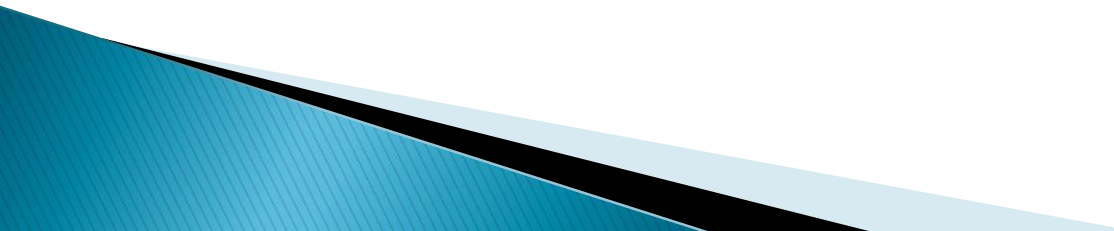


Pemodelan Sistem

Identifikasi Sistem

- ▶ White Box → Analisis Matematis dari parameter system.
 - ▶ Grey Box → Kombinasi.
 - ▶ Black Box → Menggunakan analisis dari input-output system.
- 

Review Matematika

- ▶ *Transformasi Laplace*

Problem dalam sistem kontrol adalah problem dinamik yg biasanya dideskripsikan dalam persamaan diferensial. Dengan transformasi Laplace, solusi persamaan diferensial lebih sederhana dan mudah

- ▶ *Partial Fraction Expansion*

karena berhadapan dengan pecahan simbolik (fungsi rasional) maka perlu metode ini untuk menyederhanakan persamaan

Transformasi Laplace

Jika terdapat fungsi $f(t)$ maka Transformasi Laplace dari $f(t)$ adalah $F(s)$

$$\mathcal{L}[f(t)] = F(s) = \int_{0-}^{\infty} f(t)e^{-st} dt$$

$$\mathcal{L}^{-1}[F(s)] = \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} F(s)e^{st} ds = f(t)u(t)$$

Jika terdapat fungsi $F(s)$ maka inverse Transformasi Laplace kembali ke $f(t)$ adalah

$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

Transformasi Laplace

PROBLEM: Find the Laplace transform of $f(t) = Ae^{-at}u(t)$.

SOLUTION: Since the time function does not contain an impulse function, we can replace the lower limit of Eq. (2.1) with 0. Hence,

$$\begin{aligned} F(s) &= \int_0^{\infty} f(t)e^{-st} dt = \int_0^{\infty} Ae^{-at}e^{-st} dt = A \int_0^{\infty} e^{-(s+a)t} dt \\ &= -\frac{A}{s+a} e^{-(s+a)t} \Big|_{t=0}^{\infty} = \frac{A}{s+a} \quad (2.3) \end{aligned}$$

TABLE 2.1 Laplace transform table

Item no.	$f(t)$	$F(s)$
1.	$\delta(t)$	1
2.	$u(t)$	$\frac{1}{s}$
3.	$tu(t)$	$\frac{1}{s^2}$
4.	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5.	$e^{-at}u(t)$	$\frac{1}{s+a}$
6.	$\sin \omega t u(t)$	$\frac{\omega}{s^2 + \omega^2}$
7.	$\cos \omega t u(t)$	$\frac{s}{s^2 + \omega^2}$

PROBLEM: Find the inverse Laplace transform of $F_1(s) = 1/(s+3)^2$.

SOLUTION: For this example we make use of the frequency shift theorem, Item 4 of Table 2.2, and the Laplace transform of $f(t) = tu(t)$, Item 3 of Table 2.1. If the inverse transform of $F(s) = 1/s^2$ is $tu(t)$, the inverse transform of $F(s+a) = 1/(s+a)^2$ is $e^{-at}tu(t)$. Hence, $f_1(t) = e^{-3t}tu(t)$.

Sifat-sifat Transformasi Laplace

Item no.	Theorem	Name
1.	$\mathcal{L}\{f(t)\} = F(s) = \int_{0-}^{\infty} f(t)e^{-st} dt$	Definition
2.	$\mathcal{L}\{kf(t)\} = kF(s)$	Linearity theorem
3.	$\mathcal{L}\{f_1(t) + f_2(t)\} = F_1(s) + F_2(s)$	Linearity theorem
4.	$\mathcal{L}\{e^{-at}f(t)\} = F(s + a)$	Frequency shift theorem
5.	$\mathcal{L}\{f(t - T)\} = e^{-sT}F(s)$	Time shift theorem
6.	$\mathcal{L}\{f(at)\} = \frac{1}{a}F\left(\frac{s}{a}\right)$	Scaling theorem
7.	$\mathcal{L}\left[\frac{df}{dt}\right] = sF(s) - f(0-)$	Differentiation theorem
8.	$\mathcal{L}\left[\frac{d^2f}{dt^2}\right] = s^2F(s) - sf(0-) - f'(0-)$	Differentiation theorem
9.	$\mathcal{L}\left[\frac{d^nf}{dt^n}\right] = s^nF(s) - \sum_{k=1}^n s^{n-k}f^{k-1}(0-)$	Differentiation theorem
10.	$\mathcal{L}\left[\int_{0-}^t f(\tau)d\tau\right] = \frac{F(s)}{s}$	Integration theorem
11.	$f(\infty) = \lim_{s \rightarrow 0} sF(s)$	Final value theorem ¹
12.	$f(0+) = \lim_{s \rightarrow \infty} sF(s)$	Initial value theorem ²

