Linear Momentum and Collisions

- Conservation of Energy
- Momentum
- Impulse
- Conservation of Momentum
- 1-D Collisions
- 2-D Collisions
- The Center of Mass

Conservation of Energy

- $\Delta E = \Delta K + \Delta U = 0$ if conservative forces are the only forces that do work on the system.
- The total amount of energy in the system is constant.

$$\frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}kx_f^2 = \frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}kx_i^2$$

- $\Delta E = \Delta K + \Delta U = -f_k d$ if friction forces are doing work on the system.
- The total amount of energy in the system is still constant, but the change in mechanical energy goes into "internal energy" or heat.

$$-f_k d = \left(\frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}kx_f^2\right) - \left(\frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}kx_i^2\right)$$

Linear Momentum

- This is a new fundamental quantity, like force, energy. It is a vector quantity (points in same direction as velocity).
- The linear momentum **p** of an object of mass *m* moving with a velocity **v** is defined to be the product of the mass and velocity:

$$\vec{p} = m\vec{v}$$

- The terms momentum and linear momentum will be used interchangeably in the text
- Momentum depend on an object's mass and velocity

Momentum and Energy

- Two objects with masses m_1 and m_2 have equal kinetic energy. How do the magnitudes of their momenta compare?

  (A) $p_1 < p_2$

  (B) $p_1 = p_2$

  (C) $p_1 > p_2$

  (D) Not enough information is given

Linear Momentum, cont'd

- Linear momentum is a vector quantity $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$
  - Its direction is the same as the direction of the velocity
- The dimensions of momentum are ML/T
- The SI units of momentum are kg m / s
- Momentum can be expressed in component form:

$$p_x = mv_x \qquad p_y = mv_y \qquad p_z = mv_z$$

Newton's Law and Momentum

- Newton's Second Law can be used to relate the momentum of an object to the resultant force acting on it

$$\vec{F}_{net} = m\vec{a} = m\frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta(m\vec{v})}{\Delta t}$$

- The change in an object's momentum divided by the elapsed time equals the constant net force acting on the object

$$\frac{\Delta\vec{p}}{\Delta t} = \frac{\text{change in momentum}}{\text{time interval}} = \vec{F}_{net}$$

Impulse

- When a single, constant force acts on the object, there is an **impulse** delivered to the object

$$\vec{I} = \vec{F}\Delta t$$

  - $\vec{\mathbf{I}}$ is defined as the *impulse*
  - The equality is true even if the force is not constant
  - Vector quantity, the direction is the same as the direction of the force

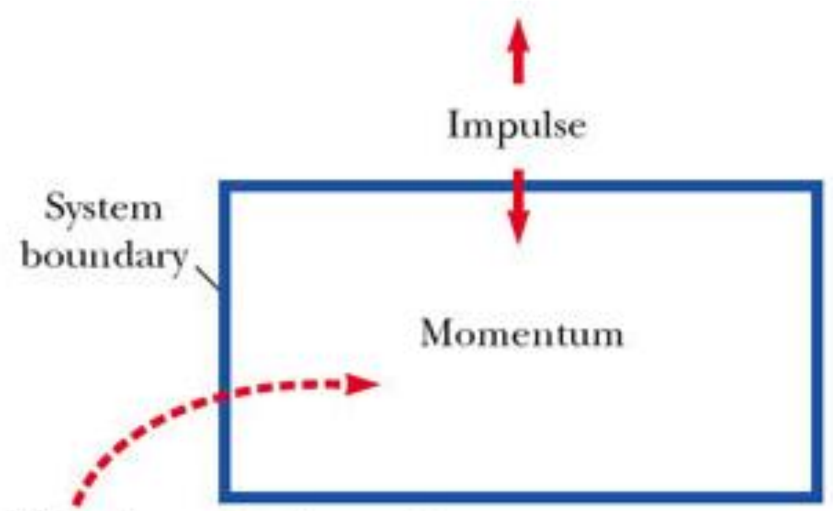
$$\frac{\Delta\vec{p}}{\Delta t} = \frac{\text{change in momentum}}{\text{time interval}} = \vec{F}_{net}$$

Impulse-Momentum Theorem

- The theorem states that the impulse acting on a system is equal to the change in momentum of the system

$$\Delta\vec{p} = \vec{F}_{net}\Delta t = \vec{I}$$

$$\vec{I} = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_i$$



The change in the total momentum of the system is equal to the total impulse on the system.

Calculating the Change of Momentum

$$\Delta\vec{p} = \vec{p}_{after} - \vec{p}_{before}$$

$$= mv_{after} - mv_{before}$$

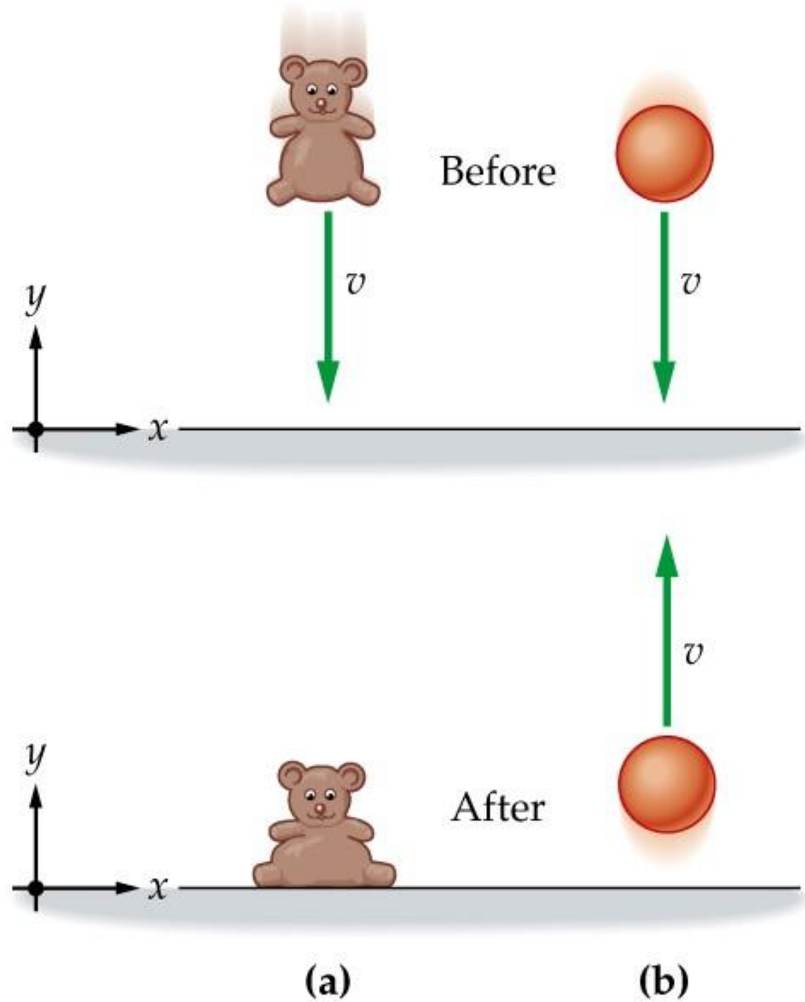
$$= m(v_{after} - v_{before})$$

For the teddy bear

$$\Delta p = m\left[0-(-v)\right] = mv$$

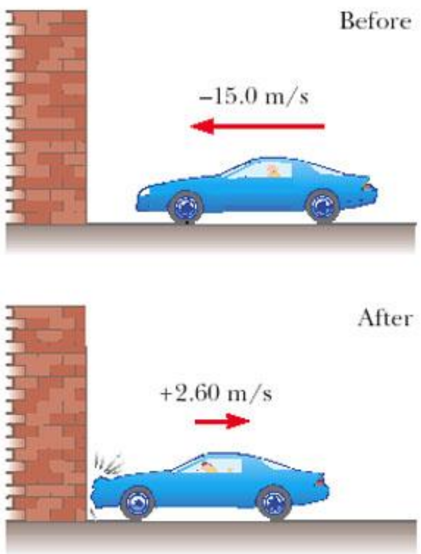
For the bouncing ball

$$\Delta p = m\left[v-(-v)\right] = 2mv$$



How Good Are the Bumpers?

- In a crash test, a car of mass 1.5×10^3 kg collides with a wall and rebounds as in figure. The initial and final velocities of the car are v_i=-15 m/s and v_f = 2.6 m/s, respectively. If the collision lasts for 0.15 s, find
(a) the impulse delivered to the car due to the collision
(b) the size and direction of the average force exerted on the car



How Good Are the Bumpers?

