Partial Fraction Expansion

1. Jika pole bilangan real dan berbeda

$$G(s) = \frac{Q(s)}{P(s)} = \frac{Q(s)}{(s+s_1)(s+s_2)\cdots(s+s_n)}$$

$$s_1 \neq s_2 \neq \cdots \neq s_n.$$

maka

$$G(s) = \frac{K_{s1}}{s+s_1} + \frac{K_{s2}}{s+s_2} + \cdots + \frac{K_{sn}}{s+s_n}$$

$$K_{s1} = \left[(s+s_1)\frac{Q(s)}{P(s)}\right]\Bigg|_{s=-s_1} = \frac{Q(-s_1)}{(s_2-s_1)(s_3-s_1)\cdots(s_n-s_1)}$$

K_{s2}, K_{s3} dst diperoleh dengan cara yang sama

Ilustrasi

$$G(s) = \frac{5s+3}{(s+1)(s+2)(s+3)} = \frac{5s+3}{s^3+6s^2+11s+6}$$

Dapat dituliskan $$G(s) = \frac{K_{-1}}{s+1} + \frac{K_{-2}}{s+2} + \frac{K_{-3}}{s+3}$$

$$K_{-1} = [(s+1)G(s)]\Big|_{s=-1} = \frac{5(-1)+3}{(2-1)(3-1)} = -1$$

$$K_{-2} = [(s+2)G(s)]\Big|_{s=-2} = \frac{5(-2)+3}{(1-2)(3-2)} = 7$$

$$K_{-3} = [(s+3)G(s)]\Big|_{s=-3} = \frac{5(-3)+3}{(1-3)(2-3)} = -6$$

→ $$G(s) = \frac{-1}{s+1} + \frac{7}{s+2} - \frac{6}{s+3}$$

↑

Bentuk yg mudah untuk diubah lagi ke t dengan Inverse transformasi Laplace

Partial Fraction Expansion

2. Jika terdapat akar yang berulang

$$G(s)=\frac{Q(s)}{P(s)}=\frac{Q(s)}{(s+s_1)(s+s_2)\cdots(s+s_{n-r})(s+s_i)^r}$$

$$G(s)=\frac{K_{s1}}{s+s_1}+\frac{K_{s2}}{s+s_2}+\cdots+\frac{K_{s(n-r)}}{s+s_{n-r}}$$

$$|\leftarrow n-r \text{ terms of simple poles} \rightarrow|$$

$$+\frac{A_1}{s+s_i}+\frac{A_2}{(s+s_i)^2}+\cdots+\frac{A_r}{(s+s_i)^r}$$

$$|\leftarrow r \text{ terms of repeated poles} \rightarrow|$$

$$A_r=\left[(s+s_i)^r G(s)\right]\Big|_{s=-s_i}$$

$$A_{r-1}=\frac{d}{ds}\left[(s+s_i)^r G(s)\right]\Big|_{s=-s_i}$$

$$A_{r-2}=\frac{1}{2!}\frac{d^2}{ds^2}\left[(s+s_i)^r G(s)\right]\Big|_{s=-s_i}$$

$$\vdots$$

$$A_1=\frac{1}{(r-1)!}\frac{d^{r-1}}{ds^{r-1}}\left[(s+s_i)^r G(s)\right]\Big|_{s=-s_i}$$

Ilustrasi

$$G(s) = \frac{1}{s(s+1)^3(s+2)} = \frac{1}{s^5 + 5s^4 + 9s^3 + 7s^2 + 2s}$$

$$G(s) = \frac{K_0}{s} + \frac{K_{-2}}{s+2} + \frac{A_1}{s+1} + \frac{A_2}{(s+1)^2} + \frac{A_3}{(s+1)^3}$$

$$K_0 = \left[sG(s)\right]\Big|_{s=0} = \frac{1}{2}$$

$$K_{-2} = \left[(s+2)G(s)\right]\Big|_{s=-2} = \frac{1}{2}$$

$$A_3 = \left[(s+1)^3G(s)\right]\Big|_{s=-1} = -1$$

$$A_2 = \frac{d}{ds}\left[(s+1)^3G(s)\right]\Big|_{s=-1} = \frac{d}{ds}\left[\frac{1}{s(s+2)}\right]\Big|_{s=-1} = 0$$

$$A_1 = \frac{1}{2!}\frac{d^2}{ds^2}\left[(s+1)^3G(s)\right]\Big|_{s=-1} = \frac{1}{2}\frac{d^2}{ds^2}\left[\frac{1}{s(s+2)}\right]\Big|_{s=-1} = -1$$

maka hasilnya

$$G(s) = \frac{1}{2s} + \frac{1}{2(s+2)} - \frac{1}{s+1} - \frac{1}{(s+1)^3}$$

Partial Fraction Expansion

3. Jika polenya adalah pasangan bilangan kompleks

$$s = -\sigma + j\omega \quad \text{and} \quad s = -\sigma - j\omega$$

Koefisien bisa dicari dengan →

$$K_{-\sigma+j\omega} = (s + \sigma - j\omega)G(s)\Big|_{s=-\sigma+j\omega}$$

$$K_{-\sigma-j\omega} = (s + \sigma + j\omega)G(s)\Big|_{s=-\sigma-j\omega}$$

Ilustrasi

$$F(s) = \frac{3}{s(s^2+2s+5)} = \frac{3}{s(s+1+j2)(s+1-j2)}$$
$$= \frac{K_1}{s} + \frac{K_2}{s+1+j2} + \frac{K_3}{s+1-j2}$$

$$K_2 = \left.\frac{3}{s(s+1-j2)}\right|_{s\to -1-j2} = -\frac{3}{20}(2+j1)$$

K_3 adalah pasangan bilangan kompleks dari K_2

K_1 bisa dicari dari cara sebelumnya

$$F(s) = \frac{3/5}{s} - \frac{3}{20}\left(\frac{2+j1}{s+1+j2} + \frac{2-j1}{s+1-j2}\right)$$

Dengan inverse transformasi Laplace →

$$f(t) = \frac{3}{5} - \frac{3}{20}\left[(2+j1)e^{-(1+j2)t} + (2-j1)e^{-(1-j2)t}\right]$$
$$= \frac{3}{5} - \frac{3}{20}e^{-t}\left[4\left(\frac{e^{j2t}+e^{-j2t}}{2}\right) + 2\left(\frac{e^{j2t}+e^{-j2t}}{2j}\right)\right]$$

Latihan

PROBLEM: Find the Laplace transform of $f(t) = te^{-5t}$.

