## CHAPTER 5

## Equilibrium of a Rigid Body Under Coplanar Forces

The Torque ( $\tau$ ) about an axis, due to a force, is a measure of the effectiveness of the force in producing rotation about that axis. The word comes from the French for "twist." It is defined in the following way:

$$
\begin{equation*}
\text { Torque }=\tau=r F \sin \theta \tag{5.1}
\end{equation*}
$$

where $r$ is the radial distance from the axis of rotation to the point of application of the force, and $\theta$ is the acute angle between the lines-of-action of and $\overrightarrow{\mathbf{r}}$ and $\overrightarrow{\mathbf{F}}$, as shown in Fig. 5-1(a). Often this definition is written in terms of the lever arm of the force, which is the perpendicular distance from the axis of rotation to the line-ofaction of the force, as shown in Fig. 5-1 (b). Because the lever arm is simply $r \sin \theta$, the torque becomes

$$
\begin{equation*}
\tau=(F)(\text { lever arm }) \tag{5.2}
\end{equation*}
$$

The units of torque are newton-meters $(\mathrm{N} \cdot \mathrm{m})$. Plus and minus signs will be assigned to torques; for example, a torque that tends to cause counterclockwise rotation about the axis might be positive, whereas one causing clockwise rotation would then be negative. This will allow us to sum the influences of several torques acting simultaneously.


Fig. 5-1
The Two Conditions for Equilibrium of a rigid object under the action of coplanar forces are
(1) The first or force condition: The vector sum of all forces acting on the body must be zero:

$$
\begin{equation*}
\sum F_{x}=0 \quad \sum F_{y}=0 \tag{5.3}
\end{equation*}
$$

where the plane of the coplanar forces is taken to be the $x y$-plane.
(2) The second or torque condition: Take any axis perpendicular to the plane of the coplanar forces. Call the torques that tend to cause clockwise rotation about that axis negative, and counterclockwise torques positive; then the sum of all the torques acting on the object must be zero:

$$
\begin{equation*}
\AA \Sigma \tau=0 \tag{5.4}
\end{equation*}
$$

The Center-of-Gravity (c.g.) of an object is the point at which the entire weight of the object may be considered concentrated-that is, the line-ofaction of the weight passes through the center-of-gravity. A single vertical upwardly directed force, equal in magnitude to the weight of the object, applied through its center-of-gravity, will keep the object in equilibrium.

The Position of the Axis Is Arbitrary: If the sum of the torques is zero about one axis for a body that obeys the force condition, it is zero about all other axes parallel to the first. To make the math a little simpler, we can often choose the axis in such a way that the line-of-action of an unknown
force passes through the intersection of the axis and the plane of the forces. The angle $\theta$ between $\overrightarrow{\mathbf{r}}$ and $\overrightarrow{\mathbf{F}}$ is then zero; hence, that particular unknown force exerts zero torque and therefore does not appear in the torque equation.

## PROBLEM SOLVING GUIDE

Start the analysis of each problem by carefully reading it, several times if necessary. Once you know what was given and what you must find, write those quantities down with their appropriate symbols. The most important equations in this chapter are (5.3) and (5.4). Two equations will allow you to solve for two unknowns. Again-try doing the [I]-level worked-out problems first. Cover the solutions and look at them only after you're finished or you get stuck. Wait a day or two and then go back to any problem you could not do and try again, and again if need be, until you really master it.

## SOLVED PROBLEMS

5.1 [I] Imagine a bar of steel 80 cm long pivoted horizontally at its left end, as depicted in Fig. 5-2. Find the torque about axis- $A$ (which is perpendicular to the page) due to each of the forces shown acting at its right end.