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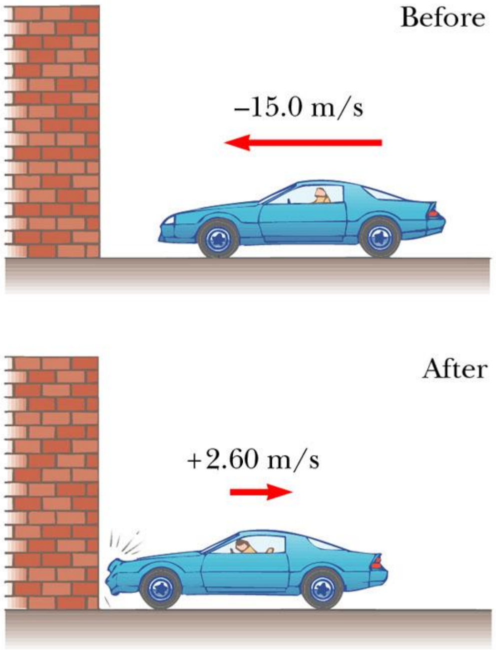
(a) the impulse delivered to the car due to the collision
(b) the size and direction of the average force exerted on the car

$$p_i = mv_i = (1.5\times10^3 kg)(-15 m/s) = -2.25\times10^4 kg\cdot m/s$$

$$p_f = mv_f = (1.5\times10^3 kg)(+2.6 m/s) = +0.39\times10^4 kg\cdot m/s$$

$$\begin{aligned} I &= p_f - p_i = mv_f - mv_i \\ &= (0.39\times10^4 kg\cdot m/s) - (-2.25\times10^4 kg\cdot m/s) \\ &= 2.64\times10^4 kg\cdot m/s \end{aligned}$$

$$F_{av} = \frac{\Delta p}{\Delta t} = \frac{I}{\Delta t} = \frac{2.64\times10^4 kg\cdot m/s}{0.15 s} = 1.76\times10^5 N$$



Impulse-Momentum Theorem

- A child bounces a 100 g superball on the sidewalk. The velocity of the superball changes from 10 m/s downward to 10 m/s upward. If the contact time with the sidewalk is 0.1s, what is the magnitude of the impulse imparted to the superball?

(A) *0*
(B) *2* kg-m/s
(C) *20* kg-m/s
(D) *200* kg-m/s
(E) *2000* kg-m/s

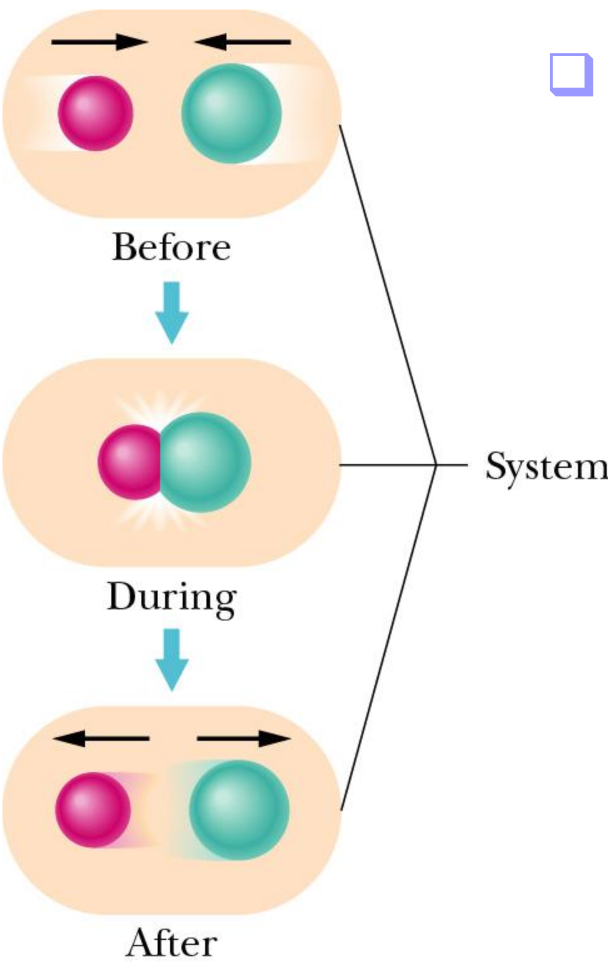
$$\vec{I} = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_i$$

Impulse-Momentum Theorem 2

- A child bounces a 100 g superball on the sidewalk. The velocity of the superball changes from 10 m/s downward to 10 m/s upward. If the contact time with the sidewalk is 0.1s, what is the magnitude of the force between the sidewalk and the superball?
  (A) *0*
  (B) *2* N
  (C) *20* N
  (D) *200* N
  (E) *2000* N

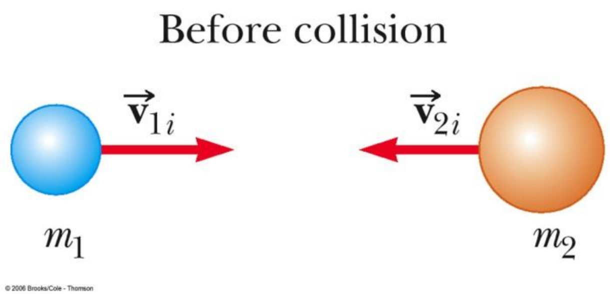
$$\vec{F} = \frac{\vec{I}}{\Delta t} = \frac{\Delta \vec{p}}{\Delta t} = \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t}$$

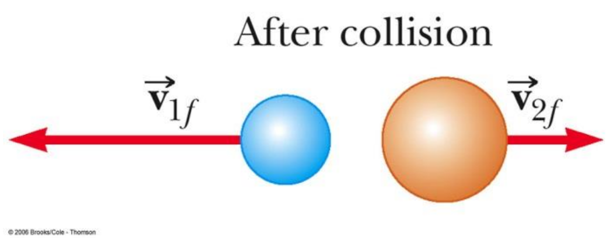
Conservation of Momentum

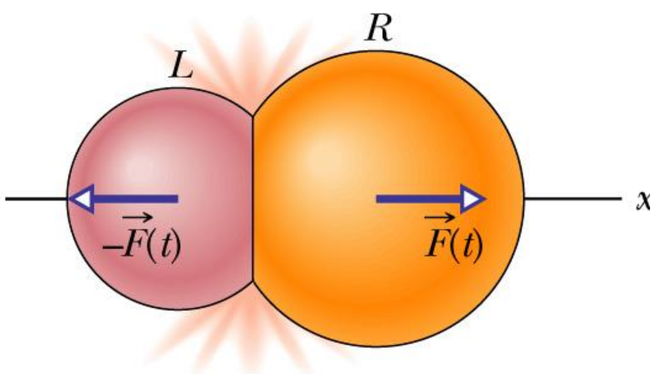


- In an isolated and closed system, the total momentum of the system remains constant in time.
  - Isolated system: no external forces
  - Closed system: no mass enters or leaves
  - The linear momentum of each colliding body may change
  - The total momentum *P* of the system cannot change.

Conservation of Momentum







- Start from impulse-momentum theorem

$$\vec{F}_{21}\Delta t = m_1\vec{v}_{1f} - m_1\vec{v}_{1i}$$

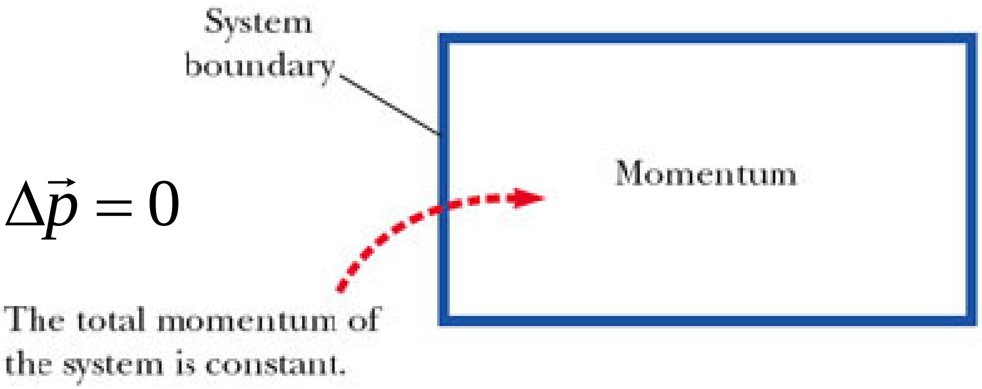
$$\vec{F}_{12}\Delta t = m_2\vec{v}_{2f} - m_2\vec{v}_{2i}$$

- Since $\vec{F}_{21}\Delta t = -\vec{F}_{12}\Delta t$
- Then $m_1\vec{v}_{1f} - m_1\vec{v}_{1i} = -(m_2\vec{v}_{2f} - m_2\vec{v}_{2i})$
- So $m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$

Conservation of Momentum

- When no external forces act on a system consisting of two objects that collide with each other, the total momentum of the system remains constant in time

$$\vec{F}_{net}\Delta t = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$$



- When $\vec{F}_{net} = 0$ then $\Delta\vec{p} = 0$
- For an isolated system

$$\vec{p}_f = \vec{p}_i$$

- Specifically, the total momentum before the collision will equal the total momentum after the collision

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

The Archer

- An archer stands at rest on frictionless ice and fires a 0.5-kg arrow horizontally at 50.0 m/s. The combined mass of the archer and bow is 60.0 kg. With what velocity does the archer move across the ice after firing the arrow?