PROBLEM: Find the inverse Laplace transform of $F(s) = 10/[s(s+2)(s+3)^2]$.

Fungsi Transfer

$$\frac{C(s)}{R(s)} = G(s) = \frac{(b_m s^m + b_{m-1} s^{m-1} + \cdots + b_0)}{(a_n s^n + a_{n-1} s^{n-1} + \cdots + a_0)}$$

G(s) adalah fungsi transfer

Output dapat dicari dengan

$$C(s) = R(s)G(s)$$

Ilustrasi

Tentukan fungsi transfer dari

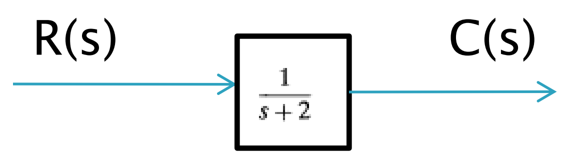
$$\frac{dc(t)}{dt} + 2c(t) = r(t)$$

Dengan transformasi Laplace

$$sC(s) + 2C(s) = R(s)$$

Fungsi transfer

$$G(s) = \frac{C(s)}{R(s)} = \frac{1}{s+2}$$

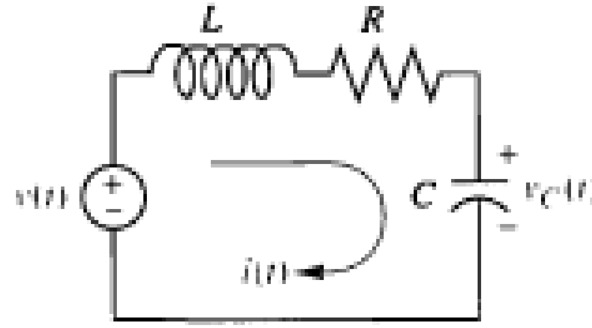


Pemodelan Sistem Elektrik

Component	Voltage-current	Current-voltage	Voltage-charge	Impedance $Z(s) = V(s)/I(s)$	Admittance $Y(s) = I(s)/V(s)$
Capacitor	$v(t) = \frac{1}{C}\int_0^t i(\tau)d\tau$	$i(t) = C\frac{dv(t)}{dt}$	$v(t) = \frac{1}{C}q(t)$	$\frac{1}{Cs}$	Cs
Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R}v(t)$	$v(t) = R\frac{dq(t)}{dt}$	R	$\frac{1}{R} = G$
Inductor	$v(t) = L\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int_0^t v(\tau)d\tau$	$v(t) = L\frac{d^2q(t)}{dt^2}$	Ls	$\frac{1}{Ls}$

Ilustrasi

input



output

Dari hukum Kirchoff (loop) tegangan

$$L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int_0^t i(\tau)d\tau = v(t)$$

$i(t) = dq(t)/dt$

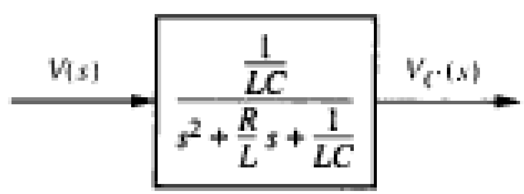
$$L\frac{d^2q(t)}{dt^2} + R\frac{dq(t)}{dt} + \frac{1}{C}q(t) = v(t)$$

Dengan transformasi Laplace

$q(t) = Cv_C(t)$

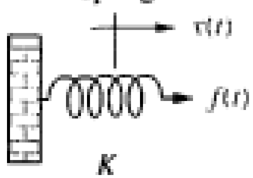
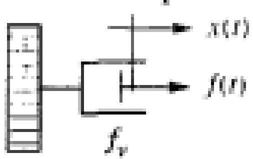
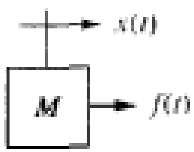
$$LC\frac{d^2v_C(t)}{dt^2} + RC\frac{dv_C(t)}{dt} + v_C(t) = v(t) \rightarrow (LCs^2 + RCs + 1)V_C(s) = V(s)$$

$$\frac{V_C(s)}{V(s)} = \frac{1/LC}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$



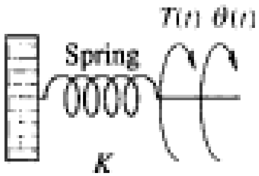
Pemodelan Sistem Mekanik

Sistem Translasi

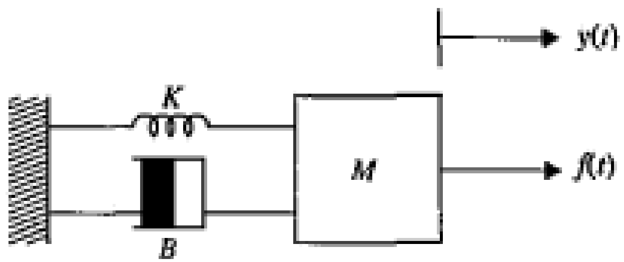
Component	Force-velocity	Force-displacement	Impedence $Z_M(s) = F(s)/X(s)$
Spring 	$f(t) = K\int_0^t v(\tau)d\tau$	$f(t) = Kx(t)$	K
Viscous damper 	$f(t) = f_v v(t)$	$f(t) = f_v \frac{dx(t)}{dt}$	$f_v s$
Mass 	$f(t) = M\frac{dv(t)}{dt}$	$f(t) = M\frac{d^2x(t)}{dt^2}$	Ms^2