Fig. 5-2
We use $\tau=r F \sin \theta$, taking clockwise torques to be negative while counterclockwise torques are positive. The individual torques due to the three forces are

For $10 \mathrm{~N}: \quad \tau=-(0.80 \mathrm{~m})(10 \mathrm{~N})\left(\sin 90^{\circ}\right)=-8.0 \mathrm{~N} \cdot \mathrm{~m}$
For $25 \mathrm{~N}: \quad \tau=+(0.80 \mathrm{~m})(25 \mathrm{~N})\left(\sin 25^{\circ}\right)=+8.5 \mathrm{~N} \cdot \mathrm{~m}$
For $20 \mathrm{~N}: \quad \tau= \pm(0.80 \mathrm{~m})(20 \mathrm{~N})\left(\sin 0^{\circ}\right)=0$
The line of the $20-\mathrm{N}$ force goes through the axis, and so $\theta=0^{\circ}$ for it. Or, put another way, because the line of the force passes through the axis, its lever arm is zero. Either way, the torque is zero for this (and any) force whose line-of-action passes through the axis. If you had trouble seeing which way the torques act, redraw the diagram on a piece of paper and imagine a pin stuck downward at A . Then put your finger at the right end of the rod and push the paper in the direction of the $10-\mathrm{N}$ force. The paper will rotate clockwise around the pin. That's the angular direction of the torque due to that force.
5.2 [II] A uniform metal beam of length $L$ weighs 200 N and holds a 450-N object as shown in Fig. 5-3. Find the magnitudes of the forces exerted on the beam by the two supports at its ends. Assume the lengths are exact.

Rather than draw a separate free-body diagram, we show the forces on the object being considered (the beam) in Fig. 5-3. Because the beam is uniform, its center of gravity is at its geometric center. Thus, the weight of the beam ( 200 N ) is shown acting downward at the beam's center. The forces $F_{1}$ and $F_{2}$ are exerted on the beam by the supports. Because there are no $x$ directed forces acting on the beam, we have only two equations to write for this equilibrium situation: $\Sigma F_{y}=0$ and $\Sigma \tau=0$.


Fig. 5-3

$$
+\uparrow \sum F_{y}=0 \quad \text { becomes } \quad F_{1}+F_{2}-200 \mathrm{~N}-450 \mathrm{~N}=0
$$

Before the torque equation is written, an axis must be chosen. We choose it at $A$, so that the unknown force $F_{1}$ will pass through it and exert no torque. The torque equation is then

$$
\AA \sum \tau_{A}=-(L / 2)(200 \mathrm{~N})\left(\sin 90^{\circ}\right)-(3 L / 4)(450 \mathrm{~N})\left(\sin 90^{\circ}\right)+L F_{2} \sin 90^{\circ}=0
$$

Dividing through the equation by $L$ and solving for $F_{2}$, we find that $F_{2}=438 \mathrm{~N}$.

To determine $F_{1}$, substitute the value of $F_{2}$ in the force equation, thereby obtaining $F_{1}=212 \mathrm{~N}$.
5.3 [II] A uniform, horizontal, $100-\mathrm{N}$ pipe is used as a lever, as shown in Fig. 5-4. Where must the fulcrum (the support point) be placed if a $500-\mathrm{N}$ weight at one end is to balance a $200-\mathrm{N}$ weight at the other end? What is the upward reaction force exerted by the support on the pipe?

The forces in question are shown in Fig. 5-4, where $F_{R}$ is the reaction force of the support on the pipe. The weight of the pipe acts downward at its center. We assume that the support point is at a distance $x$ from one end. Take the axis of rotation to be at the support point. Then the torque equation, $₫ \sum \sum \tau=0$, about that point becomes

$$
+(x)(200 \mathrm{~N})\left(\sin 90^{\circ}\right)+(x-L / 2)(100 \mathrm{~N})\left(\sin 90^{\circ}\right)-(L-x)(500 \mathrm{~N})\left(\sin 90^{\circ}\right)=
$$

$$
0
$$

This simplifies to

$$
(800 \mathrm{~N})(x)=(550 \mathrm{~N})(L)
$$

and so $x=0.69 L$. The support should be placed 0.69 of the way from the lighter-loaded end. To find $F_{R}$ use $+\uparrow \sum F_{y}=0$,

$$
F_{R}-200 \mathrm{~N}-100 \mathrm{~N}-500 \mathrm{~N}=0
$$

from which $F_{R}=800 \mathrm{~N}$.


