CHAPTER 4

Equilibrium Under the Action of Concurrent Forces

Concurrent Forces are forces whose lines of action all pass through a common point. The forces acting on a point object are obviously concurrent because they are all applied at that same point.

An Object Is in Equilibrium under the action of concurrent forces provided it is not accelerating. It may be traveling at a constant speed, and yet as long as $a = 0$, the object is in equilibrium.

The First Condition for Equilibrium is the requirement that $\sum \vec{\mathbf{F}} = 0$ or, in component form,

$$\sum F_x = \sum F_y = \sum F_z = 0 \tag{4.1}$$

That is, the resultant of all external forces acting on the object must be zero. This condition is sufficient for equilibrium when the external forces are concurrent. A second condition must also be satisfied if an object is to be in equilibrium under nonconcurrent forces; it is discussed in Chapter 5.

Problem Solution Method (Concurrent Forces):

(1) Isolate the object for discussion.

(2) Show the forces acting on the isolated object in a diagram (the *free-body diagram*).

(3) Find the rectangular components of each force.

(4) Write the first condition for equilibrium in equation form.

(5) Solve for the required quantities.

The Weight of an Object ($\vec{F}_W$) is essentially the force with which gravity pulls downward upon it. Recall from the previous chapter that $F_W = mg$.

The Tensile Force ($\vec{F}_T$) acting on a string or cable or chain (or, indeed, on any structural member) is the applied force tending to stretch it. The scalar magnitude of the tensile force is the *tension* (F_T). When an object is in tension, the forces acting *on* it point outward away from its center, and the forces it exerts point inward toward its center. Remember that ropes, cables, and chains can only function in tension.

The Friction Force ($\vec{F}_f$) is a tangential force acting on an object that opposes the sliding of that object across an adjacent surface with which it is in contact. The friction force is parallel to the surface and opposite to the direction of motion or of impending motion.

The Normal Force ($\vec{F}_N$) on an object that is interacting with a surface is the component of the force exerted by the surface that is perpendicular to the surface.

Pulleys: When a system of several frictionless light-weight pulleys in equilibrium has a single continuous rope wound around it, the tension *in each length of the rope* is the same and it equals the force applied to the end of the rope (F) by some external agency (usually a person). Thus, when a load is supported by N lengths of a continuous rope, the net force delivered to the load, the output force, is NF. Often the pulley attached to the load moves with the load and we need only count up the number of lengths of rope (N) acting on that pulley to determine the output force. There's more material on pulleys in Chapter 7; see, for example, Problems 7.5 and 7.12.

PROBLEM SOLVING GUIDE

(1) Determine the object (a hook, a knot, a body, etc.) on which the forces of interest act.

(2) Draw a free-body diagram of that object.

(3) Find the x- and y-components of all the forces.

(4) Apply Eq. (4.1) with the appropriate signs.

(5) Solve for the required quantities. In two dimensions, you will have two equations and can solve for two unknowns.

Do not round off numbers in the middle of a calculation.

SOLVED PROBLEMS

4.1 [II] In Fig. 4-1(a), the tension in the horizontal cord is 30 N as shown. Find the weight of the hanging body.

The tension in cord-1 is equal to the weight of the body hanging from it. Therefore, $F_{T1} = F_W$, and we wish to find F_{T1} or F_W.

Notice that the unknown force F_{T1} and the known force of 30 N both pull on the knot at point P. It therefore makes sense to isolate the knot at P as our point object for which we will write the two sum-of-the-forces-equals-zero equations. The free-body diagram showing the forces on the knot is drawn as in Fig. 4-1(b). The force components are also shown there.

We next write the first condition for equilibrium for the knot. From the free-body diagram,

$$\overset{+}{\rightarrow}\sum F_x = 0 \quad \text{becomes} \quad 30\text{ N} - F_{T2}\cos 40^\circ = 0$$
$$+\uparrow\sum F_y = 0 \quad \text{becomes} \quad F_{T2}\sin 40^\circ - F_W = 0$$

Solving the first equation for F_{T2} gives F_{T2} = 39.2 N. Substituting this value in the second equation yields F_W = 25 N as the weight of the hanging body.