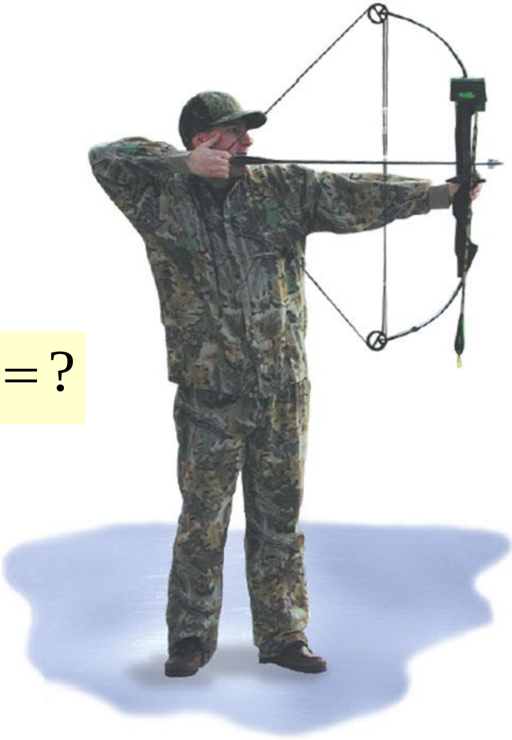
$$p_i = p_f$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$m_1 = 60.0kg, m_2 = 0.5kg, v_{1i} = v_{2i} = 0, v_{2f} = 50m/s, v_{1f} = ?$$

$$0 = m_1 v_{1f} + m_2 v_{2f}$$

$$v_{1f} = -\frac{m_2}{m_1} v_{2f} = -\frac{0.5kg}{60.0kg}(50.0m/s) = -0.417m/s$$



Conservation of Momentum

- A 100 kg man and 50 kg woman on ice skates stand facing each other. If the woman pushes the man backwards so that his final speed is 1 m/s, at what speed does she recoil?
  (A) 0
  (B) 0.5 m/s
  (C) 1 m/s
  (D) 1.414 m/s
  (E) 2 m/s

Types of Collisions

- Momentum is conserved in any collision
- **Inelastic collisions:** *rubber ball and hard ball*
  - Kinetic energy is not conserved
  - **Perfectly inelastic** collisions occur when the objects stick together
- **Elastic collisions:** *billiard ball*
  - both momentum and kinetic energy are conserved
- **Actual collisions**
  - Most collisions fall between elastic and perfectly inelastic collisions

Collisions Summary

- In an elastic collision, both momentum and kinetic energy are conserved
- In a non-perfect inelastic collision, momentum is conserved but kinetic energy is not. Moreover, the objects do not stick together
- In a perfectly inelastic collision, momentum is conserved, kinetic energy is not, and the two objects stick together after the collision, so their final velocities are the same
- Elastic and perfectly inelastic collisions are limiting cases, most actual collisions fall in between these two types
- Momentum is conserved in all collisions

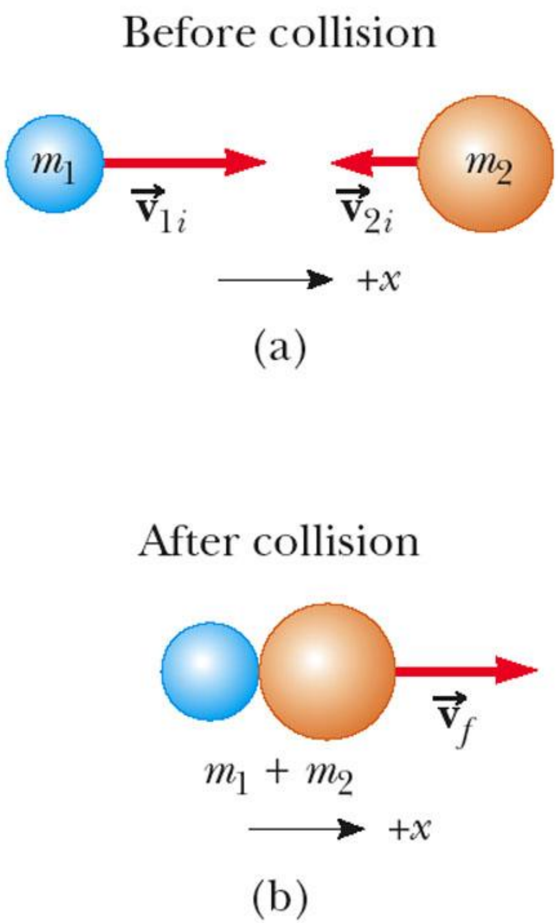
More about Perfectly Inelastic Collisions

- When two objects stick together after the collision, they have undergone a perfectly inelastic collision
- Conservation of momentum

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

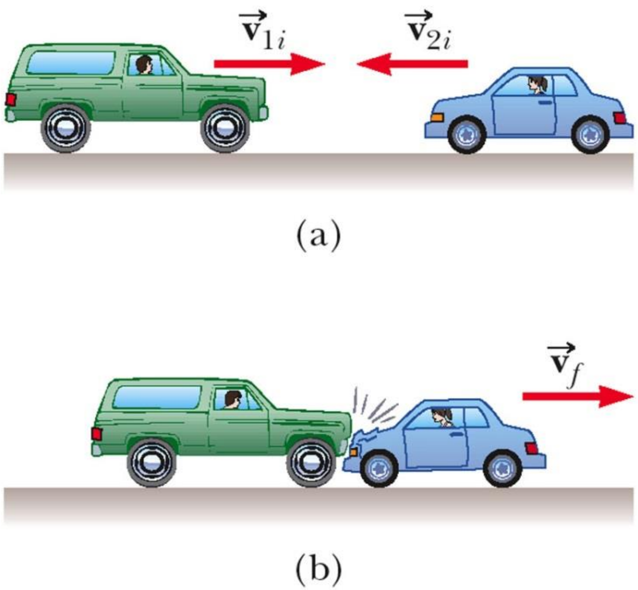
- Kinetic energy is NOT conserved



An SUV Versus a Compact

- An SUV with mass 1.80×10^3 kg is travelling eastbound at +15.0 m/s, while a compact car with mass 9.00×10^2 kg is travelling westbound at -15.0 m/s. The cars collide head-on, becoming entangled.

(a) Find the speed of the entangled cars after the collision.
(b) Find the change in the velocity of each car.
(c) Find the change in the kinetic energy of the system consisting of both cars.



An SUV Versus a Compact

(a) Find the speed of the entangled cars after the collision.

$$m_1 = 1.80 \times 10^3 kg, v_{1i} = +15 m/s$$

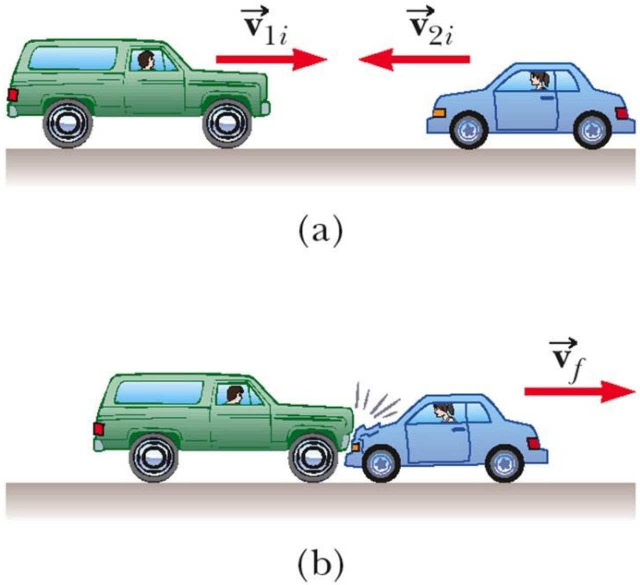
$$m_2 = 9.00 \times 10^2 kg, v_{2i} = -15 m/s$$

$$p_i = p_f$$

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

$$v_f = +5.00 m/s$$



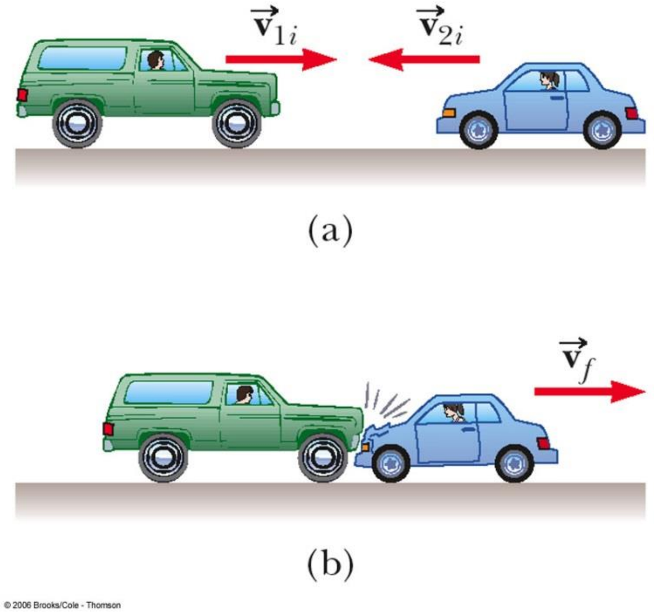
An SUV Versus a Compact

(b) Find the change in the velocity of each car.

$$m_1 = 1.80\times10^3 kg, v_{1i} = +15m/s$$
$$m_2 = 9.00\times10^2 kg, v_{2i} = -15m/s$$

$$v_f = +5.00m/s$$

$$\Delta v_1 = v_f - v_{1i} = -10.0m/s$$
$$\Delta v_2 = v_f - v_{2i} = +20.0m/s$$



$$m_1\Delta v_1 = m_1(v_f - v_{1i}) = -1.8\times10^4 kg\cdot m/s$$
$$m_2\Delta v_2 = m_2(v_f - v_{2i}) = +1.8\times10^4 kg\cdot m/s$$

$$m_1\Delta v_1 + m_2\Delta v_2 = 0$$

An SUV Versus a Compact

(c) Find the change in the kinetic energy of the system consisting of both cars.