Fig. 5-4
5.4 [II] Where must a $0.80-\mathrm{kN}$ object be hung on a uniform, horizontal, rigid $100-\mathrm{N}$ pole so that a girl pushing up at one end supports onethird as much as a woman pushing up at the other end?

The situation is shown in Fig. 5-5, where the weight of the pole acts down at its center. We represent the force exerted by the girl as $F$, and that by the woman as $3 F$. There are two unknowns, $F$ and $x$, and we will need two equations. To avoid the possibility of writing equations that turn out not to be independent, it's a good practice to write one sum-of-the-torques equation and one sum-of-the-forces equation. Take the rotational axis point at the left end. Then the torque equation becomes

$$
-(x)(800 \mathrm{~N})\left(\sin 90^{\circ}\right)-(L / 2)(100 \mathrm{~N})\left(\sin 90^{\circ}\right)+(L)(F)\left(\sin 90^{\circ}\right)=0
$$

For the second equation write

$$
+\uparrow \Sigma F_{y}=3 F-800 \mathrm{~N}-100 \mathrm{~N}+F=0
$$

from which $F=225 \mathrm{~N}$. Substitution of this value in the torque equation yields

$$
(800 \mathrm{~N})(x)=(225 \mathrm{~N})(L)-(100 \mathrm{~N})(L / 2)
$$

and so $x=0.22 L$. The load should be hung 0.22 of the way from
the woman to the girl.


Fig. 5-5


Fig. 5-6
5.5 [II] A uniform, horizontal, $0.20-\mathrm{kN}$ board of length $L$ has two objects hanging from it with weights of 300 N at exactly $L / 3$ from one end and 400 N at exactly $3 \mathrm{~L} / 4$ from the same end. What single additional force acting on the board will cause the board to be in equilibrium?

The situation is drawn in Fig. 5-6, where $F$ is the force we wish to find. For equilibrium, $\Sigma F_{y}=0$ and so

$$
F=400 \mathrm{~N}+200 \mathrm{~N}+300 \mathrm{~N}=900 \mathrm{~N}
$$

Because the board is to be in equilibrium, we are free to locate the axis of rotation anywhere. Choose it at point- $A$ at the left end of the board, since all the forces are measured (as to location) from that end in the diagram. Then $\Sigma \tau=0$, and taking counterclockwise as positive,

$$
\begin{gathered}
+(x)(F)\left(\sin 90^{\circ}\right)-(3 L / 4)(400 \mathrm{~N})\left(\sin 90^{\circ}\right)-(L / 2)(200 \mathrm{~N})\left(\sin 90^{\circ}\right)-(L / 3) \\
(300 \mathrm{~N})\left(\sin 90^{\circ}\right)=0
\end{gathered}
$$

Using $F=900 \mathrm{~N}$, we find that $x=0.56 L$. The required force is 0.90 kN upward at 0.56 L from the left end.
5.6 [III] The right-angle rule (or square) depicted in Fig. 5-7 hangs at rest from a peg as shown. It is made of a uniform metal sheet. One arm is $L \mathrm{~cm}$ long, while the other is $2 L \mathrm{~cm}$ long. Find (to two significant figures) the angle $\theta$ at which it will hang.


Fig. 5-7
If the rule is not too wide, we can approximate it as two thin rods of lengths $L$ and $2 L$ joined perpendicularly at $A$. Let $\gamma$ be the weight of each centimeter of rule. The forces acting are indicated in Fig. 5-7, where $F_{R}$ is the upward reaction force of the peg.