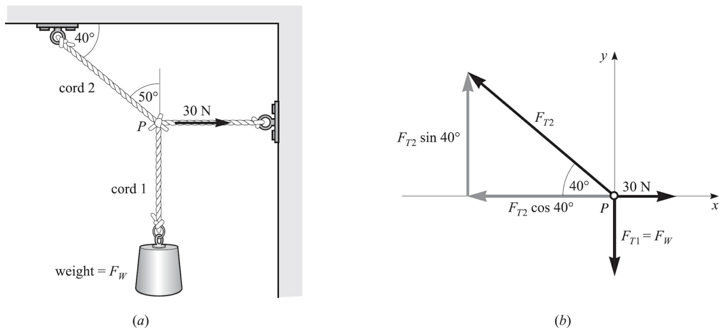

Fig. 4-1

4.2 [II] A rope extends between two poles. A 90-N boy hangs from it as shown in Fig. 4-2(*a*). Find the tensions in the two parts of the rope.

Label the two tensions F_{T1} and F_{T2}, and isolate the piece of rope at the boy's hands as the point object. That's the place where the three forces of interest act. And doing the analysis at that location will therefore allow F_{T1}, F_{T2}, and F_W, the boy's weight, to enter the equations. The free-body diagram for the object is found in Fig. 4-2(*b*).

After resolving the forces into their components as shown, write the first condition for equilibrium:

$$\overset{+}{\rightarrow}\sum F_x = 0 \quad \text{becomes} \quad F_{T2}\cos 5.0^\circ - F_{T1}\cos 10^\circ = 0$$
$$+\uparrow\sum F_y = 0 \quad \text{becomes} \quad F_{T2}\sin 5.0^\circ + F_{T1}\sin 10^\circ - 90\ \text{N} = 0$$

Evaluating the sines and cosines, these equations become

$$0.996F_{T2} - 0.985F_{T1} = 0 \quad \text{and} \quad 0.087F_{T2} + 0.174F_{T1} - 90 = 0$$

Solving the first for F_{T2} gives $F_{T2} = 0.990F_{T1}$. Substituting this in the second equation yields

$$0.086F_{T1} + 0.174F_{T1} - 90 = 0$$

from which $F_{T1} = 0.35$ kN. Then, because $F_{T2} = 0.990F_{T1}$, it follows that $F_{T2} = 0.34$ kN.

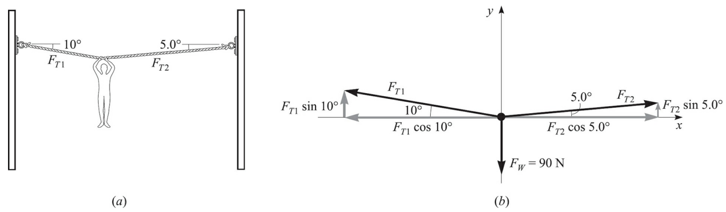

Fig. 4-2

4.3 [II] A 50-N box is slid straight across the floor at constant speed by a force of 25 N, as depicted in Fig. 4-3(*a*). How large a friction force impedes the motion of the box? (*b*) How large is the normal force? (*c*) Find μ_k between the box and the floor.