Partial Fraction Expansion

Misalnya terdapat fungsi dalam s dari hasil transformasi Laplace

$$\longrightarrow G(s) = \frac{Q(s)}{P(s)}$$

di mana $Q(s)$ dan $P(s)$ adalah polinom dalam s

Akar-akar (yang membuat persamaan menjadi nol) dari $Q(s)$ disebut *zero* dari $G(s)$

Akar-akar (yang membuat persamaan menjadi nol) dari $P(s)$ disebut *pole* dari $G(s)$

Jika diasumsikan orde (pangkat tertinggi) dari $P(s)$ lebih besar dari $Q(s)$ maka

$$P(s) = s^n + a_{n-1}s^{n-1} + \dots + a_1s + a_0$$

Dimana a_1, a_0 dst adalah koefisien real

maka terdapat beberapa jenis penyederhanaan

Partial Fraction Expansion

1. Jika pole bilangan real dan berbeda

$$G(s) = \frac{Q(s)}{P(s)} = \frac{Q(s)}{(s + s_1)(s + s_2) \cdots (s + s_n)}$$

$$s_1 \neq s_2 \neq \cdots \neq s_n.$$

maka

$$G(s) = \frac{K_{s1}}{s + s_1} + \frac{K_{s2}}{s + s_2} + \cdots + \frac{K_{sn}}{s + s_n}$$

$$K_{s1} = \left[(s + s_1) \frac{Q(s)}{P(s)} \right] \Big|_{s=-s_1} = \frac{Q(-s_1)}{(s_2 - s_1)(s_3 - s_1) \cdots (s_n - s_1)}$$

K_{s2} , K_{s3} dst diperoleh dengan cara yang sama

Ilustrasi

$$G(s) = \frac{5s + 3}{(s + 1)(s + 2)(s + 3)} = \frac{5s + 3}{s^3 + 6s^2 + 11s + 6}$$

Dapat dituliskan

$$G(s) = \frac{K_{-1}}{s + 1} + \frac{K_{-2}}{s + 2} + \frac{K_{-3}}{s + 3}$$

$$K_{-1} = [(s + 1)G(s)] \Big|_{s=-1} = \frac{5(-1) + 3}{(2 - 1)(3 - 1)} = -1$$

$$K_{-2} = [(s + 2)G(s)] \Big|_{s=-2} = \frac{5(-2) + 3}{(1 - 2)(3 - 2)} = 7$$

$$K_{-3} = [(s + 3)G(s)] \Big|_{s=-3} = \frac{5(-3) + 3}{(1 - 3)(2 - 3)} = -6$$

$$G(s) = \frac{-1}{s + 1} + \frac{7}{s + 2} - \frac{6}{s + 3}$$

Bentuk yg mudah untuk diubah lagi ke t dengan Inverse transformasi Laplace

Partial Fraction Expansion

2. Jika terdapat akar yang berulang

$$G(s) = \frac{Q(s)}{P(s)} = \frac{Q(s)}{(s + s_1)(s + s_2) \cdots (s + s_{n-r})(s + s_l)^r}$$

$$G(s) = \frac{K_{s1}}{s + s_1} + \frac{K_{s2}}{s + s_2} + \cdots + \frac{K_{s(n-r)}}{s + s_{n-r}}$$

| ← $n - r$ terms of simple poles → |

$$+ \frac{A_1}{s + s_l} + \frac{A_2}{(s + s_l)^2} + \cdots + \frac{A_r}{(s + s_l)^r}$$

| ← r terms of repeated poles → |

$$A_r = [(s + s_l)^r G(s)] \Big|_{s=-s_l}$$

$$A_{r-1} = \frac{d}{ds} [(s + s_l)^r G(s)] \Big|_{s=-s_l}$$

$$A_{r-2} = \frac{1}{2!} \frac{d^2}{ds^2} [(s + s_l)^r G(s)] \Big|_{s=-s_l}$$

⋮

$$A_1 = \frac{1}{(r-1)!} \frac{d^{r-1}}{ds^{r-1}} [(s + s_l)^r G(s)] \Big|_{s=-s_l}$$

Ilustrasi

$$G(s) = \frac{1}{s(s+1)^3(s+2)} = \frac{1}{s^5 + 5s^4 + 9s^3 + 7s^2 + 2s}$$

$$\longrightarrow G(s) = \frac{K_0}{s} + \frac{K_{-2}}{s+2} + \frac{A_1}{s+1} + \frac{A_2}{(s+1)^2} + \frac{A_3}{(s+1)^3}$$

$$K_0 = [sG(s)] \Big|_{s=0} = \frac{1}{2}$$

$$K_{-2} = [(s+2)G(s)] \Big|_{s=-2} = \frac{1}{2}$$

$$A_3 = [(s+1)^3G(s)] \Big|_{s=-1} = -1$$

$$A_2 = \frac{d}{ds} [(s+1)^3G(s)] \Big|_{s=-1} = \frac{d}{ds} \left[\frac{1}{s(s+2)} \right] \Big|_{s=-1} = 0$$

$$A_1 = \frac{1}{2!} \frac{d^2}{ds^2} [(s+1)^3G(s)] \Big|_{s=-1} = \frac{1}{2} \frac{d^2}{ds^2} \left[\frac{1}{s(s+2)} \right] \Big|_{s=-1} = -1$$