Write the torque equation using point- $A$ as the axis of rotation. Because $\tau=r F \sin \theta$ and because the torque about $A$ due to $F_{R}$ is zero, the torque equation becomes

$$
\Omega \sum \tau_{A}=+(L / 2)(\gamma L)\left[\sin \left(90^{\circ}-\theta\right)\right]-(L)(2 \gamma L)(\sin \theta)=0
$$

where the moment arm of the counterclockwise torque (due to $\gamma L$ ) is $(L / 2) \sin \left(90^{\circ}-\theta\right)$ and that of the clockwise torque (due to $2 \gamma L$ ) is $L \sin \theta$. Recall that $\sin \left(90^{\circ}-\theta\right)=\cos \theta$. After making this substitution and dividing by $2 \gamma L^{2} \cos \theta$,

$$
\frac{\sin \theta}{\cos \theta}=\tan \theta=\frac{1}{4}
$$

which yields $\theta=14^{\circ}$.
5.7 [II] Consider the situation illustrated in Fig. 5-8(a). The uniform 0.60kN beam is hinged at $P$. Find the tension in the tie rope and the components of the reaction force exerted by the hinge on the beam. Give your answers to two significant figures.

(a)

(b)

Fig. 5-8
The reaction forces acting on the beam are shown in Fig. 5-8(b), where the force exerted by the hinge is represented by its horizontal and vertical components, $F_{R H}$ and $F_{R V}$. The torque equation about $P$ is

$$
\curvearrowright \sum \tau_{P}=+(3 L / 4)\left(F_{T}\right)\left(\sin 40^{\circ}\right)-(L)(800 \mathrm{~N})\left(\sin 90^{\circ}\right)-(L / 2)(600 \mathrm{~N})\left(\sin 90^{\circ}\right)=0
$$

(We take the axis at $P$ because then $F_{R H}$ and $F_{R V}$ do not appear in the torque equation.) Solving this equation yields $F_{T}=2280 \mathrm{~N}$ or,
to two significant figures, $F_{T}=2.3 \mathrm{kN}$.
To find $F_{R H}$ and $F_{R V}$, write

$$
\begin{array}{lll} 
\pm \sum F_{x}=0 & \text { or } & -F_{T} \cos 40^{\circ}+F_{R H}=0 \\
+\uparrow \sum F_{y}=0 & \text { or } & F_{T} \sin 40^{\circ}+F_{R V}-600-800=0
\end{array}
$$

Since we know $F_{T}$, these equations lead to $F_{R H}=1750 \mathrm{~N}$ or 1.8 kN and $F_{R V}=65.6 \mathrm{~N}$ or 66 N .
5.8 [II] A uniform, $0.40-\mathrm{kN}$ boom is supported as shown in Fig. 5-9 (a). Find the tension in the tie rope and the force exerted on the boom by the pin at $P$.

The forces acting on the boom are shown in Fig. 5-9(b). Take the pin as the axis of rotation. The torque equation is then
$丹+\sum \tau_{P}=+(3 L / 4)\left(F_{T}\right)\left(\sin 50^{\circ}\right)-(L / 2)(400 \mathrm{~N})\left(\sin 40^{\circ}\right)-(L)(2000 \mathrm{~N})\left(\sin 40^{\circ}\right)=0$

(a)


Fig. 5-9
from which it follows that $F_{T}=2460 \mathrm{~N}$ or 2.5 kN . Now write

$$
\pm \sum F_{x}=0 \quad \text { or } \quad F_{R H}-F_{T}=0
$$

and so $F_{R H}=2.5 \mathrm{kN}$. Also,

$$
+\uparrow \sum F_{y}=0 \quad \text { or } \quad F_{R V}-2000 \mathrm{~N}-400 \mathrm{~N}=0
$$

and so $F_{R V}=2.4 \mathrm{kN} . F_{R V}$ and $F_{R H}$ are the components of the reaction force at the pin. The magnitude of this force is

$$
\sqrt{(2400)^{2}+(2460)^{2}}=3.4 \mathrm{kN}
$$

The tangent of the angle it makes with the horizontal is $\tan \theta=$ $2400 / 2460$, and so $\theta=44^{\circ}$.
5.9 [II] As indicated in Fig. 5-10, hinges $A$ and $B$ hold a uniform, 400-N door in place. If the upper hinge happens to support the entire weight of the door, find the forces exerted on the door at both hinges. The width of the door is exactly $h / 2$, where $h$ is the distance between the hinges.