The forces acting on the box are shown in Fig. 4-3(*a*). The friction force is F_f, and the normal force, the supporting force exerted by the floor, is F_N. The free-body diagram and components are drawn in Fig. 4-3(*b*). Because the box is moving with constant velocity, it is in equilibrium. The first condition for equilibrium, taking to the right as positive,

$$\overset{+}{\rightarrow}\sum F_x = 0 \qquad \text{or} \qquad 25\cos 40° - F_f = 0$$

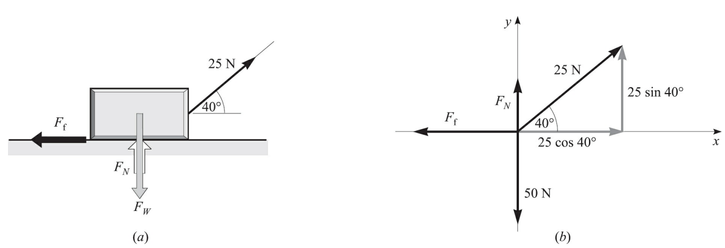

Fig. 4-3

(*a*) We can solve for the friction force F_f at once to find that F_f = 19.2 N, or to two significant figures, F_f = 19 N.

(*b*) To find F_N, use the fact that

$$+\uparrow\sum F_y = 0 \qquad \text{or} \qquad F_N + 25\sin 40° - 50 = 0$$

Solving gives the normal force as F_N = 33.9 N or, to two significant figures, F_N = 34 N.

(*c*) From the definition of μ_k,

$$\mu_k = \frac{F_f}{F_N} = \frac{19.2\text{ N}}{33.9\text{ N}} = 0.57$$

4.4 [II] Find the tensions in the ropes illustrated in Fig. 4-4(*a*) if the supported body weighs 600 N.

Select as our first point object the knot at *A* because we know one force acting on it. The weight of the hanging body pulls down on *A* with a force of 600 N, and so the free-body diagram for the knot is as shown in Fig. 4-4(*b*). Notice that the system is symmetrical and that will make things a lot simpler. Applying the first condition for equilibrium to point object *A*,

$$\overset{+}{\rightarrow}\sum F_x = 0 \qquad \text{or} \qquad F_{T2}\cos 60^\circ - F_{T1}\cos 60^\circ = 0$$
$$+\uparrow\sum F_y = 0 \qquad \text{or} \qquad F_{T1}\sin 60^\circ + F_{T2}\sin 60^\circ - 600 = 0$$

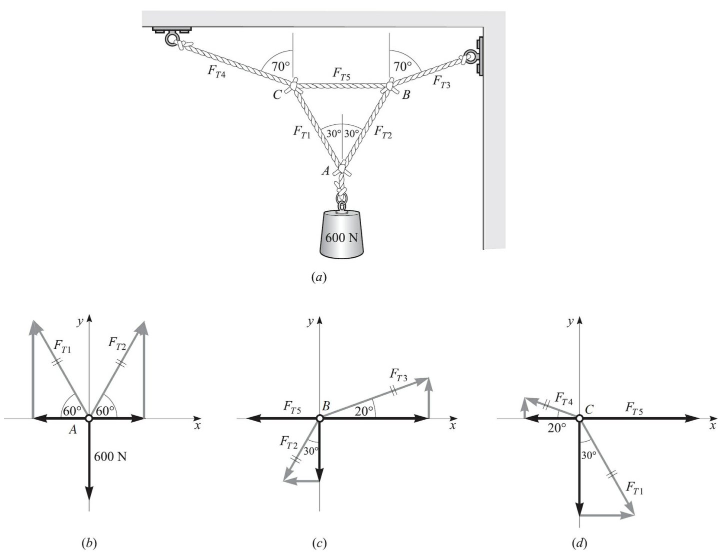

Fig. 4-4

The first equation yields $F_{T1} = F_{T2}$. (We could have inferred this from the symmetry of the system. Also from symmetry, $F_{T3} = F_{T4}$.) Substitution of F_{T1} for F_{T2} in the second equation gives $F_{T1} = 346$ N or 0.35 kN and so $F_{T2} = 346$ N or 0.35 kN.