Pemodelan Sistem Mekanik

Sistem Rotasi

Component	Torque-angular velocity	Torque-angular displacement	Impedence $Z_M(s) = T(s)/\theta(s)$
	$T(t) = K\int_0^t \omega(\tau)d\tau$	$T(t) = K\theta(t)$	K
	$T(t) = D\omega(t)$	$T(t) = D\frac{d\theta(t)}{dt}$	Ds
	$T(t) = J\frac{d\omega(t)}{dt}$	$T(t) = J\frac{d^2\theta(t)}{dt^2}$	Js^2

Ilustrasi



Sistem massa pegas peredam

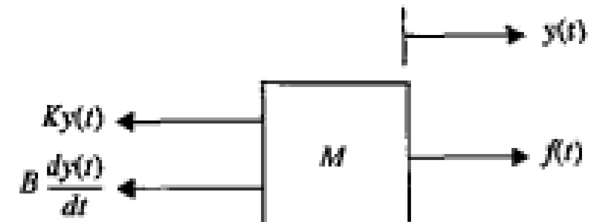


Diagram Benda Bebas

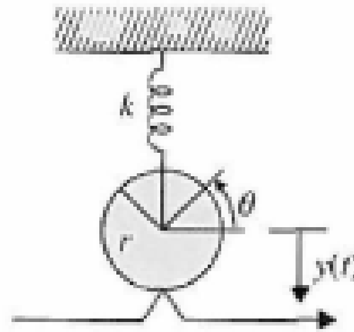
Persamaan kesetimbangan

$$f(t) - B\frac{dy(t)}{dt} - Ky(t) = M\frac{d^2y(t)}{dt^2}$$

Dengan transformasi Laplace dengan asumsi kondisi awal nol

$$\frac{Y(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K}$$

Latihan

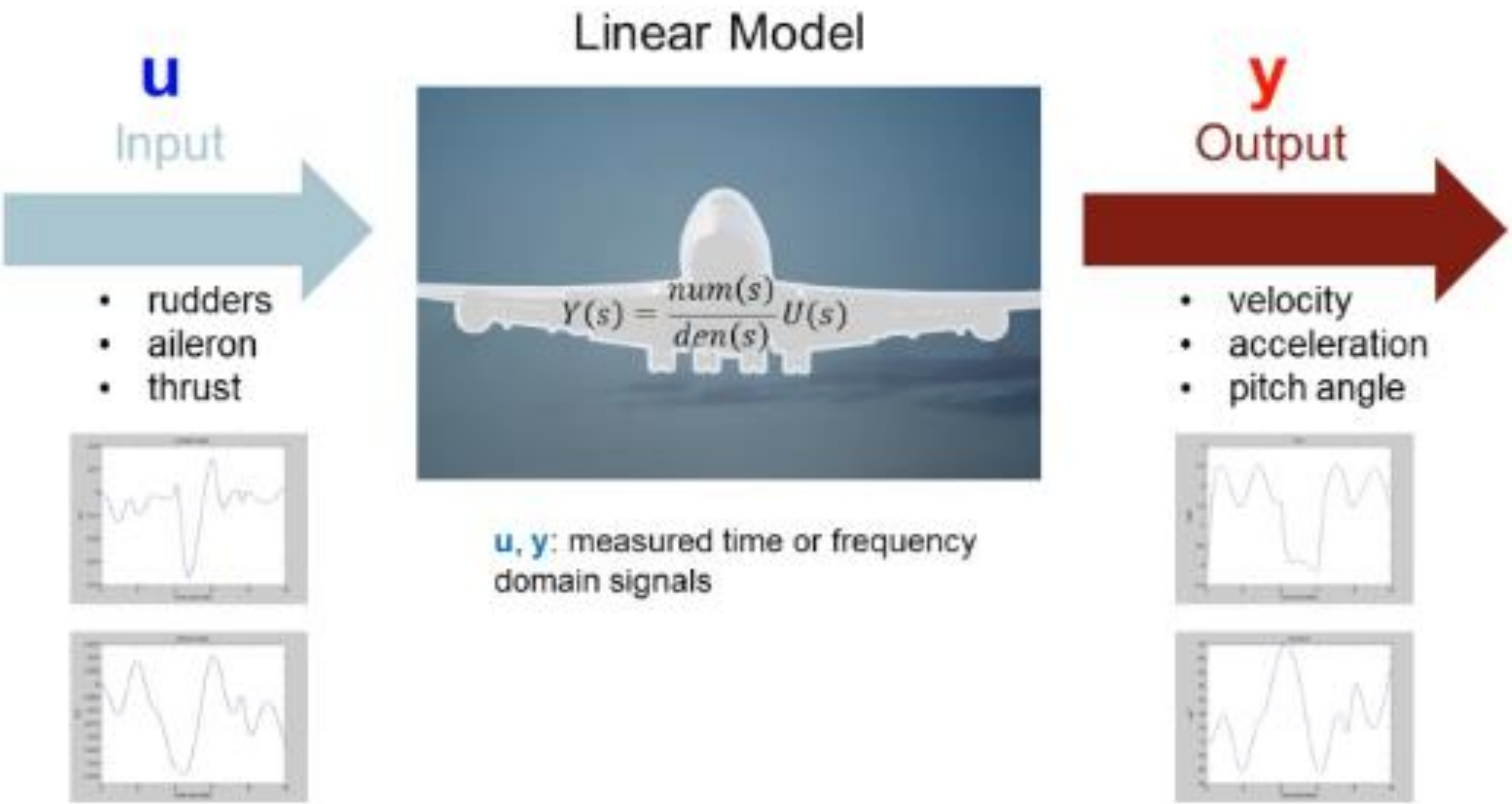


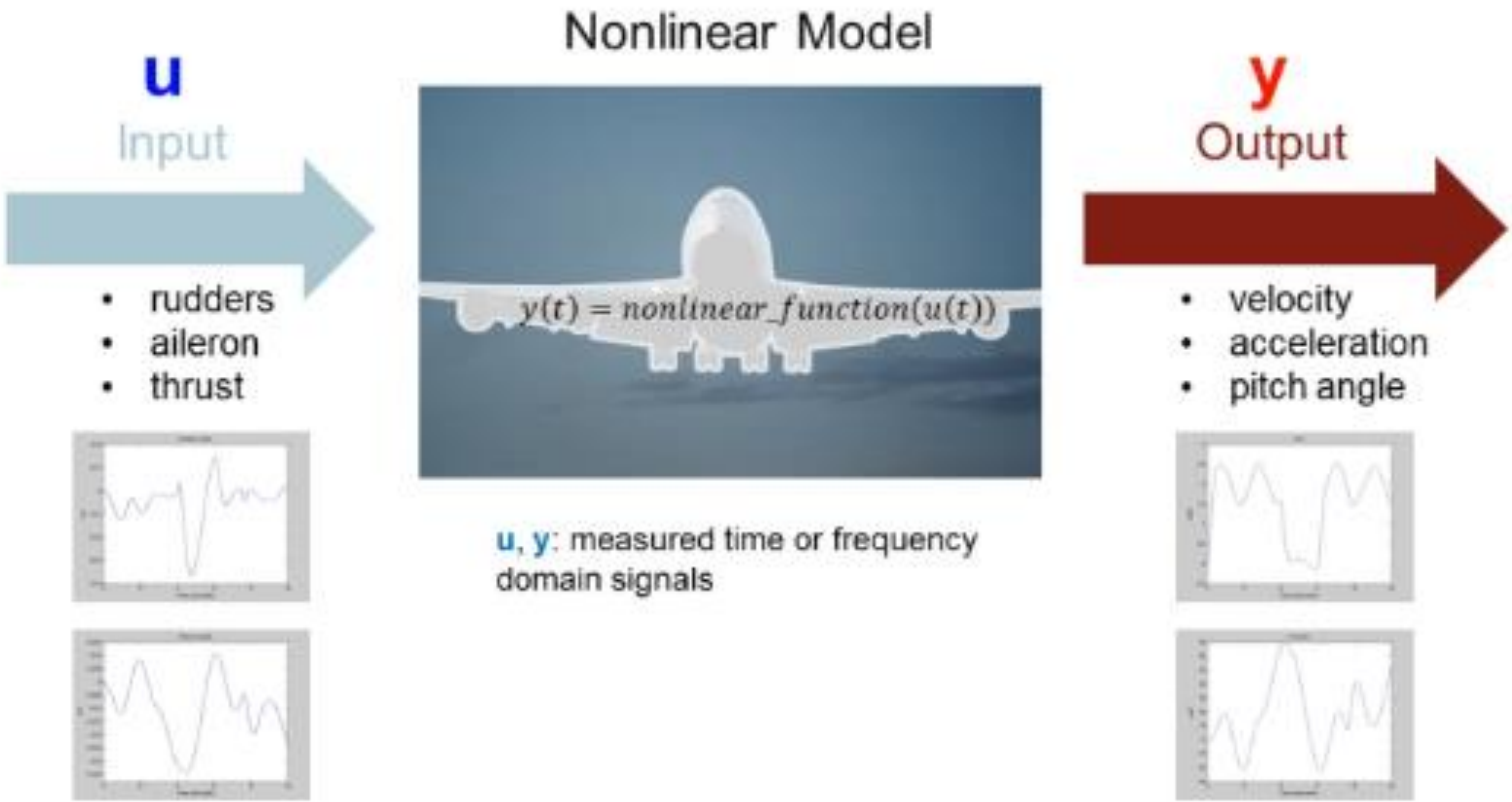
Sistem suspensi kendaraan yang melewati jalan yang bergelombang,
Tentukan fungsi transfer sistem tersebut (output y(t))

Kesimpulan

- Transformasi Laplace digunakan untuk mencari solusi persamaan diferensial dengan menjadikannya menjadi persamaan aljabar yang dapat dimanipulasi dengan mudah
- Partial Fraction Expansion digunakan untuk memecahkan fungsi rasional ke dalam komponen-komponen akar-akarnya
- Pemodelan sistem elektrik : Hukum Kirchoff dan Hukum Ohm
- Pemodelan sistem mekanik : Hukum Newton

Black-Box Methode


Linear Model
u
Input
rudders
aileron
thrust
$Y(s) = \frac{num(s)}{den(s)} U(s)$
y
Output
velocity
acceleration
pitch angle
u, y: measured time or frequency domain signals


Nonlinear Model
u
Input
rudders
aileron
thrust
y(t) = nonlinear_function(u(t))
y
Output
velocity
acceleration
pitch angle
u, y: measured time or frequency domain signals


G(s)