Fig. 5-10
The forces acting on the door are shown in Fig. 5-10. Only a horizontal force acts at $B$, because the upper hinge is assumed to support the door's weight. Take torques about point- $A$ as the axis of rotation:

$$
\oiint \sum \tau_{A}=0 \quad \text { becomes } \quad+(h)(F)\left(\sin 90.0^{\circ}\right)-(h / 4)(400 \mathrm{~N})\left(\sin 90.0^{\circ}\right)=0
$$

from which $F=100 \mathrm{~N}$. We also have

$$
\begin{array}{lll} 
\pm \sum F_{x}=0 & \text { or } & F-F_{R H}=0 \\
+\uparrow \sum F_{y}=0 & \text { or } & F_{R V}-400 \mathrm{~N}-0
\end{array}
$$

We find from these that $F_{R H}=100 \mathrm{~N}$ and $F_{R V}=400 \mathrm{~N}$.
For the resultant reaction force $F_{R}$ on the hinge at $A$, we have

$$
F_{R}=\sqrt{(400)^{2}+(100)^{2}}=412 \mathrm{~N}
$$

The tangent of the angle that $\overrightarrow{\mathbf{F}}_{R}$ makes with the negative $x$ direction is $F_{R V} / F_{R H}$, and so the angle is arctan $4.00=76.0^{\circ}$.
5.10 [II] A ladder leans against a smooth wall, as can be seen in Fig. 5-11. (By a "smooth" wall, we mean that the wall exerts on the ladder only a force that is perpendicular to the wall. There is no friction force.) The ladder weighs 200 N , and its center of gravity is 0.40 L from the base, where $L$ is the ladder's length. (a) How large a friction force must exist at the base of the ladder if it is not to slip? (b) What is the necessary coefficient of static friction?


Fig. 5-11
(a) We wish to find the friction force $F_{f}$. Notice that no friction force exists at the top of the ladder. Taking torques about point- $A$ gives the torque equation

$$
\AA \sum \tau_{A}=-(0.40 L)(200 \mathrm{~N})\left(\sin 40^{\circ}\right)+(L)\left(F_{N 2}\right)\left(\sin 50^{\circ}\right)=0
$$

Solving leads to $F_{N 2}=67.1 \mathrm{~N}$. We can also write

$$
\begin{array}{lll} 
\pm \sum F_{x}=0 & \text { or } & F_{\mathrm{f}}-F_{N 2}=0 \\
+\uparrow \sum F_{y}=0 & \text { or } & F_{N 1}-200=0
\end{array}
$$

and so $F_{\mathrm{f}}=67 \mathrm{~N}$ and $F_{N 1}=0.20 \mathrm{kN}$.
(b)

$$
\mu_{s}=\frac{F_{\mathrm{f}}}{F_{N 1}}=\frac{67.1}{200}=0.34
$$

5.11 [III] For the situation drawn in Fig. 5-12 (a), find $F_{T 1}, F_{T 2}$, and $F_{T 3}$. The boom is uniform and weighs 800 N .

First apply the force condition to point-A. The appropriate freebody diagram is shown in Fig. 5-12(b). We then have

$$
F_{T 2} \cos 50.0^{\circ}-2000 \mathrm{~N}=0 \quad \text { and } \quad F_{T 1}-F_{T 2} \sin 50.0^{\circ}=0
$$

From the first of these we find $F_{T 2}=3.11 \mathrm{kN}$; then the second equation gives $F_{T 1}=2.38 \mathrm{kN}$.

Let us now isolate the boom and apply the equilibrium conditions to it. The appropriate free-body diagram is found in Fig. 5-12(c). The torque equation, for torques taken about point- $C$, is

$$
\AA \sum \tau_{C}=+(L)\left(F_{T 3}\right)\left(\sin 20.0^{\circ}\right)-(L)(3110 \mathrm{~N})\left(\sin 90.0^{\circ}\right)-(L / 2)(800 \mathrm{~N})\left(\sin 40.0^{\circ}\right)=0
$$

Solving for $F_{T 3}$, we compute it to be 9.84 kN . If it were required, we could find $F_{R H}$ and $F_{R V}$ by using the $x$ - and $y$-force equations.