Now isolate knot-*B* as our point object. Its free-body diagram is shown in Fig. 4-4(*c*). We have already found that F_{T2} = 346 N and so the equilibrium equations are

$$\overset{+}{\rightarrow}\sum F_x = 0 \quad \text{or} \quad F_{T3}\cos 20^\circ - F_{T5} - 346\sin 30^\circ = 0$$
$$+\uparrow\sum F_y = 0 \quad \text{or} \quad F_{T3}\sin 20^\circ - 346\cos 30^\circ = 0$$

The last equation yields F_{T3} = 876 N or 0.88 kN. Substituting this in the prior equation leads to F_{T5} = 650 N or 0.65 kN. As stated previously, from symmetry, $F_{T4} = F_{T3}$ = 876 N or 0.88 kN. How could you have found F_{T4} without recourse to symmetry? [*Hint*: See Fig. 4-4(*d*).]

4.5 [I] Each of the objects in Fig. 4-5 is in equilibrium. Find the normal force F_N in each case.

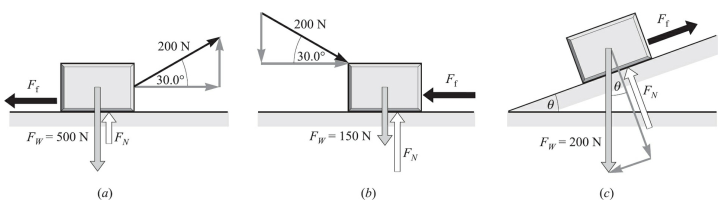

Fig. 4-5

Apply $+\uparrow\sum F_y = 0$ in each case.

(*a*) $F_N + (200\text{ N})\sin 30.0^\circ - 500 = 0$ from which $F_N = 400$ N

(*b*) $F_N - (200\text{ N})\sin 30.0^\circ - 150 = 0$ from which $F_N = 250$ N

(*c*) $F_N - (200\text{ N})\cos\theta = 0$ from which $F_N = (200\cos\theta)$ N

4.6 [I] For the situations of Problem 4.5, find the coefficient of kinetic friction if the object is moving with constant speed. Round off your answers to two significant figures.

We have already found F_N for each case in Problem 4.5. To determine F_f, the sliding-friction force, use $\overset{+}{\rightarrow}\sum F_x = 0$. Then employ the definition of μ_k.

(*a*) $200\cos 30.0^\circ - F_f = 0$ so that $F_f = 173$ N and $\mu_k = F_f/F_N = 173/400 = 0.43$.

(*b*) $200\cos 30.0^\circ - F_f = 0$ so that $F_f = 173$ N and $\mu_k = F_f/F_N = 173/250 = 0.69$.

(*c*) $-200\sin\theta + F_f = 0$ so that $F_f = (200\sin\theta)$ N and $\mu_k = F_f/F_N = (200\sin\theta)/(200\cos\theta) = \tan\theta$.

4.7 [II] Suppose that in Fig. 4-5(*c*) the block is at rest. The angle of the incline is slowly increased. At an angle $\theta = 42°$, the block begins to slide. What is the coefficient of static friction between the block and the incline? (The block and surface are not the same as in Problems 4.5 and 4.6.)

At the instant the block begins to slide, the friction force has its critical value. Therefore, $\mu_s = F_f/F_N$ at that instant. Following the method of Problems 4.5 and 4.6,

$$F_N = F_W \cos\theta \qquad \text{and} \qquad F_f = F_W \sin\theta$$

Therefore, when sliding just starts,

$$\mu_s = \frac{F_f}{F_N} = \frac{F_W \sin\theta}{F_W \cos\theta} = \tan\theta$$

But θ was found by experiment to be 42°. Therefore, $\mu_s = \tan 42° = 0.90$.

4.8 [II] Pulled by the 8.0-N load shown in Fig. 4-6(*a*), the 20-N block slides to the right at a constant velocity. Find μ_k between the block and the table. Assume the pulley to be both light and frictionless.

Because it is moving at a constant velocity, the 20-N block is in equilibrium. Since the pulley is frictionless, the tension in the continuous rope is the same on both sides of the pulley. Thus, $F_{T1} = F_{T2} = 8.0$ N.