maka hasilnya

$$G(s) = \frac{1}{2s} + \frac{1}{2(s+2)} - \frac{1}{s+1} - \frac{1}{(s+1)^3}$$

Partial Fraction Expansion

3. Jika polanya adalah pasangan bilangan kompleks

$$s = -\sigma + j\omega \quad \text{and} \quad s = -\sigma - j\omega$$

Koefisien bisa dicari dengan →

$$K_{-\sigma+j\omega} = (s + \sigma - j\omega)G(s) \Big|_{s=-\sigma+j\omega}$$

$$K_{-\sigma-j\omega} = (s + \sigma + j\omega)G(s) \Big|_{s=-\sigma-j\omega}$$

Ilustrasi

$$F(s) = \frac{3}{s(s^2 + 2s + 5)} = \frac{3}{s(s + 1 + j2)(s + 1 - j2)}$$
$$= \frac{K_1}{s} + \frac{K_2}{s + 1 + j2} + \frac{K_3}{s + 1 - j2}$$

$$K_2 = \frac{3}{s(s + 1 - j2)} \Big|_{s \rightarrow -1 - j2} = -\frac{3}{20}(2 + j1)$$

K_3 adalah pasangan bilangan kompleks dari K_2

K_1 bisa dicari dari cara sebelumnya

$$F(s) = \frac{3/5}{s} - \frac{3}{20} \left(\frac{2 + j1}{s + 1 + j2} + \frac{2 - j1}{s + 1 - j2} \right)$$

Dengan inverse transformasi Laplace \longrightarrow

$$f(t) = \frac{3}{5} - \frac{3}{20} \left[(2 + j1)e^{-(1+j2)t} + (2 - j1)e^{-(1-j2)t} \right]$$
$$= \frac{3}{5} - \frac{3}{20} e^{-t} \left[4 \left(\frac{e^{j2t} + e^{-j2t}}{2} \right) + 2 \left(\frac{e^{j2t} + e^{-j2t}}{2j} \right) \right]$$

Latihan

PROBLEM: Find the Laplace transform of $f(t) = te^{-5t}$.

PROBLEM: Find the inverse Laplace transform of $F(s) = 10/[s(s + 2)(s + 3)^2]$.

Fungsi Transfer



$$\frac{C(s)}{R(s)} = G(s) = \frac{(b_m s^m + b_{m-1} s^{m-1} + \dots + b_0)}{(a_n s^n + a_{n-1} s^{n-1} + \dots + a_0)}$$

$G(s)$ adalah fungsi transfer

Output dapat dicari dengan

$$C(s) = R(s)G(s)$$

Ilustrasi

Tentukan fungsi transfer dari

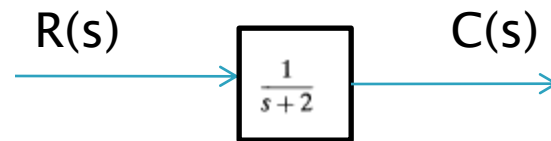
$$\frac{dc(t)}{dt} + 2c(t) = r(t)$$

Dengan transformasi Laplace



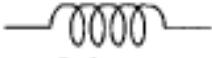
$$sC(s) + 2C(s) = R(s)$$

Fungsi transfer

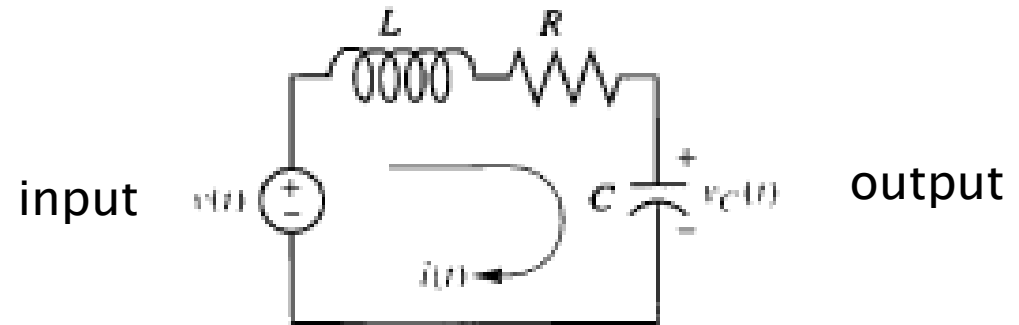
$$G(s) = \frac{C(s)}{R(s)} = \frac{1}{s+2}$$



Pemodelan Sistem Elektrik

Component	Voltage-current	Current-voltage	Voltage-charge	Impedance $Z(s) = V(s)/I(s)$	Admittance $Y(s) = I(s)/V(s)$
 Capacitor	$v(t) = \frac{1}{C} \int_0^1 i(\tau) d\tau$	$i(t) = C \frac{dv(t)}{dt}$	$v(t) = \frac{1}{C} q(t)$	$\frac{1}{Cs}$	Cs
 Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R} v(t)$	$v(t) = R \frac{dq(t)}{dt}$	R	$\frac{1}{R} = G$
 Inductor	$v(t) = L \frac{di(t)}{dt}$	$i(t) = \frac{1}{L} \int_0^1 v(\tau) d\tau$	$v(t) = L \frac{d^2 q(t)}{dt^2}$	Ls	$\frac{1}{Ls}$