Fig. 5-12

## SUPPLEMENTARY PROBLEMS

5.12 [I] A steering wheel has a diameter of 40.0 cm . A force of 30.0 N is applied to its rim on the right, tangent to the wheel and in the plane of it. Determine the size of the resulting torque. [Hint: The moment arm is the radius. Watch out for units.]
5.13 [I] A wrench is 50.0 cm long. It is placed on a nut, and a force of 100 N is applied perpendicular to the wrench handle. This force is in the plane of the wrench and nut, at a distance of 30.0 cm from the center of the nut. Determine the size of the torque twisting the nut. [Hint: Draw a diagram and label the moment arm. Watch out for units.]
5.14 [I] A horizontal essentially weightless lever is pivoted so it can rotate freely in a vertical plane. A downward force of 30.0 N is applied perpendicularly to the lever at a point 25.0 cm from and to the right of the pivot. Determine the torque on the lever, about the pivot. [Hint: Draw a diagram and specify the direction of the torque.]
5.15 [I] A horizontal essentially weightless lever is pivoted at its center so it can rotate freely in a vertical plane. A downward force of 80.0 N is applied perpendicularly to the lever at a point 35.0 cm from and to the right of the pivot. Another downward force of 100.0 N is applied perpendicularly to the lever at a point 15.0 cm from and to the left of the pivot. Determine the net torque on the lever. [Hint: Draw a diagram.]
5.16 [I] A seesaw is 5.00 m long with a fulcrum at its center. The uniform plank is balanced horizontally when a $40.0-\mathrm{kg}$ kid sits at the very end on the right and an $80.0-\mathrm{kg}$ kid sits somewhere on the left. Locate that second kid. [Hint: Draw a diagram.]
5.17 [I] A force of 1000 N is applied downward at the right end of a $1.50-\mathrm{m}$ long, essentially weightless horizontal crowbar. The bar is pivoted on a rock 1.25 m from the right end. What is the maximum
amount of weight that can be supported on the left end before the bar moves? [Hint: Draw a diagram. Watch out for significant figures.]
5.18 [I] An essentially weightless shovel is 120 cm long. Someone holds it horizontally, supporting it with his left hand at the shovel's center of gravity and his right hand 80.0 cm to the right of the $c . g$. The shovel contains a $20.0-\mathrm{N}$ rock whose $c . g$. is 8.00 cm to the right of the edge of the shovel. How much force does the person exert down on the handle? [Hint: Draw a diagram and take the torques around the left hand to avoid the force of the left hand.]
5.19 [I] An 800-N painter stands on a uniform horizontal 100-N plank resting on the rungs of two separated stepladders. The plank is 4.00 m long, and it is supported at its very ends (not a very safe arrangement). The painter stands on the plank 1.00 m from its right end. Determine the upward force exerted by the ladder on the left. [Hint: Draw a diagram and locate the weight of the plank at its $c . g$. and take the torques around the right end.]
$\underline{5.20}$ [II] As depicted in Fig. 5-13, two people sit in a car that weighs 8000 N . The person in front weighs 700 N , while the one in the back weighs 900 N . Call $L$ the distance between the front and back wheels. The car's center of gravity is a distance 0.400 L behind the front wheels. How much force does each front wheel and each back wheel support if the people are seated along the centerline of the car?


Fig. 5-13
5.21 [I] Two people, one at each end of a uniform beam that weighs 400 N , hold the beam at an angle of $25.0^{\circ}$ to the horizontal. How large a vertical force must each person exert on the beam?
5.22 [II] Repeat Problem 5.13 if a $140-\mathrm{N}$ child sits on the beam at a point one-fourth of the way along the beam from its lower end.
5.23 [II] Shown in Fig. 5-14 is a uniform 1600-N beam hinged at one end and held by a horizontal tie rope at the other. Determine the tension $F_{T}$ in the rope and the force components at the hinge.


Fig. 5-14
5.24 [II] The uniform horizontal beam illustrated in Fig. 5-15 weighs 500 N and supports a $700-\mathrm{N}$ load. Find the tension in the tie rope and the reaction force of the hinge on the beam.