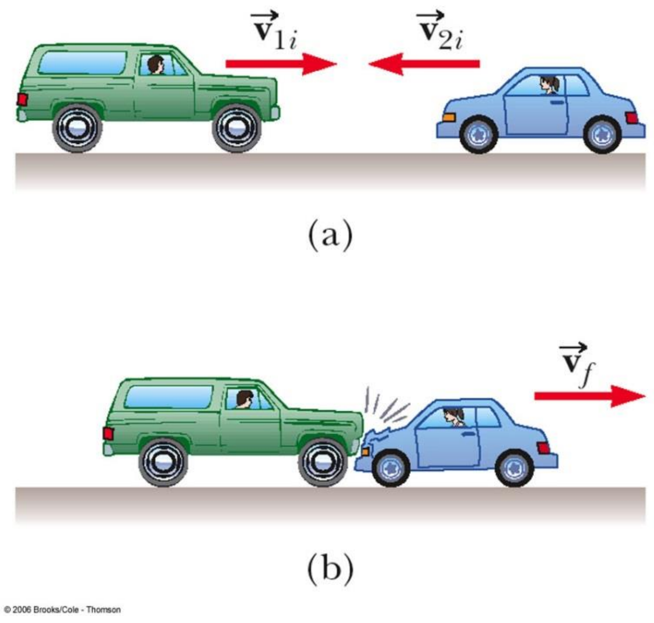
$$m_1 = 1.80\times10^3 kg, v_{1i} = +15 m/s$$
$$m_2 = 9.00\times10^2 kg, v_{2i} = -15 m/s$$

$$v_f = +5.00 m/s$$

$$KE_i = \frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = 3.04\times10^5 J$$

$$KE_f = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2 = 3.38\times10^4 J$$

$$\Delta KE = KE_f - KE_i = -2.70\times10^5 J$$



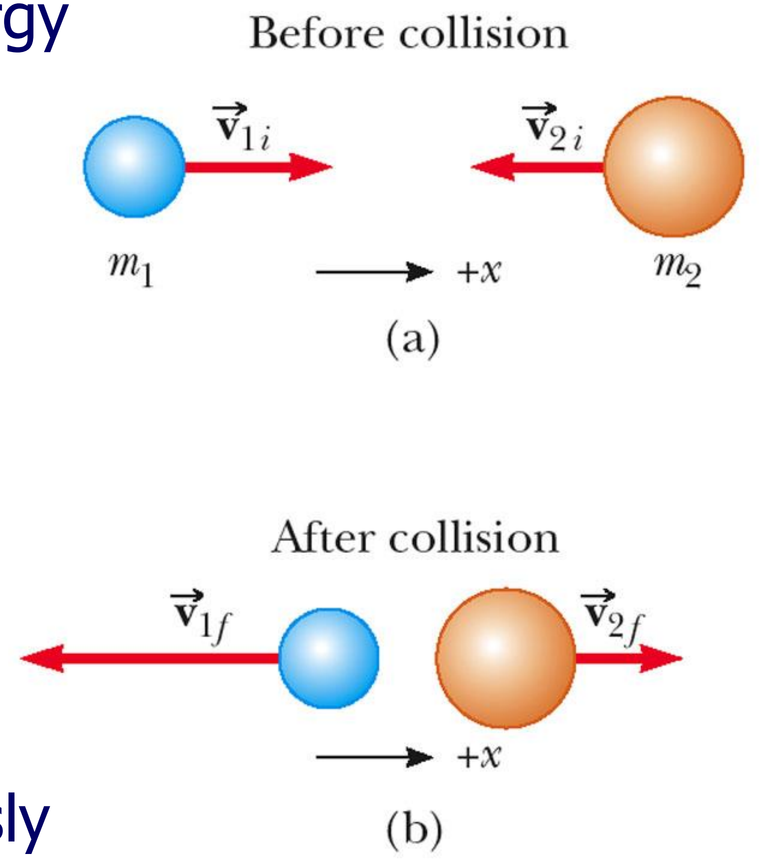
More About Elastic Collisions

- Both momentum and kinetic energy are conserved

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

- Typically have two unknowns
- Momentum is a vector quantity
  - Direction is important
  - Be sure to have the correct signs
- Solve the equations simultaneously



Elastic Collisions

- A simpler equation can be used in place of the KE equation

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

$$m_1(v_{1i}^2 - v_{1f}^2) = m_2(v_{2f}^2 - v_{2i}^2)$$

$$m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) = m_2(v_{2f} - v_{2i})(v_{2f} + v_{2i})$$

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

$$m_1(v_{1i} - v_{1f}) = m_2(v_{2f} - v_{2i})$$

$$\boxed{v_{1i} + v_{1f} = v_{2f} + v_{2i}}$$

$$\boxed{m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}}$$

Summary of Types of Collisions

- In an elastic collision, both momentum and kinetic energy are conserved

$$v_{1i} + v_{1f} = v_{2f} + v_{2i}$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

- In an inelastic collision, momentum is conserved but kinetic energy is not

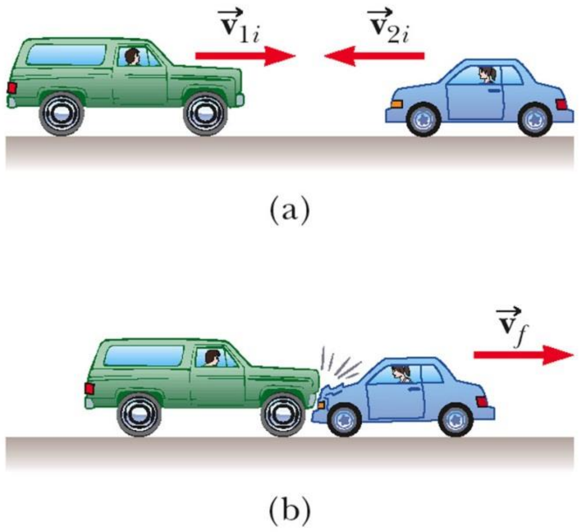
$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

- In a *perfectly* inelastic collision, momentum is conserved, kinetic energy is not, and the two objects stick together after the collision, so their final velocities are the same

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

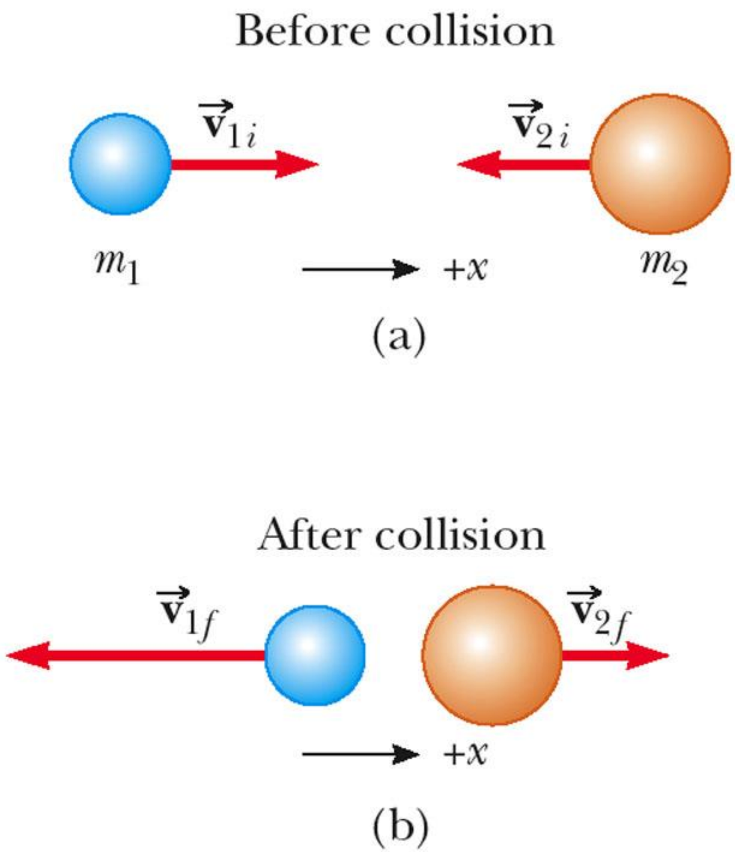
Conservation of Momentum

- An object of mass m moves to the right with a speed v. It collides head-on with an object of mass $3m$ moving with speed $v/3$ in the opposite direction. If the two objects stick together, what is the speed of the combined object, of mass $4m$, after the collision?
  (A) 0
  (B) v/2
  (C) v
  (D) 2v
  (E) 4v