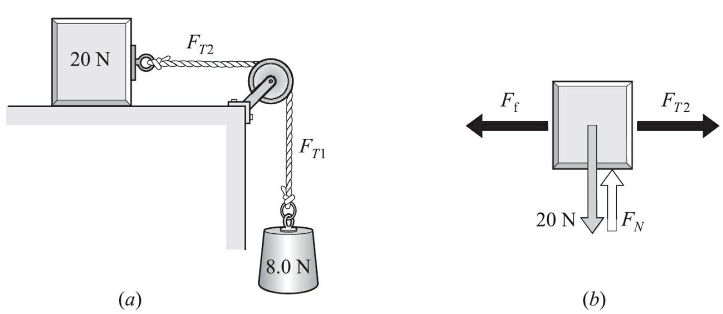

Fig. 4-6

Looking at the free-body diagram in Fig. 4-6(*b*) and recalling that the block is in equilibrium,

$$\overset{+}{\rightarrow}\sum F_x = 0 \qquad \text{or} \qquad F_f = F_{T2} = 8.0\ \text{N}$$
$$+\uparrow\sum F_y = 0 \qquad \text{or} \qquad F_N = 20\ \text{N}$$

Then, from the definition of μ_k,

$$\mu_k = \frac{F_f}{F_N} = \frac{8.0\ \text{N}}{20\ \text{N}} = 0.40$$

SUPPLEMENTARY PROBLEMS

4.9 [I] A person stands on a scale, which then reads 600 N. (*a*) What force is exerted on the scale by the person? (*b*) What force is exerted on the person by the scale? (*c*) What would happen to the reading as the person began to jump straight up?

4.10 [I] Two evenly matched teams of youngsters are having a tug-of-war. At a given moment each team pulls with a force of 2000 N. What is the tension in the rope at that instant?

4.11 [I] A rope is tied to a hook fastened to a brick wall. Someone then pulls horizontally on the rope with a force of 400 N, keeping the rope perpendicular to the wall. What is the value of the force on the hook? What is the tension in the rope?

4.12 [I] An essentially weightless pulley that is effectively without friction is attached to a ceiling hook. A very lightweight rope is passed over the pulley and hangs down on both sides. A 200.0-N load is then hung from each end of the rope. What is the value of the tension in the rope? Determine the net downward force on the hook.

4.13 [I] An essentially weightless rope is slung over a frictionless lightweight pulley that is attached to a hook in the ceiling. An object weighing 500 N is hung from one end, and a student holds the other end, keeping the system in equilibrium. What force must she pull with? In what direction? What is the value of the tension in the rope?

4.14 **[I]** An essentially weightless rope is slung over a lightweight frictionless pulley that is attached to a ceiling. A 20.0-kg mass is hung from one end of the rope, and a student holds the other end, keeping the system in equilibrium. What force must she pull with? In what direction? What is the value of the tension in the rope? Determine the total downward force on the ceiling.

4.15 **[I]** A 2.00-kg block rests on a frictionless air table. Two horizontal forces act on it; one is 500 N due east, and the other is 1200 N due south. What third force will keep the block from accelerating?

4.16 **[I]** The load in Fig. 4-7 is hanging at rest. Take the ropes to all be vertical and the pulleys to be weightless and frictionless. (*a*) How many segments of rope support the combination of the lower pulley and load? (*b*) What is the downward force on the lowest pulley (the "floating" one)? (*c*) What must be the total upward force exerted on the floating pulley by the two lengths of rope? (*d*) What is the upward force exerted on the floating pulley by each length of rope supporting it? (*e*) What is the tension in the rope wound around the two pulleys? (*f*) How much force is the man exerting? (*g*) What is the net downward force acting on the uppermost pulley? (*h*) How much force acts downward on the hook in the ceiling?