Ilustrasi



Dari hukum Kirchoff (loop) tegangan

$$L \frac{di(t)}{dt} + Ri(t) + \frac{1}{C} \int_0^t i(\tau) d\tau = v(t)$$

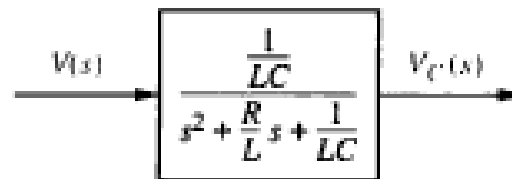
$$i(t) = dq(t)/dt$$

$$L \frac{d^2q(t)}{dt^2} + R \frac{dq(t)}{dt} + \frac{1}{C} q(t) = v(t)$$

Dengan transformasi Laplace

$$q(t) = Cv_C(t)$$

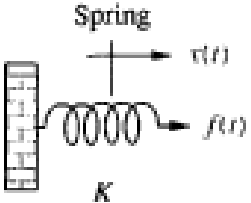
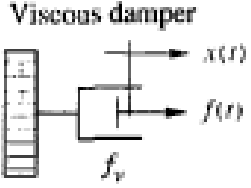
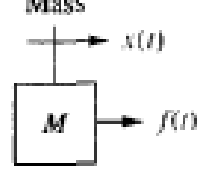
$$LC \frac{d^2v_C(t)}{dt^2} + RC \frac{dv_C(t)}{dt} + v_C(t) = v(t) \rightarrow (LCs^2 + RCs + 1)V_C(s) = V(s)$$



$$\frac{V_C(s)}{V(s)} = \frac{1/LC}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Pemodelan Sistem Mekanik

Sistem Translasi

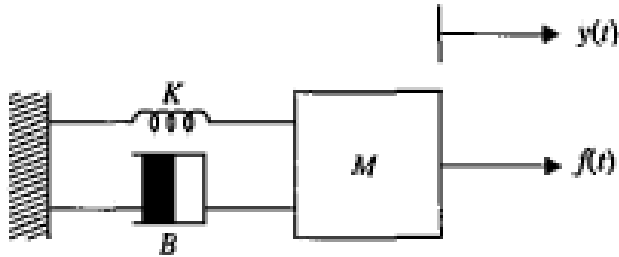
Component	Force-velocity	Force-displacement	Impedance $Z_M(s) = F(s)/X(s)$
<p>Spring</p> 	$f(t) = K \int_0^t v(\tau) d\tau$	$f(t) = Kx(t)$	K
<p>Viscous damper</p> 	$f(t) = f_v v(t)$	$f(t) = f_v \frac{dx(t)}{dt}$	$f_v s$
<p>Mass</p> 	$f(t) = M \frac{dv(t)}{dt}$	$f(t) = M \frac{d^2 x(t)}{dt^2}$	$M s^2$

Pemodelan Sistem Mekanik

Sistem Rotasi

Component	Torque-angular velocity	Torque-angular displacement	Impedance $Z_M(s) = T(s)/\theta(s)$
<p>Spring K</p>	$T(t) = K \int_0^t \omega(\tau) d\tau$	$T(t) = K\theta(t)$	K
<p>Viscous damper D</p>	$T(t) = D\omega(t)$	$T(t) = D \frac{d\theta(t)}{dt}$	Ds
<p>Inertia J</p>	$T(t) = J \frac{d\omega(t)}{dt}$	$T(t) = J \frac{d^2\theta(t)}{dt^2}$	Js^2

Ilustrasi



Sistem massa pegas peredam

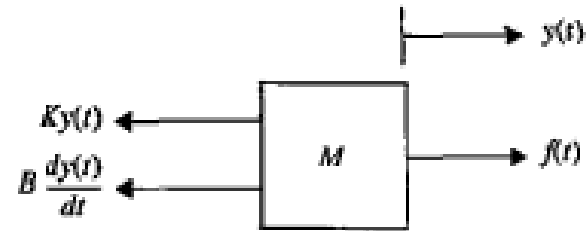


Diagram Benda Bebas

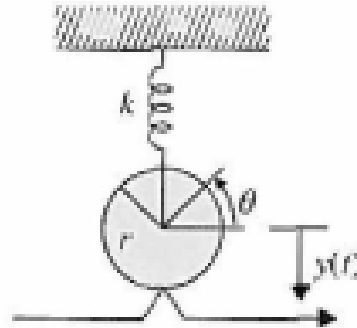
Persamaan kesetimbangan

$$f(t) - B \frac{dy(t)}{dt} - Ky(t) = M \frac{d^2y(t)}{dt^2}$$

Dengan transformasi Laplace dengan asumsi kondisi awal nol

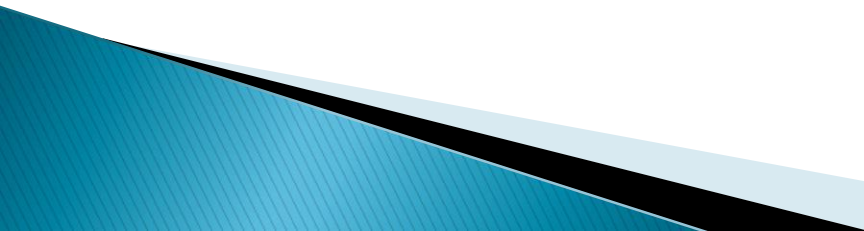
$$\longrightarrow \frac{Y(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K}$$

Latihan



Sistem suspensi kendaraan yang melewati jalan yang bergelombang,
Tentukan fungsi transfer sistem tersebut (output $y(t)$)

Kesimpulan

- ▶ Transformasi Laplace digunakan untuk mencari solusi persamaan diferensial dengan menjadikannya menjadi persamaan aljabar yang dapat dimanipulasi dengan mudah
 - ▶ Partial Fraction Expansion digunakan untuk memecahkan fungsi rasional ke dalam komponen-komponen akar-akarnya
 - ▶ Pemodelan sistem elektrik : Hukum Kirchoff dan Hukum Ohm
 - ▶ Pemodelan sistem mekanik : Hukum Newton
- 