Problem Solving for 1D Collisions, 1

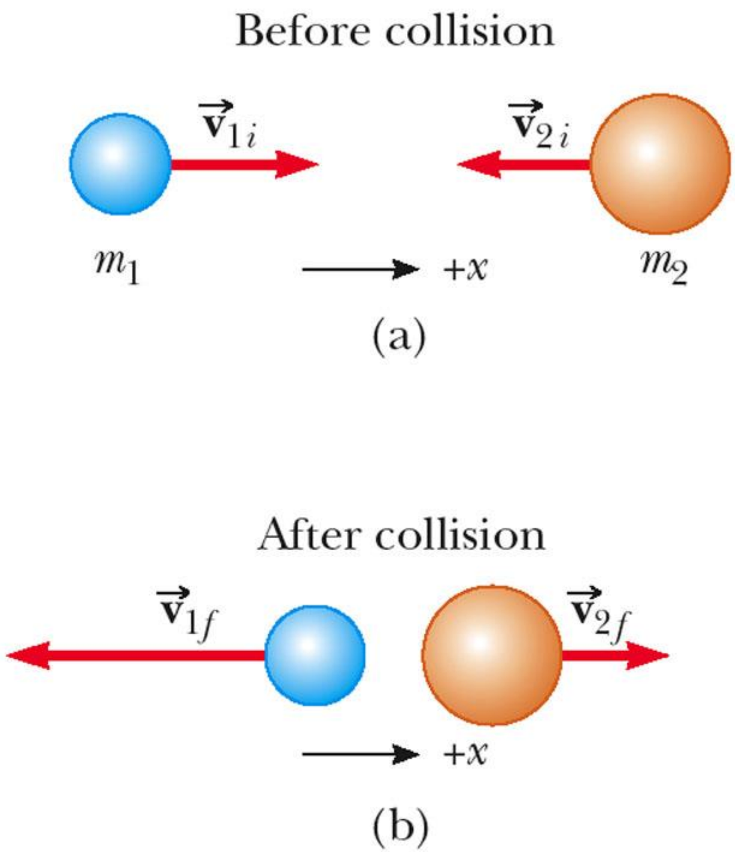
- **Coordinates:** Set up a coordinate axis and define the velocities with respect to this axis
  - It is convenient to make your axis coincide with one of the initial velocities
- **Diagram:** In your sketch, draw all the velocity vectors and label the velocities and the masses



Problem Solving for 1D Collisions, 2

- **Conservation of Momentum:** Write a general expression for the total momentum of the system *before* and *after* the collision
  - Equate the two total momentum expressions
  - Fill in the known values

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

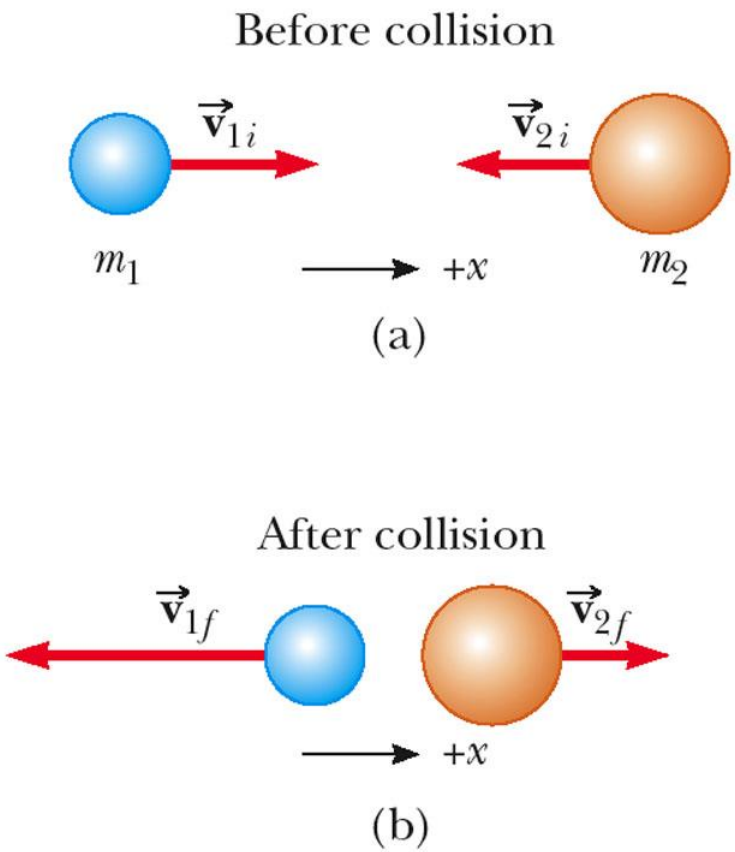


Problem Solving for 1D Collisions, 3

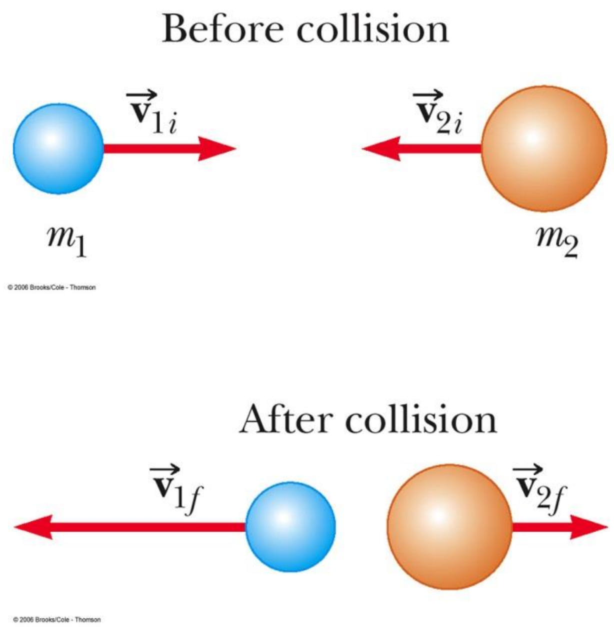
- **Conservation of Energy:** If the collision is elastic, write a second equation for conservation of KE, or the alternative equation
  - This only applies to perfectly elastic collisions

$$v_{1i} + v_{1f} = v_{2f} + v_{2i}$$

- **Solve:** the resulting equations simultaneously



One-Dimension vs Two-Dimension

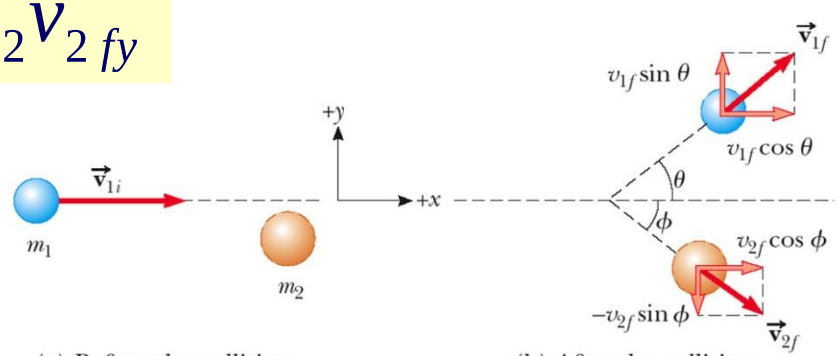


Two-Dimensional Collisions

- For a general collision of two objects in two-dimensional space, the conservation of momentum principle implies that the *total momentum of the system in each direction is conserved*

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$



(a) Before the collision (b) After the collision

Two-Dimensional Collisions

- The momentum is conserved in all directions
- Use subscripts for
  - Identifying the object
  - Indicating initial or final values
  - The velocity components

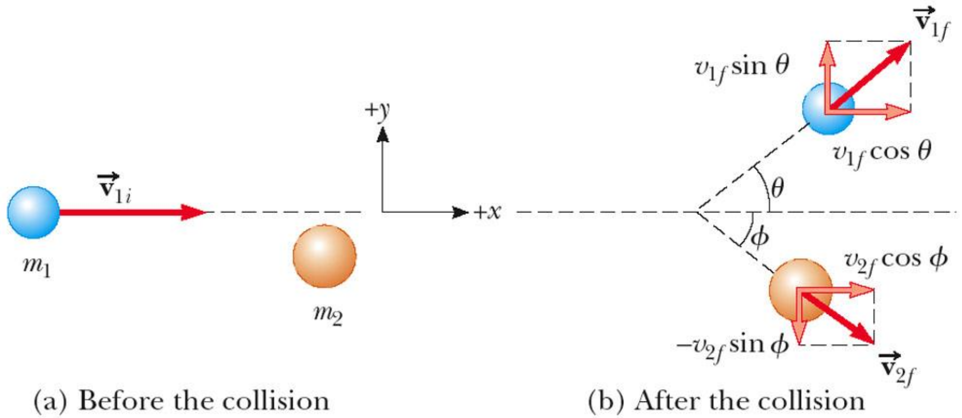
$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$