4.17 **[I]** (*a*) A 600-N load hangs motionlessly in Fig. 4-8. Assume the ropes to all be vertical and the pulleys to be weightless and frictionless. (*a*) What is the tension in the bottom hook attached, via a ring, to the load? (*b*) How many lengths of rope support the movable pulley? (*c*) What is the tension in the long rope? (*d*) How much force does the man apply? (*e*) How much force acts downward on the ceiling?

4.18 **[I]** For the situation shown in Fig. 4-9, find the values of F_{T1} and F_{T2}

if the hanging object’s weight is 600 N.

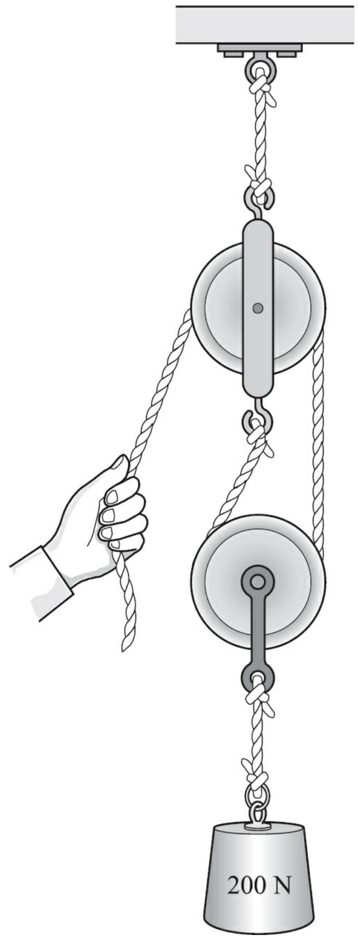

Fig. 4-7

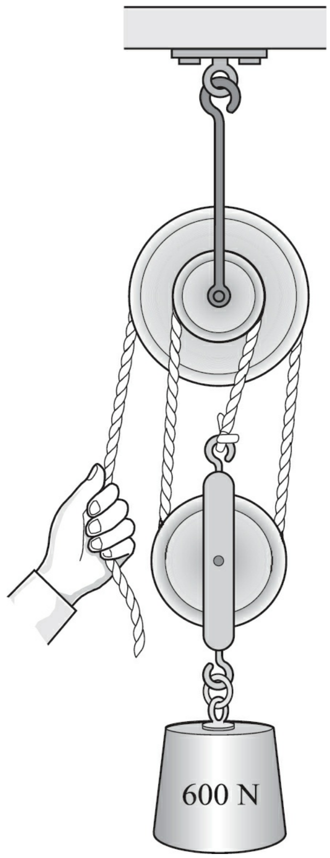

Fig. 4-8

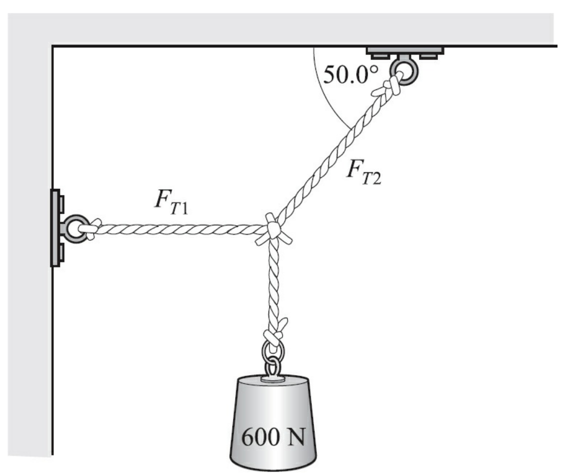

Fig. 4-9

4.19 **[I]** The following coplanar forces pull on a ring: 200 N at 30.0°, 500 N at 80.0°, 300 N at 240°, and an unknown force. Find the magnitude and direction of the unknown force if the ring is in equilibrium.

4.20 [II] In Fig. 4-10, the pulleys are frictionless and weightless and the system hangs in equilibrium. If F_{W3}, the weight of the hanging object on the right, is 200 N, what are the values of F_{W1} and F_{W2}?