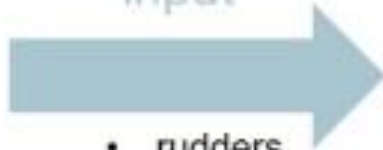
Black-Box Methode



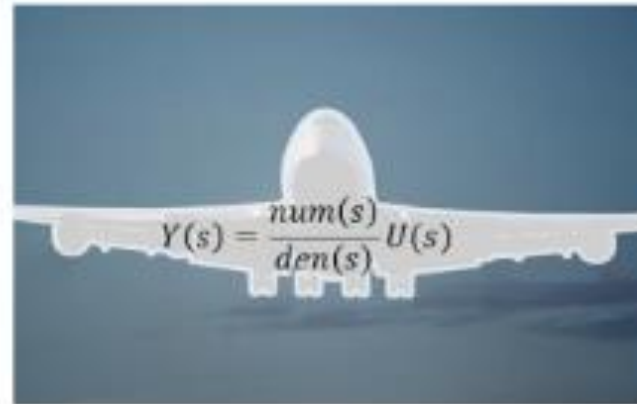
Linear Model

u

Input



- rudders
- aileron
- thrust



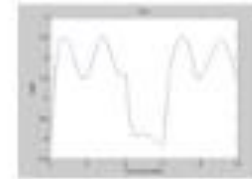
u, y: measured time or frequency domain signals