Fig. 5-15


Fig. 5-16
5.25 [II] The arm drawn in Fig. 5-16 supports a 4.0-kg sphere. The mass of the hand and forearm together is 3.0 kg and its weight acts at a point 15 cm from the elbow. Assuming all the forces are vertical, determine the force exerted by the biceps muscle.
5.26 [II] The mobile depicted in Fig. 5-17 hangs in equilibrium. It consists of objects held by vertical strings. Object-3 weighs 1.40 N , while each of the identical uniform horizontal bars weighs 0.50 N . Find (a) the weights of objects-1 and -2 , and (b) the tension in the upper string.


Fig. 5-17
5.27 [II] The hinges of a uniform door which weighs 200 N are 2.5 m apart. One hinge is a distance $d$ from the top of the door, while the other is a distance $d$ from the bottom. The door is 1.0 m wide. The weight of the door is supported by the lower hinge. Determine the forces exerted by the hinges on the door.
$\underline{5.28}$ [III] The uniform bar in Fig. 5-18 weighs 40 N and is subjected to the forces shown. Find the magnitude, location, and direction of the force needed to keep the bar in equilibrium.


Fig. 5-18
5.29 [III] The horizontal, uniform, 120-N board drawn in Fig. 5-19 is supported by two ropes as shown. A $0.40-\mathrm{kN}$ weight is suspended one-quarter of the way from the left end. Find $F_{T 1}, F_{T 2}$, and the
angle $\theta$ made by the rope on the left.


Fig. 5-19


Fig. 5-20
5.30 [III] The foot of a ladder rests against a wall, and its top is held by a horizontal tie rope, as indicated in Fig. 5-20. The ladder weighs 100 N , and its center of gravity is 0.40 of its length from the foot. A $150-\mathrm{N}$ child hangs from a rung that is 0.20 of the length from the top. Determine the tension in the tie rope and the components of the force on the foot of the ladder.
5.31 [III] A truss is made by hinging two uniform, $150-\mathrm{N}$ rafters as depicted in Fig. 5-21. They rest on an essentially frictionless floor and are held together by a horizontal tie rope. A $500-\mathrm{N}$ load is held at their apex. Find the tension in the tie rope.


Fig. 5-21
5.32 [III] A 900-N lawn roller is to be pulled over a $5.0-\mathrm{cm}$ high curb (see Fig. 5-22). The radius of the roller is 25 cm . What minimum pulling force is needed if the angle $\theta$ made by the handle is (a) $0^{\circ}$ and (b) $30^{\circ}$ ? [Hint: Find the force needed to keep the roller balanced against the edge of the curb, just clear of the ground.]


Fig. 5-22


Fig. 5-23
5.33 [II] In Fig. 5-23, the uniform horizontal beam weighs 500 N . If the tie rope can support 1800 N , what is the maximum value the load $F_{W}$ can have?
5.34 [III] The beam in Fig. 5-24 has negligible weight. If the system hangs in equilibrium when $F_{W 1}=500 \mathrm{~N}$, what is the value of $F_{W 2}$ ?


Fig. 5-24
5.35 [III] Repeat Problem 5.26, but now find $F_{W 1}$ if $F_{W 2}$ is 500 N . Here the beam weighs 300 N and is uniform.
5.36 [III] An object is subjected to the forces shown in Fig. 5-25. What single force $F$ applied at a point on the $x$-axis will balance these forces leaving the object motionless? (First find its components, and then find the force.) Where on the $x$-axis should the force be
applied? Notice that before $F$ is applied there is an unbalanced force with components to the left and upward.


Fig. 5-25
5.37 [III] The solid uniform disk of radius $b$ illustrated in Fig. 5-26 can turn freely on an axle through its center. A hole of diameter $D$ is drilled through the disk; its center is a distance $r$ from the axle. The weight of the material drilled out is $F_{W h}$. (a) Find the weight $F_{W}$ of an object hung from a string wound on the disk that will hold the disk in equilibrium in the position shown. (b) What would happen if the load $F_{W}$ vanished? Explain your answer.


Fig. 5-26