- If the collision is elastic, use conservation of kinetic energy as a second equation
  - Remember, the simpler equation can only be used for one-dimensional situations

$$v_{1i} + v_{1f} = v_{2f} + v_{2i}$$

Glancing Collisions



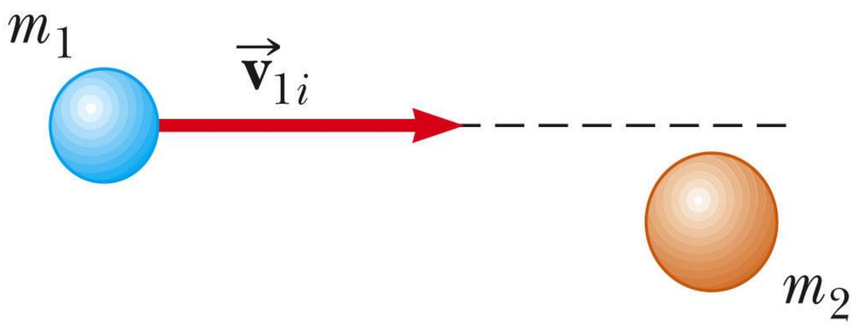
- The "after" velocities have x and y components
- Momentum is conserved in the x direction and in the y direction
- Apply conservation of momentum separately to each direction

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$

2-D Collision, example

- Particle 1 is moving at velocity $\vec{\mathbf{v}}_{1i}$ and particle 2 is at rest
- In the *x*-direction, the initial momentum is $m_1 v_{1i}$
- In the *y*-direction, the initial momentum is 0



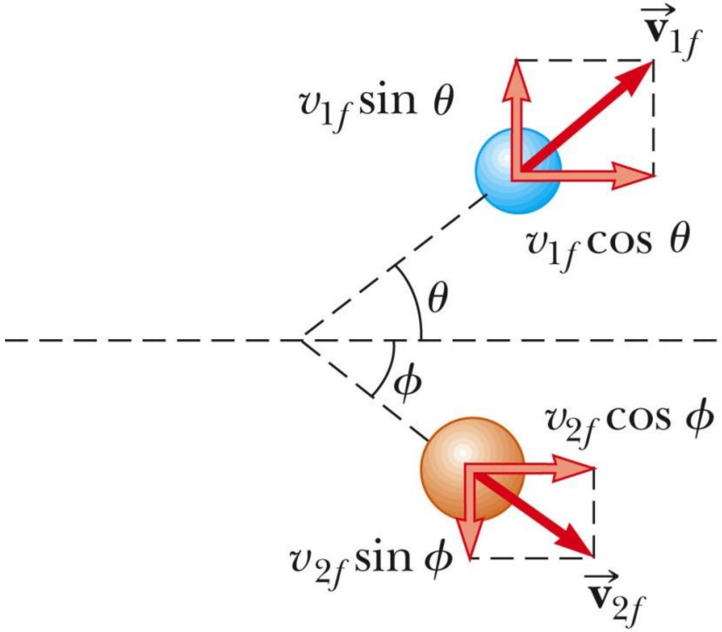
(a) Before the collision

2-D Collision, example cont

- After the collision, the momentum in the *x*-direction is $m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\phi$
- After the collision, the momentum in the *y*-direction is $m_1 v_{1f} \sin\theta + m_2 v_{2f} \sin\phi$

$$m_1 v_{1i} + 0 = m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\phi$$

$$0 + 0 = m_1 v_{1f} \sin\theta - m_2 v_{2f} \sin\phi$$



(b) After the collision

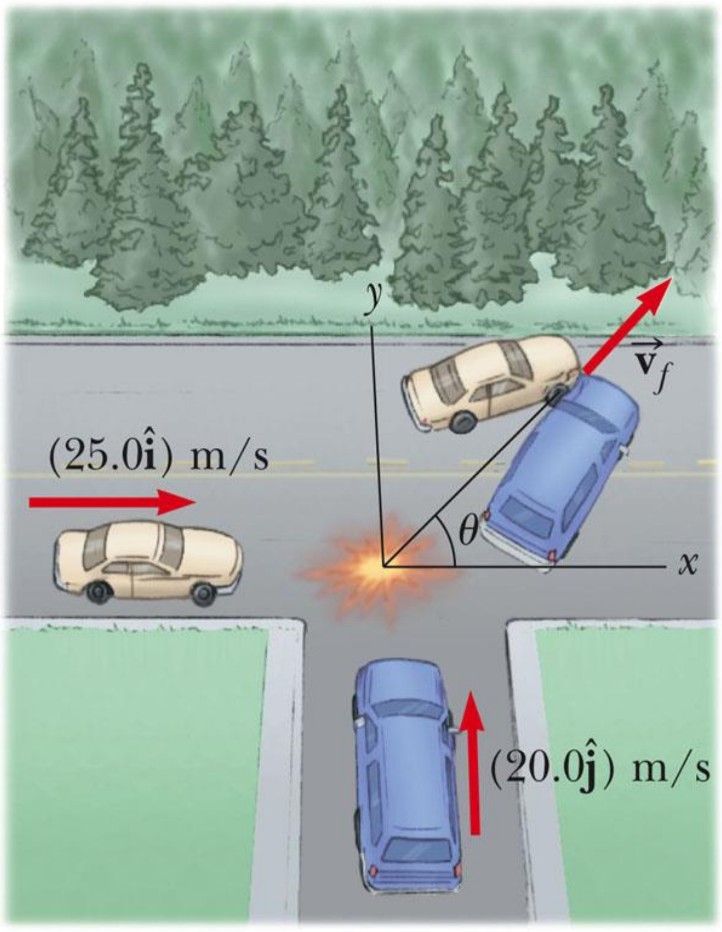
- If the collision is elastic, apply the kinetic energy equation

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

Collision at an Intersection

- A car with mass 1.5×10^3 kg traveling east at a speed of 25 m/s collides at an intersection with a 2.5×10^3 kg van traveling north at a speed of 20 m/s. Find the magnitude and direction of the velocity of the wreckage after the collision, assuming that the vehicles undergo a perfectly inelastic collision and assuming that friction between the vehicles and the road can be neglected.

$$m_c = 1.5\times10^3 kg, m_v = 2.5\times10^3 kg$$
$$v_{cix} = 25m/s, v_{viy} = 20m/s, v_f = ?\ \theta = ?$$



Collision at an Intersection

$$m_c = 1.5\times10^3\,\text{kg}, m_v = 2.5\times10^3\,\text{kg}$$

$$v_{cix} = 25\,\text{m/s}, v_{viy} = 20\,\text{m/s}, v_f = ?\,\theta = ?$$

$$\sum p_{xi} = m_c v_{cix} + m_v v_{vix} = m_c v_{cix} = 3.75\times10^4\,\text{kg}\cdot\text{m/s}$$

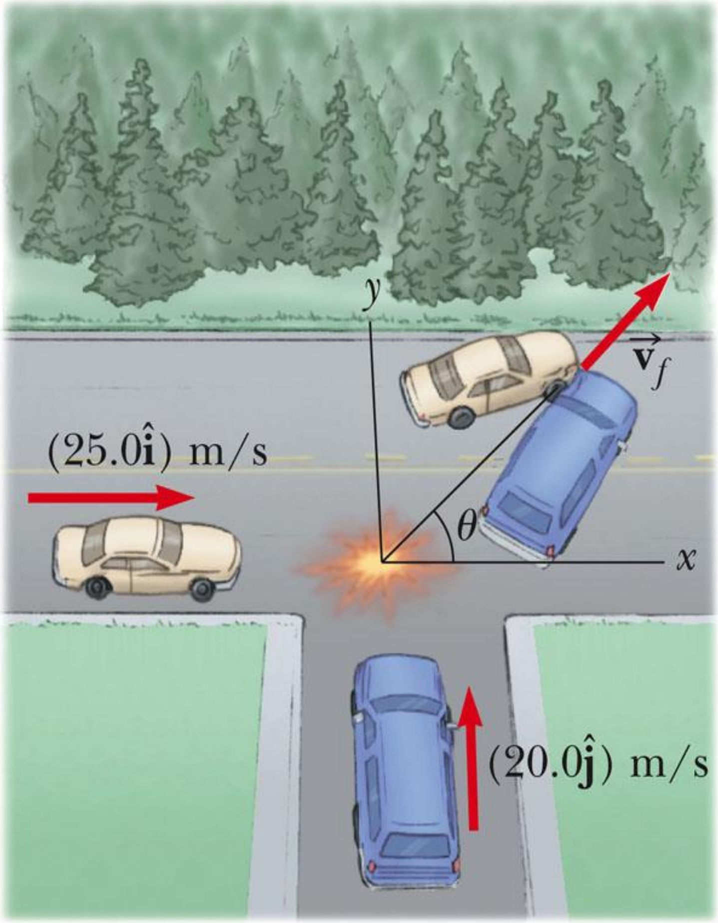
$$\sum p_{xf} = m_c v_{cfx} + m_v v_{vfx} = (m_c + m_v) v_f \cos\theta$$

$$3.75\times10^4\,\text{kg}\cdot\text{m/s} = (4.00\times10^3\,\text{kg}) v_f \cos\theta$$

$$\sum p_{yi} = m_c v_{ciy} + m_v v_{viy} = m_v v_{viy} = 5.00\times10^4\,\text{kg}\cdot\text{m/s}$$

$$\sum p_{yf} = m_c v_{cfy} + m_v v_{vfy} = (m_c + m_v) v_f \sin\theta$$

$$5.00\times10^4\,\text{kg}\cdot\text{m/s} = (4.00\times10^3\,\text{kg}) v_f \sin\theta$$