• DC MOTOR SPEED MODELING

By transfer function

$$T = K_t i$$

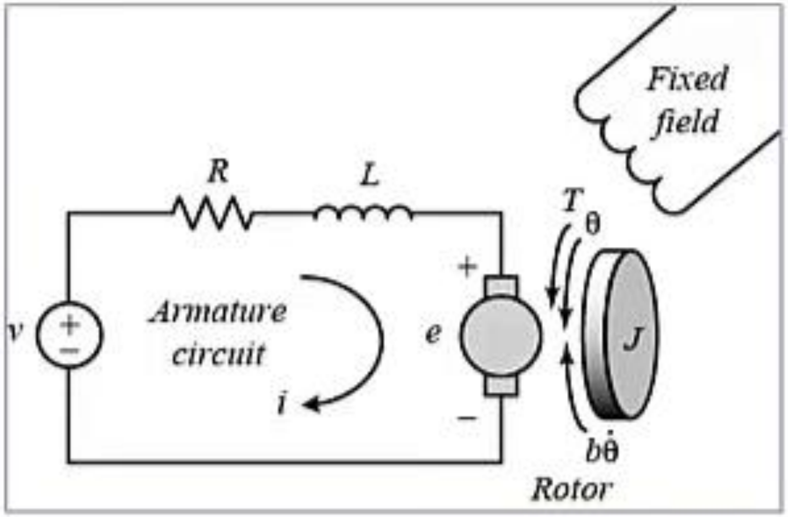
$$e = K_e \dot{\Theta}$$

$$J\ddot{\Theta} + b\dot{\Theta} = Ki$$

$$L\frac{di}{dt} + Ri = V - K\dot{\Theta}$$

$$s(Js + b)\Theta(s) = KI(s)$$

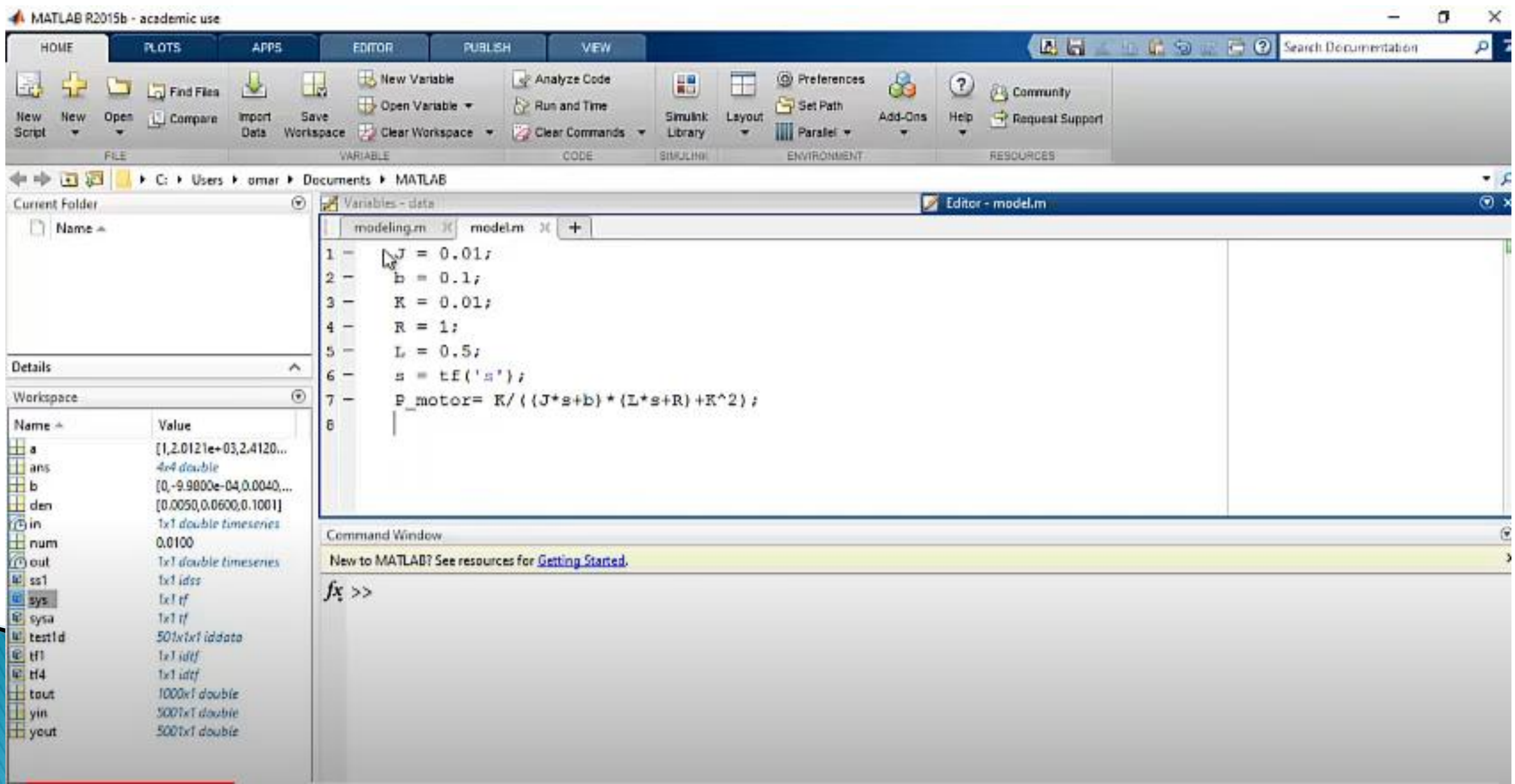
$$(Ls + R)I(s) = V(s) - Ks\Theta(s)$$



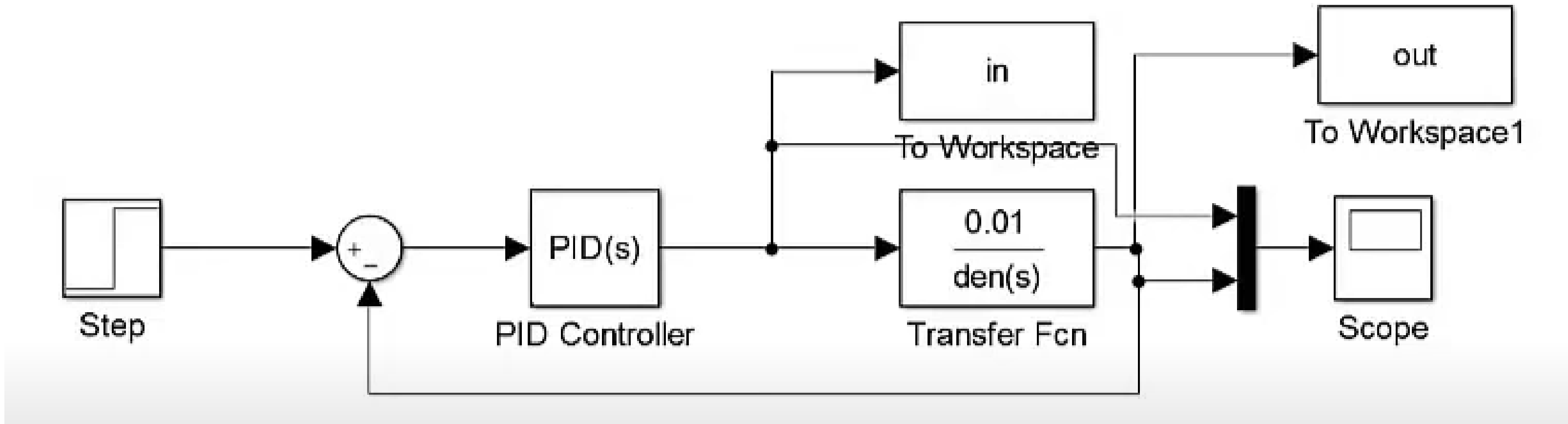