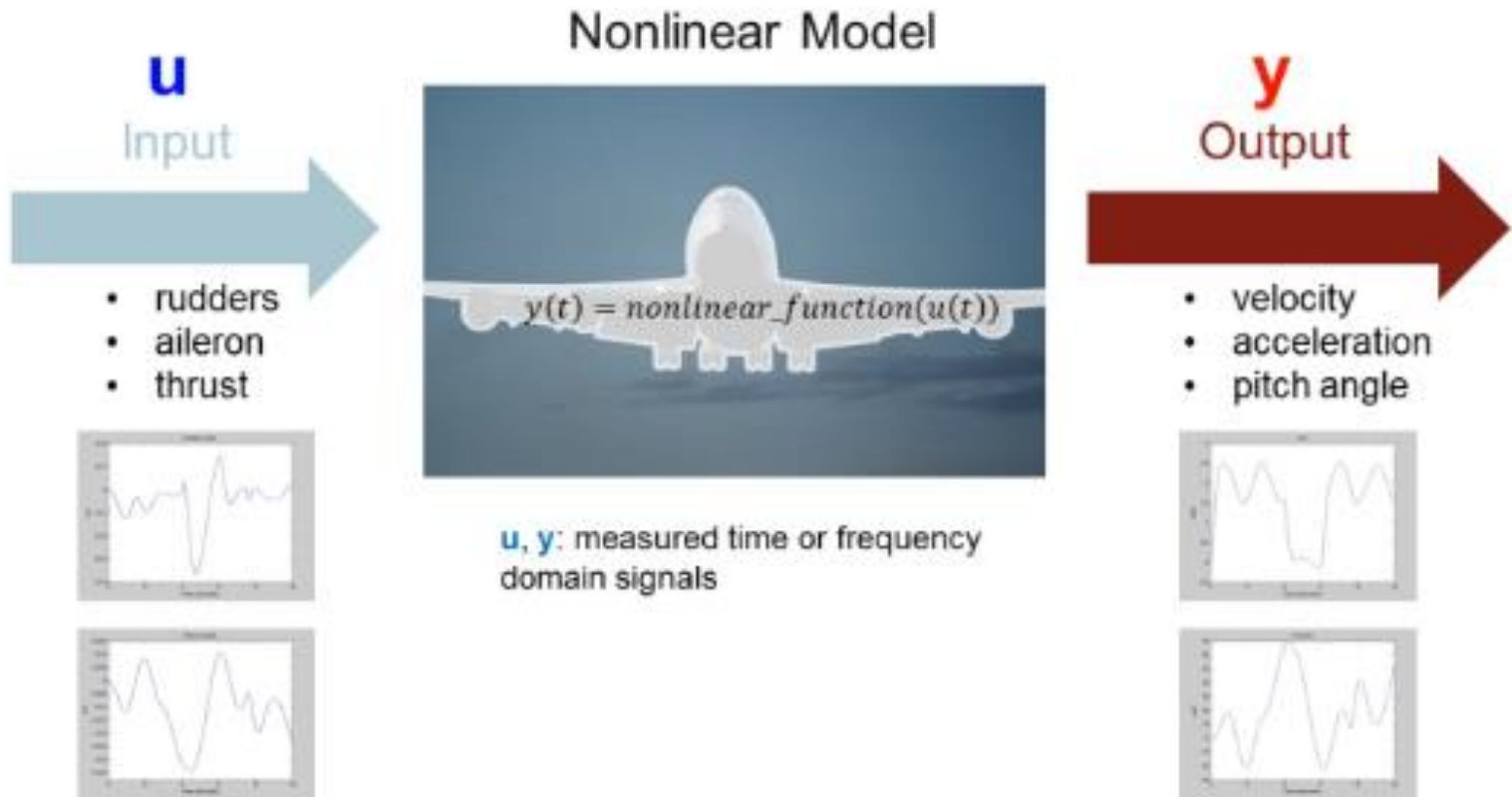
y

Output



- velocity
- acceleration
- pitch angle







• DC MOTOR SPEED MODELING

By transfer function

$$T = K_t i$$

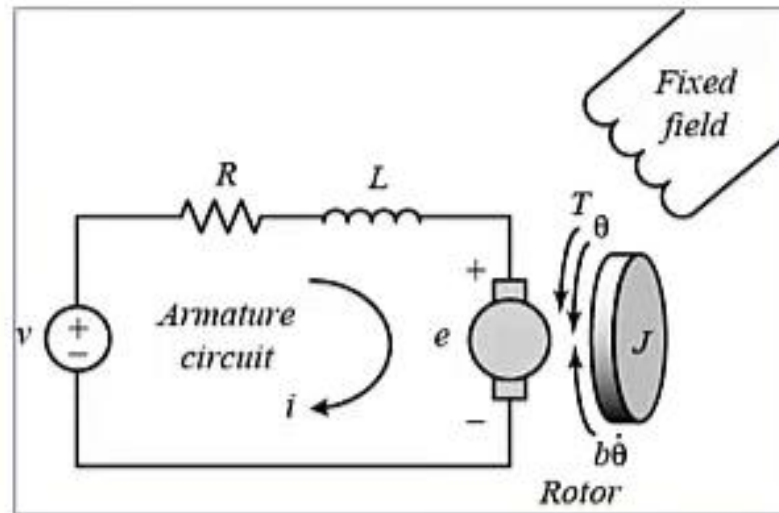
$$e = K_e \dot{\Theta}$$

$$J \ddot{\Theta} + b \dot{\Theta} = K i$$

$$L \frac{di}{dt} + Ri = V - K \dot{\Theta}$$

$$s(Js + b)\Theta(s) = KI(s)$$

$$(Ls + R)I(s) = V(s) - Ks\Theta(s)$$



$$\frac{\dot{\Theta}(s)}{V(s)} = \frac{K}{(Js + b)(Ls + R) + K^2}$$

physical parameters are:

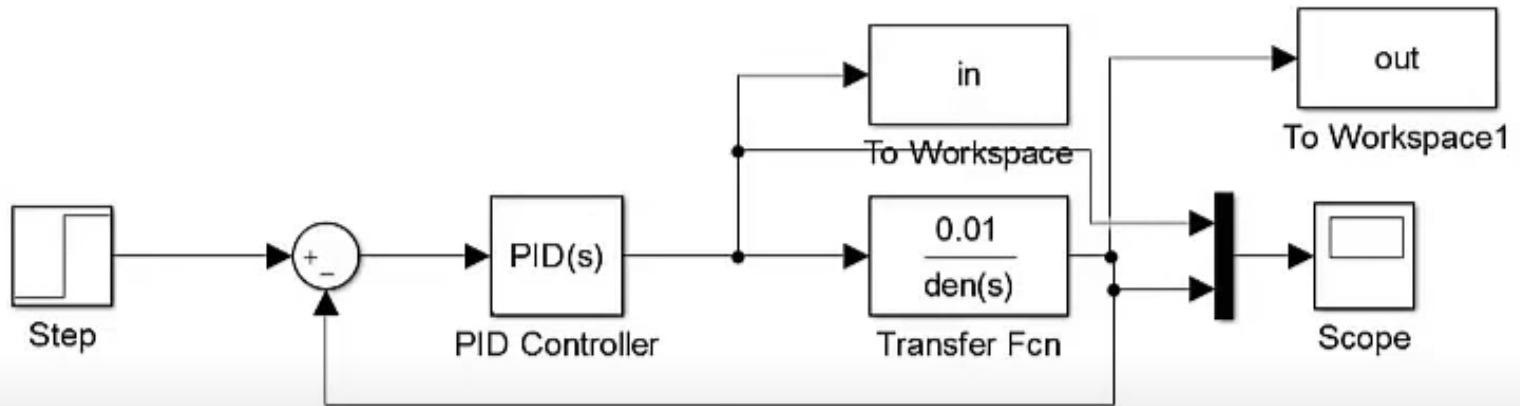
(J)	moment of inertia of the rotor	0.01 kg.m ²
(b)	motor viscous friction constant	0.1 N.m.s
(Ke)	electromotive force constant	0.01 V/rad/sec
(Kt)	motor torque constant	0.01 N.m/Amp
(R)	electric resistance	1 Ohm
(L)	electric inductance	0.5 H

The screenshot displays the MATLAB R2015b environment. The main window shows a script named 'model.m' with the following code:

```
1 - J = 0.01;  
2 - b = 0.1;  
3 - K = 0.01;  
4 - R = 1;  
5 - L = 0.5;  
6 - s = tf('s');  
7 - P_motor = K / ((J*s+b) * (L*s+R) + K^2);  
8 -
```

The Command Window at the bottom shows the prompt `>>`. The Workspace window on the left lists variables such as 'a', 'ans', 'b', 'den', 'in', 'num', 'out', 'ss1', 'sys', 'sysa', 'test1d', 't1', 't4', 'tout', 'yin', and 'yout' with their respective values and dimensions.

Retrieve Data Input-Output



The screenshot shows the MATLAB software interface. The **Command Window** at the bottom displays the following code:

```
1 - J = 0.01;  
2 - b = 0.1;  
3 - K = 0.01;  
4 - R = 1;  
5 - L = 0.5;  
6 - s = tf('s');  
7 - P_motor = K / ((J*s+b) * (L*s+R) + K^2);  
8
```

The **Workspace** window on the left shows the following variables:

Name	Value
in	1000x1 double
out	1000x1 double
tout	1000x1 double

The **in**, **out**, and **tout** variables are circled in red in the image.