Collision at an Intersection

$$m_c = 1.5\times10^3 kg, m_v = 2.5\times10^3 kg$$

$$v_{cix} = 25m/s, v_{viy} = 20m/s, v_f = ?\theta = ?$$

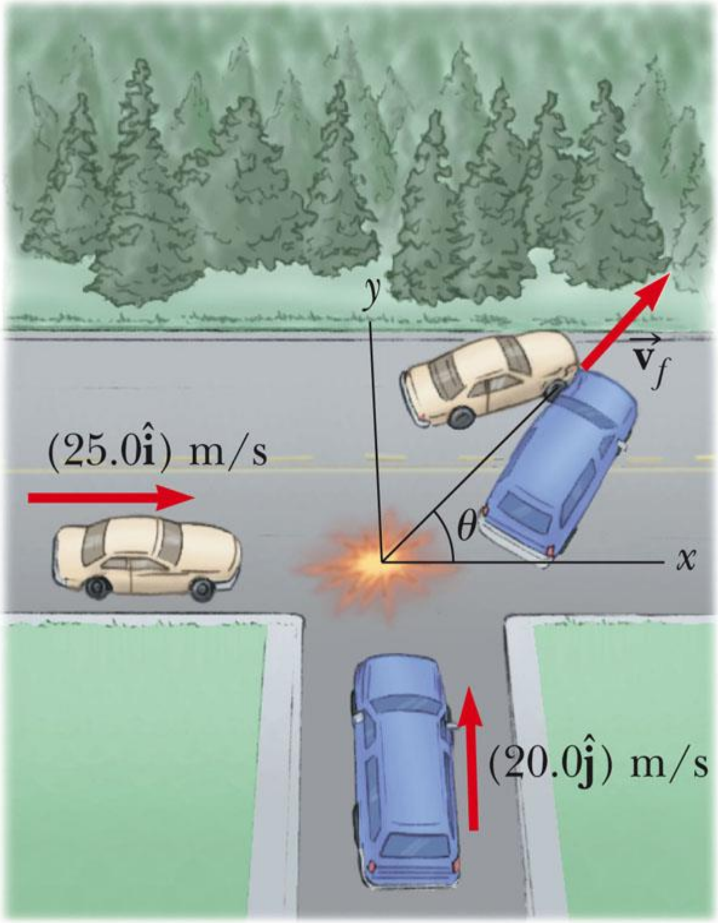
$$5.00\times10^4 \text{kg}\cdot\text{m/s} = (4.00\times10^3 \text{kg})v_f \sin\theta$$

$$3.75\times10^4 \text{kg}\cdot\text{m/s} = (4.00\times10^3 \text{kg})v_f \cos\theta$$

$$\tan\theta = \frac{5.00\times10^4 kg\cdot m/s}{3.75\times10^4 kg\cdot m/s} = 1.33$$

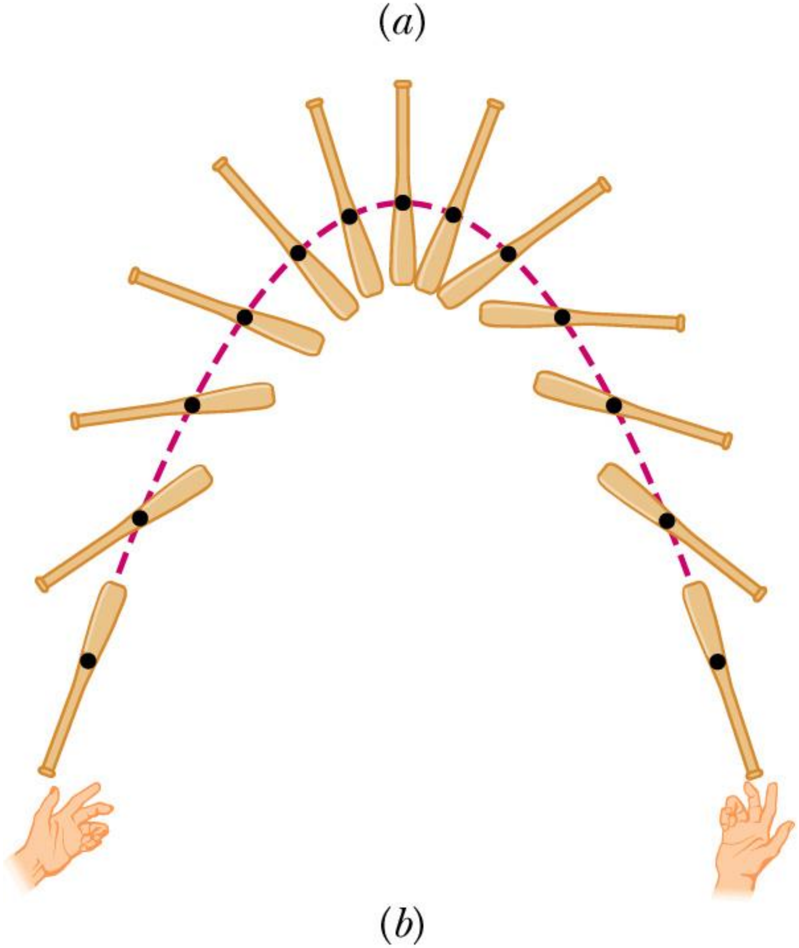
$$\theta = \tan^{-1}(1.33) = 53.1^\circ$$

$$v_f = \frac{5.00\times10^4 \text{kg}\cdot\text{m/s}}{(4.00\times10^3 \text{kg})\sin 53.1^\circ} = 15.6 \text{m/s}$$



The Center of Mass

- How should we define the position of the moving body ?
- What is y for U_g = mgy ?
- Take the average position of mass. Call "Center of Mass" (COM or CM)

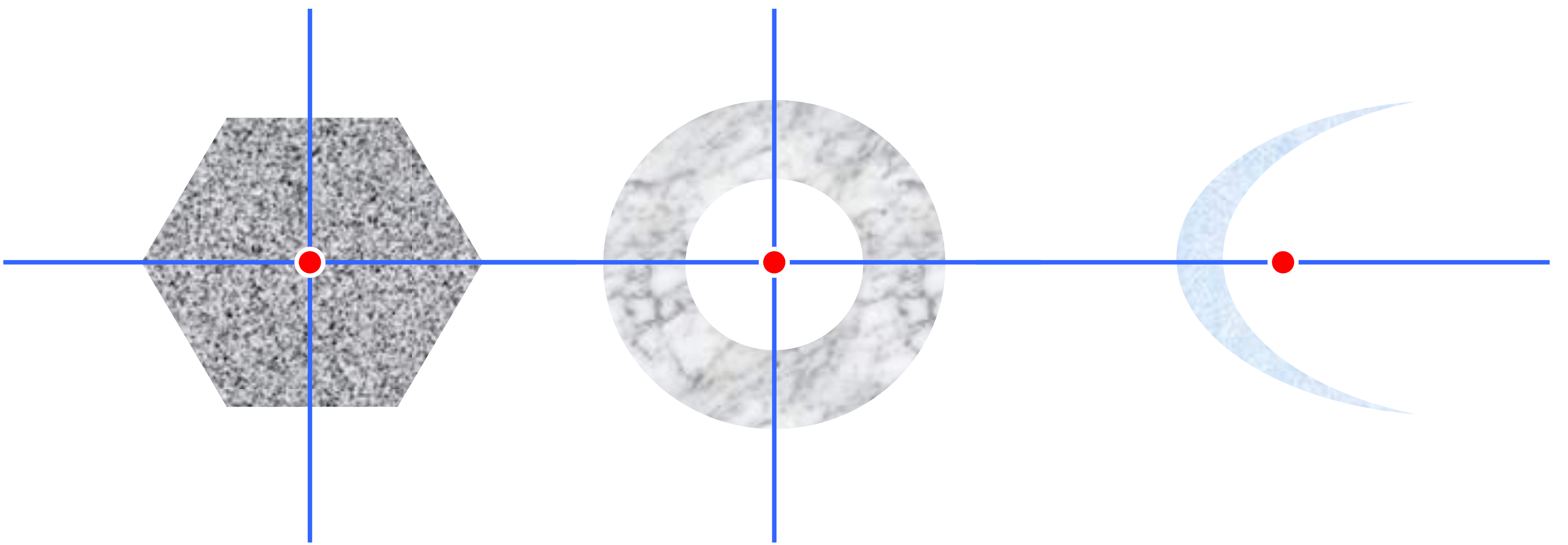


The Center of Mass

- There is a special point in a system or object, called the ***center of mass***, that moves as if all of the mass of the system is concentrated at that point
- The CM of an object or a system is the point, where the object or the system can be balanced in the uniform gravitational field