$$\frac{\dot{\Theta}(s)}{V(s)} = \frac{K}{(Js + b)(Ls + R) + K^2}$$

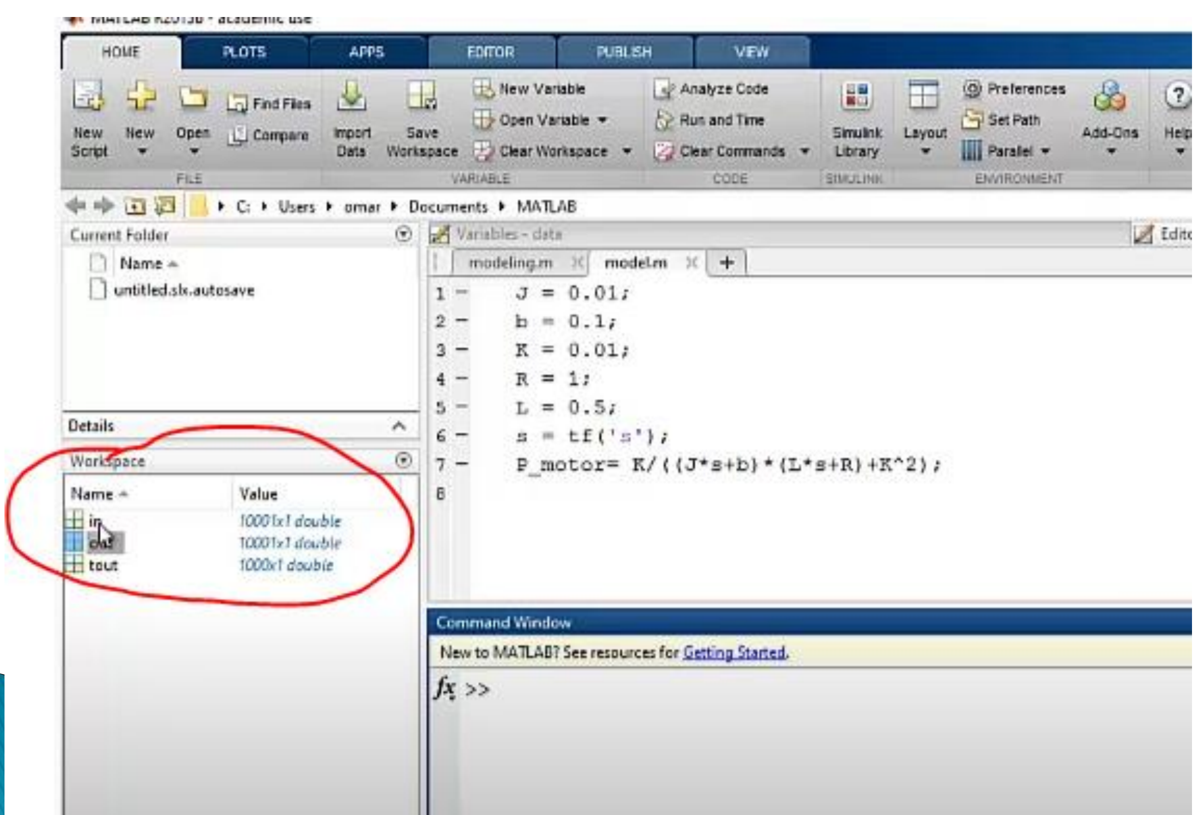
physical parameters are:

(J)	moment of inertia of the rotor	0.01 kg.m^2
(b)	motor viscous friction constant	0.1 N.m.s
(Ke)	electromotive force constant	0.01 V/rad/sec
(Kt)	motor torque constant	0.01 N.m/Amp
(R)	electric resistance	1 Ohm
(L)	electric inductance	0.5 H