Using "ident" in MATLAB

The image displays the MATLAB System Identification software interface. The main window is titled "System Identification - Untitled" and features a menu bar with "File", "Options", "Window", and "Help". The central workspace contains a workflow diagram with "Import data" and "Import models" sections, a "Preprocess" dropdown, "Working Data", and an "Estimate" dropdown. Below the diagram are "Data Views" (Time plot, Data spectra, Frequency function), "Model Views" (Model output, Model resids, Transient response, Frequency response, Zeros and poles, Noise spectrum), and "Nonlinear ARX" and "Nams-Wiener" options. A "Trash" icon and "Validation Data" input field are also present. The "Import Data" dialog box is open on the right, showing "Data Format for Signals" set to "Time-Domain Signals", "Workspace Variable" with "Input: in" and "Output: out", and "Data Information" with "Data name: test", "Starting time: 0", and "Sample time: 0.001". The "Import" button is highlighted. The Command Window at the bottom shows the command `>> ident` and a feedback icon `fx >>`.

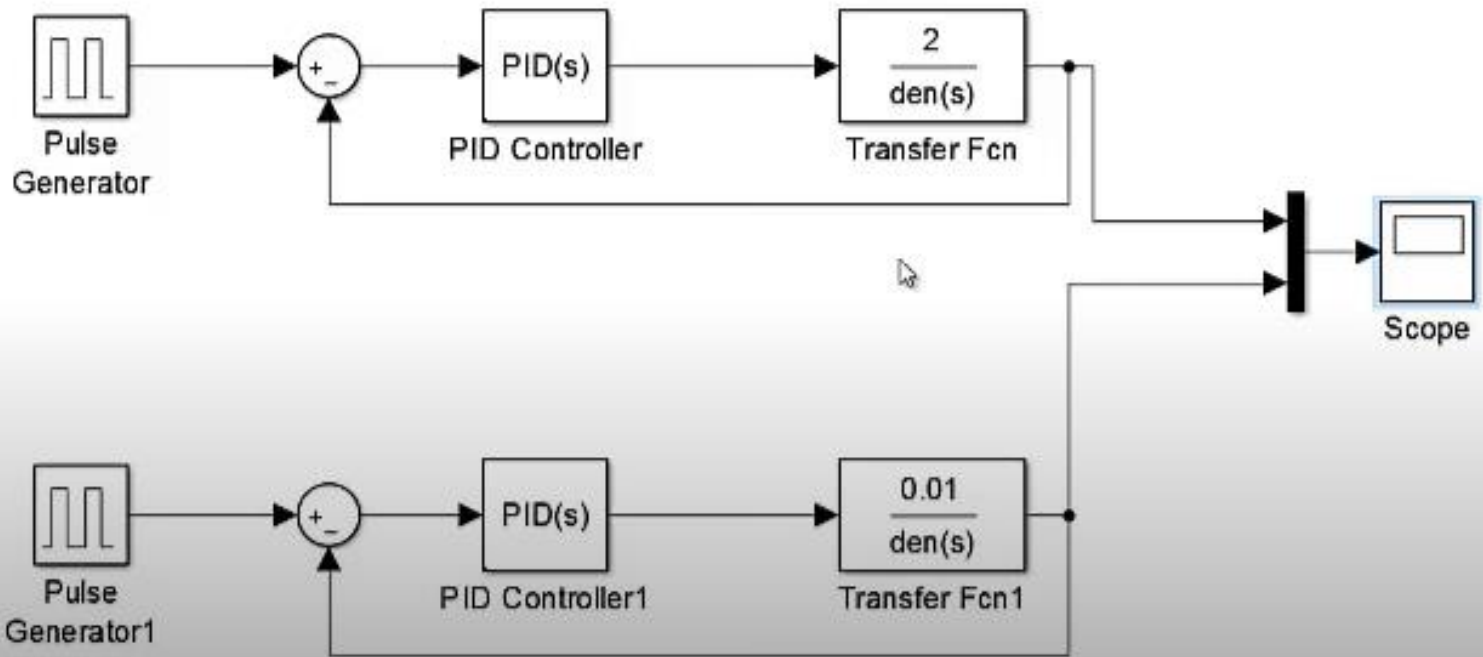
Training Data to Get Transfer Function

The image displays the MATLAB System Identification software interface. The main window, titled "Plant Identification Progress", shows the results of a transfer function identification process. The "Estimation Progress" table lists 14 iterations with various parameters. The "Result" section indicates that the estimation was successful, with a fit to estimation data of 100% and a Final Prediction Error (FPE) of 1.66906e-18. The "Data/model Info: tf1" window is open, showing the model name "tf1", color "(0.75 0.75 0.75)", and the transfer function:
$$\frac{1}{s^2 + 12.05s + 20.02}$$
 The "Diary and Notes" window shows the command used for estimation:

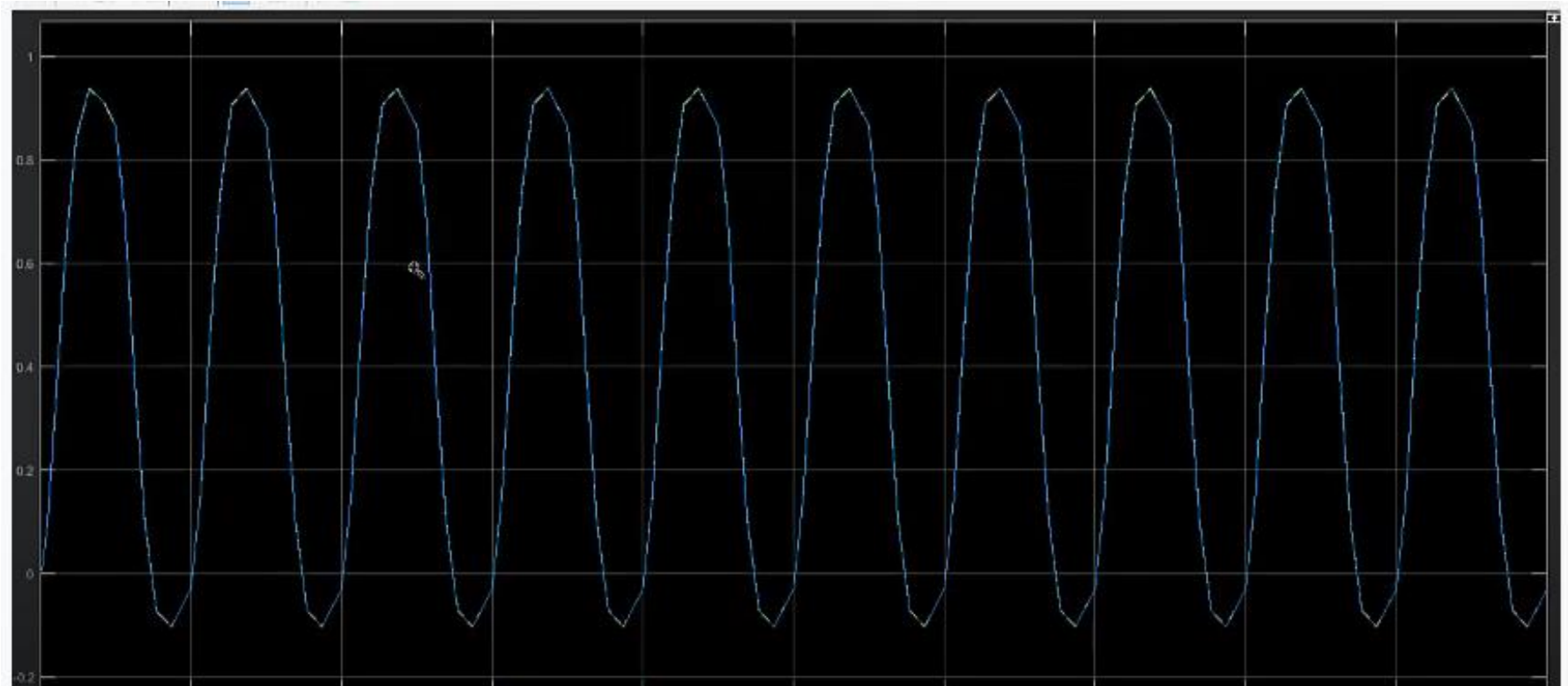
```
% Import test
% Transfer function estimation
Options = tfestOptions;
Options.Display = 'on';
% tf = tfest(test, 2, 0, Options);
```

Iteration	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Time	1.66739e-10	1.66739e-10	1.66739e-10	1.66739e-10	1.66739e-10	1.66739e-10	1.66739e-10	1.66739e-10	1.66739e-10	1.66739e-10	1.66739e-10	1.66739e-10	1.66739e-10	1.66739e-10
Time domain data test	5.49e-09	1.63e-14	5.73e-16	5.73e-16	5.73e-16	5.73e-16	5.72e-16	5.72e-16	5.72e-16	5.72e-16	5.72e-16	5.71e-16	5.71e-16	5.44e-17
Number of poles	1.26e+07	4.64e+04	4.64e+04	4.64e+04	4.64e+04	4.64e+04	4.64e+04	4.64e+04	4.64e+04	4.64e+04	4.64e+04	4.64e+04	4.64e+04	4.64e+04
Number of zeros	1.47e-16	1.05e+09	1.36e+07	1.36e+07	1.36e+07	1.36e+07	1.36e+07	1.36e+07	1.36e+07	1.36e+07	1.36e+07	1.36e+07	1.36e+07	1.36e+07
Initialization Method	0.0245	7.45e-07	1.04e-08	6e-09	5.75e-09	5.7e-09	4.02e-09	5.27e-09	4.91e-09	4.59e-09	4.71e-09	4.42e-09	4.37e-09	3.29e-09

3. Validasi



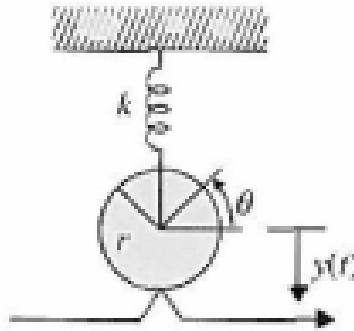
Validasi



Tutorial :

1. <https://www.mathworks.com/videos/system-identification-toolbox-overview-64920.html>
2. https://www.youtube.com/watch?v=Zez7kcbRKSo&ab_channel=MATLAB

Tugas :



1. Sistem suspensi kendaraan yang melewati jalan yang bergelombang, Tentukan fungsi transfer sistem tersebut (output $y(t)$)

2. Lakukan pemodelan Motor DC dengan System Identification Toolbox : https://www.youtube.com/watch?v=Zez7kcbRKSo&ab_channel=MATLAB