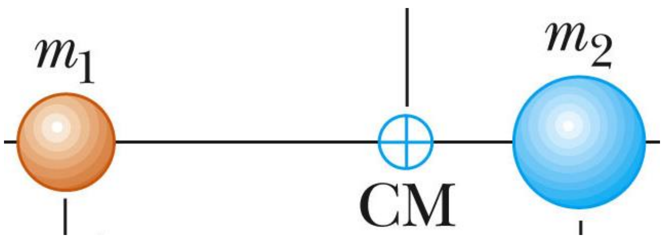
The Center of Mass

- The center of mass of any symmetric object lies on an axis of symmetry and on any plane of symmetry
  - If the object has uniform density
- The CM may reside inside the body, or outside the body

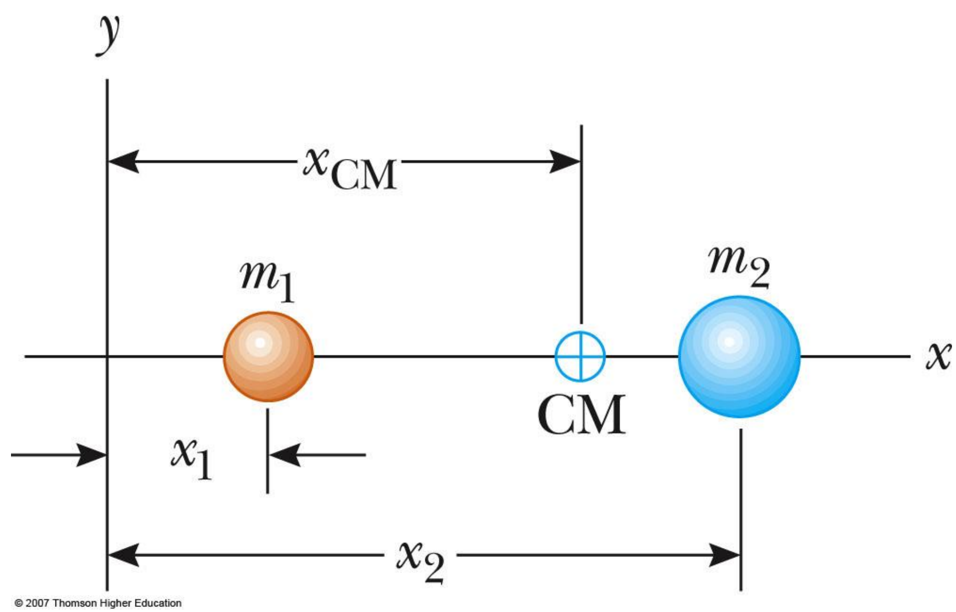


Where is the Center of Mass ?

- The center of mass of particles
- Two bodies in 1 dimension



$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$



Center of Mass for many particles in 3D?

Where is the Center of Mass ?

- Assume m_1 = 1 kg, m_2 = 3 kg, and x_1 = 1 m, x_2 = 5 m, where is the center of mass of these two objects?

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

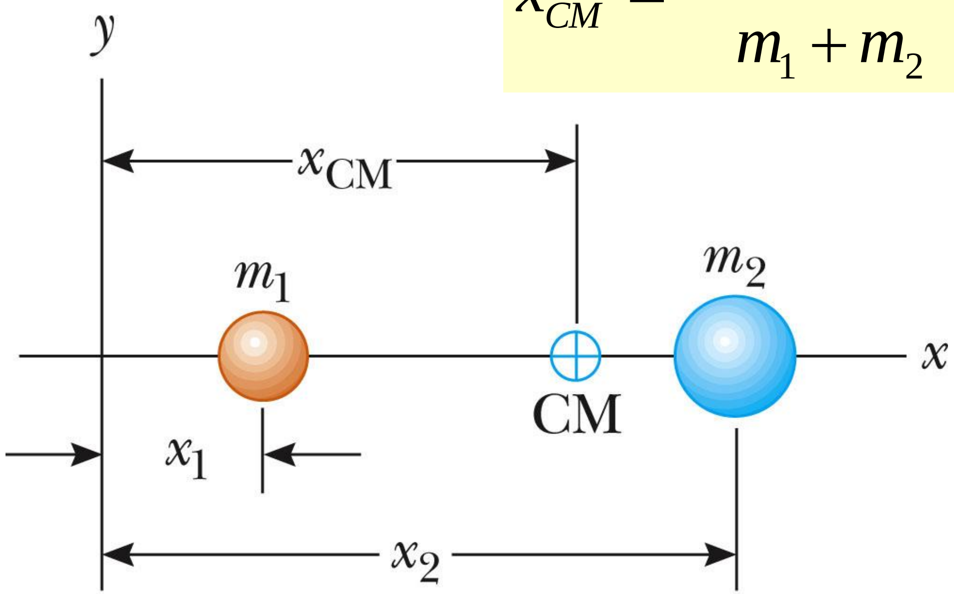
A) x_{CM} = 1 m

B) x_{CM} = 2 m

C) x_{CM} = 3 m

D) x_{CM} = 4 m

E) x_{CM} = 5 m



Center of Mass for a System of Particles

- Two bodies and one dimension

$$x_{\text{com}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

- General case: n bodies and three dimension

$$x_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i, \quad y_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i, \quad z_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i$$

  - where $M = m_1 + m_2 + m_3 + \ldots$

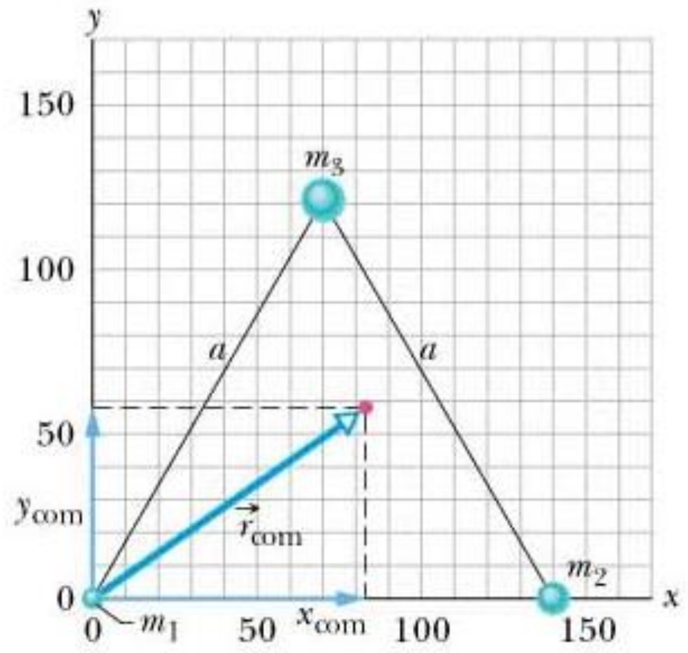
$$\vec{r}_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i$$

Sample Problem : Three particles of masses $m1 = 1.2$ kg, $m2 = 2.5$ kg, and $m3 = 3.4$ kg form an equilateral triangle of edge length $a = 140$ cm. Where is the center of mass of this system? (Hint: m1 is at (0,0), m2 is at (140 cm,0), and m3 is at (70 cm, 120 cm), as shown in the figure below.)

$$x_{CM} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$y_{CM} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

$$x_{CM} = 82.8\,\text{cm} \quad \text{and} \quad y_{CM} = 57.5\,\text{cm}$$



Motion of a System of Particles

- Assume the total mass, M, of the system remains constant
- We can describe the motion of the system in terms of the velocity and acceleration of the center of mass of the system
- We can also describe the momentum of the system and Newton’s Second Law for the system

Velocity and Momentum of a System of Particles

- The velocity of the center of mass of a system of particles is

$$\vec{\mathbf{v}}_{\text{CM}} = \frac{d\vec{\mathbf{r}}_{\text{CM}}}{dt} = \frac{1}{M}\sum_i m_i \vec{\mathbf{v}}_i$$

- The momentum can be expressed as

$$M\vec{\mathbf{v}}_{\text{CM}} = \sum_i m_i \vec{\mathbf{v}}_i = \sum_i \vec{\mathbf{p}}_i = \vec{\mathbf{p}}_{\text{tot}}$$

- The total linear momentum of the system equals the total mass multiplied by the velocity of the center of mass

Acceleration and Force of the Center of Mass

- The acceleration of the center of mass can be found by differentiating the velocity with respect to time

$$\vec{\mathbf{a}}_{\text{CM}} = \frac{d\vec{\mathbf{v}}_{\text{CM}}}{dt} = \frac{1}{M}\sum_i m_i \vec{\mathbf{a}}_i$$

- The acceleration can be related to a force

$$M\vec{\mathbf{a}}_{\text{CM}} = \sum_i \vec{\mathbf{F}}_i$$

- If we sum over all the internal forces, they cancel in pairs and the net force on the system is caused only by the external forces

Newton's Second Law for a System of Particles

- Since the only forces are external, the net external force equals the total mass of the system multiplied by the acceleration of the center of mass:

$$\sum \vec{\mathbf{F}}_{ext} = M\vec{\mathbf{a}}_{CM}$$

- The center of mass of a system of particles of combined mass *M* moves like an equivalent particle of mass *M* would move under the influence of the net external force on the system