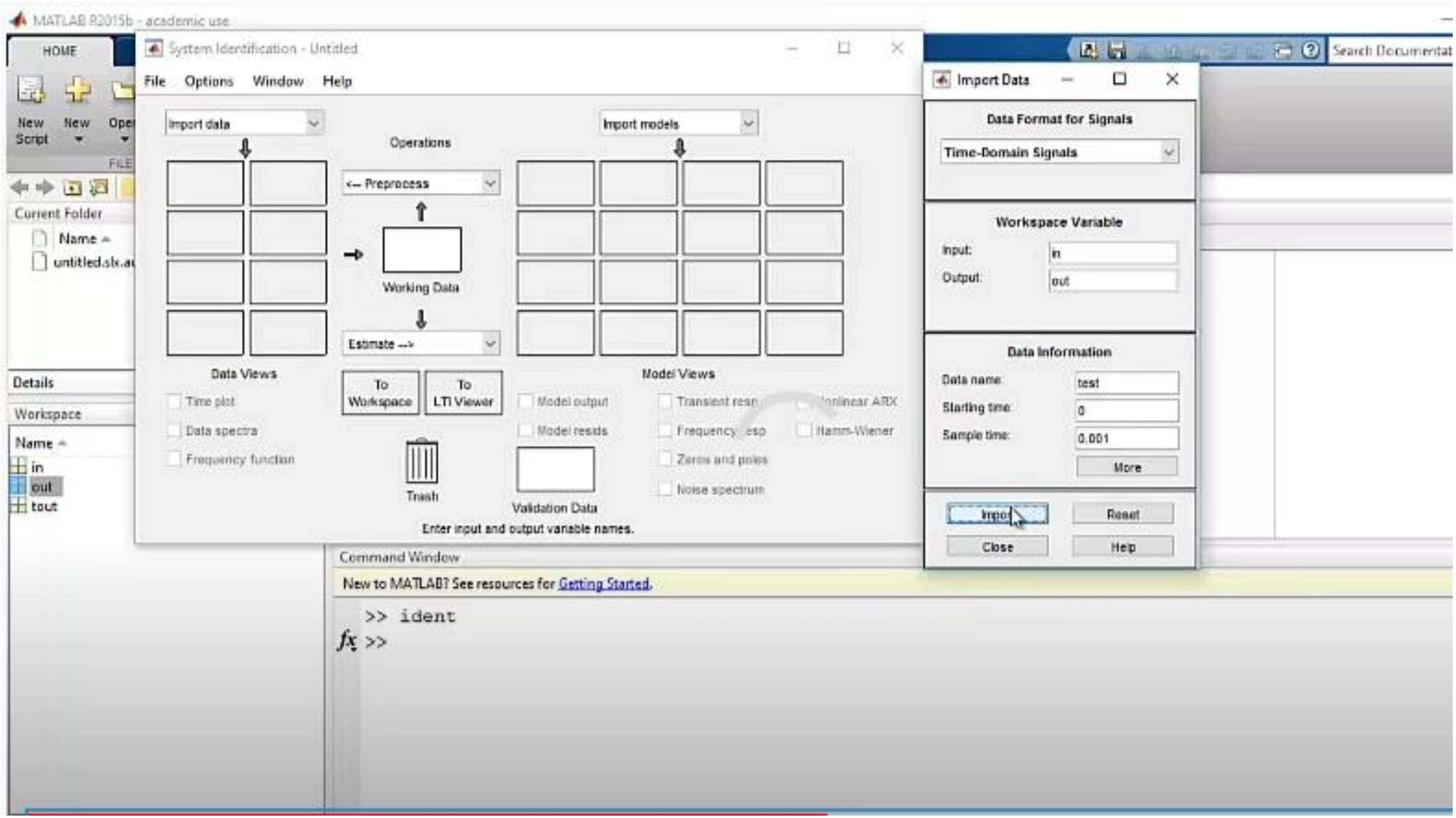


Retrive Data Input-Output

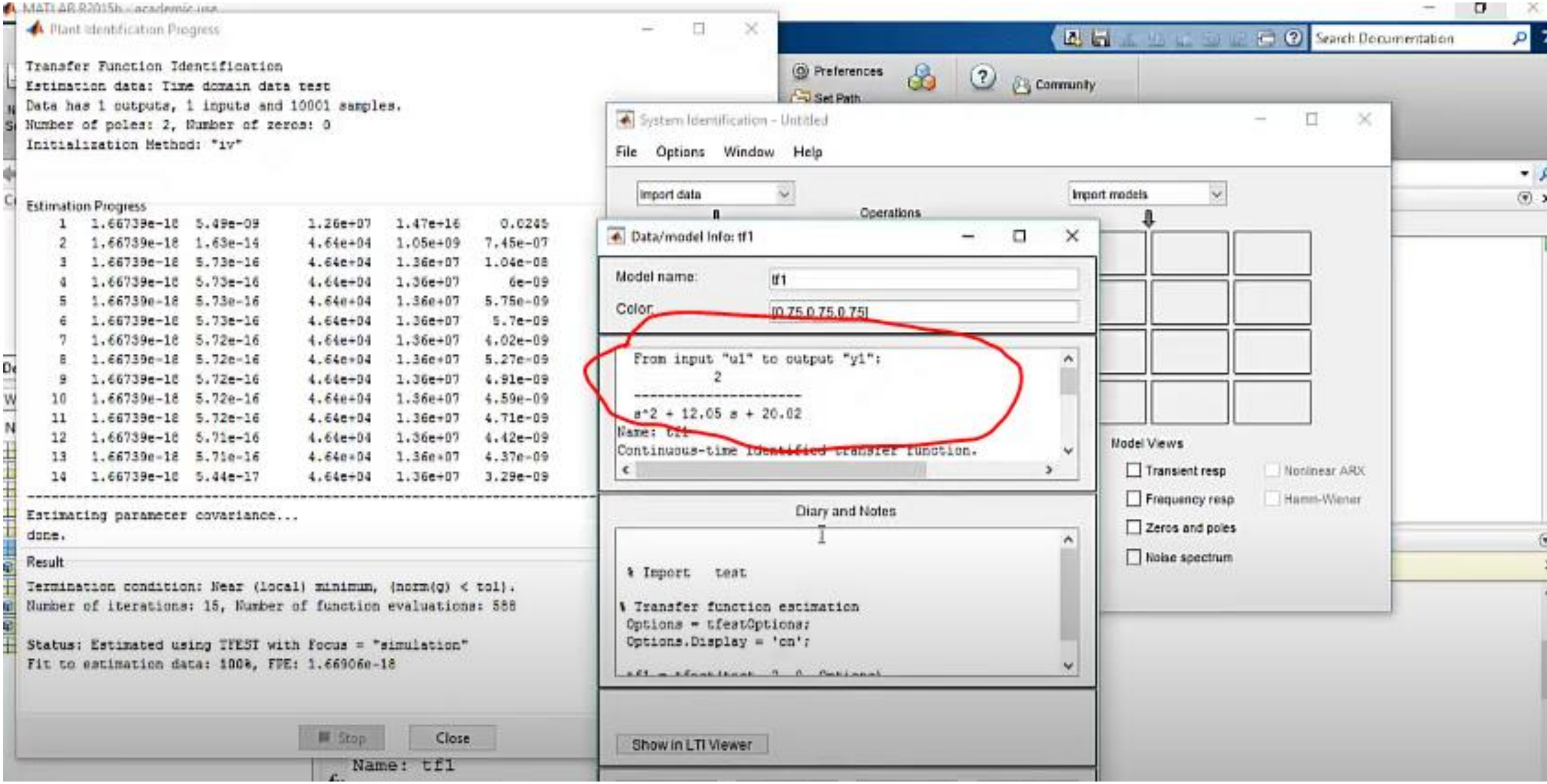




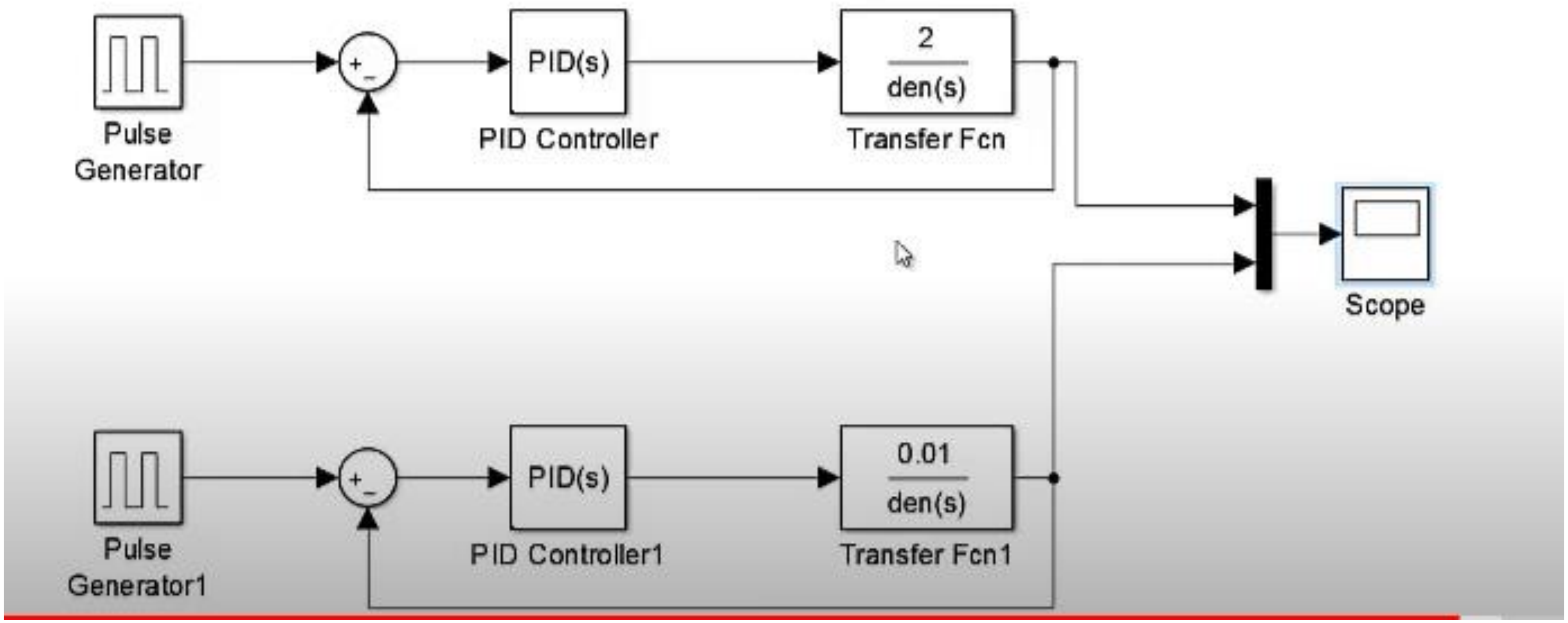
Using “ident” in MATLAB



Training Data to Get Transfer Function



3. Validasi



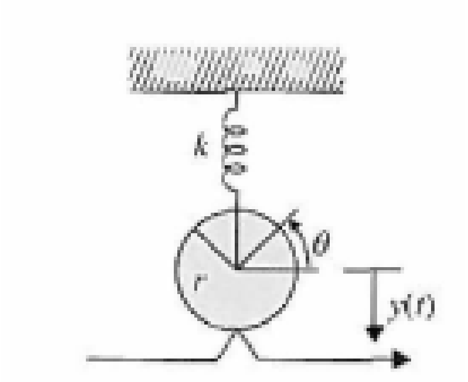
Pulse
Generator
PID(s)
PID Controller
2
den(s)
Transfer Fcn
Scope
Pulse
Generator1
PID(s)
PID Controller1
0.01
den(s)
Transfer Fcn1

Validasi

Tutorial :

1. https://www.mathworks.com/videos/system-identification-toolbox-overview-64920.html
2. https://www.youtube.com/watch?v=Zez7kcbRKSo&ab_channel=MATLAB

Tugas :



1. Sistem suspensi kendaraan yang melewati jalan yang bergelombang, Tentukan fungsi transfer sistem tersebut (output y(t))

2. Lakukan pemodelan Motor DC dengan System Identification Toolbox : https://www.youtube.com/watch?v=Zez7kcbRKSo&ab_channel=MATLAB