4.21 [II] Suppose F_{W1} in Fig. 4-10 is 500 N. Find the values of F_{W2} and F_{W3} if the system is to hang in equilibrium as shown.

4.22 [I] If in Fig. 4-11 the friction between the block and the incline is negligible, how much must the object on the right weigh if the 200-N block is to remain at rest?

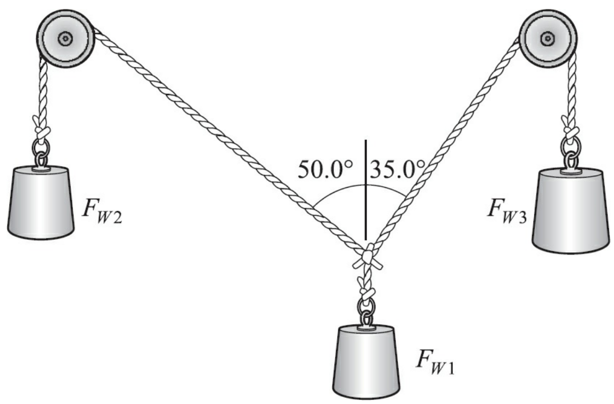

Fig. 4-10

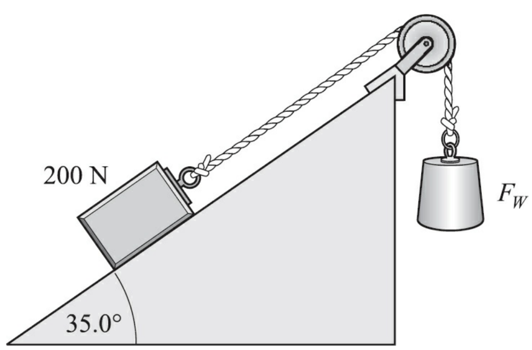

Fig. 4-11

4.23 [II] The system in Fig. 4-11 remains at rest when F_W = 220 N. What are the magnitude and direction of the friction force on the 200-N block?

4.24 **[II]** Find the normal force acting on the block in each of the equilibrium situations shown in Fig. 4-12.

4.25 **[II]** The block depicted Fig. 4-12(*a*) slides with constant speed under the action of the force shown. (*a*) How large is the retarding friction force? (*b*) What is the coefficient of kinetic friction between the block and the floor?

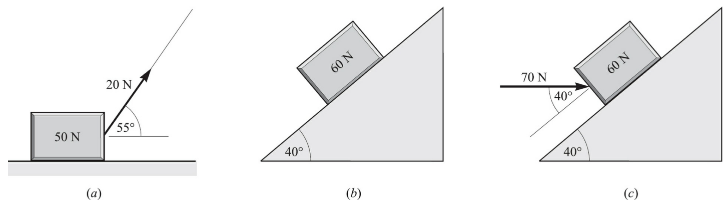

Fig. 4-12

4.26 **[II]** The block shown in Fig. 4-12(*b*) slides at a constant speed down the incline. (*a*) How large is the friction force that opposes its motion? (*b*) What is the coefficient of sliding (kinetic) friction between the block and the plane?

4.27 **[II]** The block in Fig. 4-12(*c*) just begins to slide up the incline when the pushing force shown is increased to 70 N. (*a*) What is the maximum static friction force on it? (*b*) What is the value of the coefficient of static friction

4.28 **[II]** If $F_W = 40$ N in the equilibrium situation shown in Fig. 4-13, find F_{T1} and F_{T2}.

4.29 **[III]** Refer to the equilibrium situation shown in Fig. 4-13. The cords are strong enough to withstand a maximum tension of 80 N. What is the largest value of F_W that they can support as shown?

4.30 **[III]** The hanging object in Fig. 4-14 is in equilibrium and has a weight $F_W = 80$ N. Find F_{T1}, F_{T2}, F_{T3}, and F_{T4}. Give all answers to two significant figures.

4.31 **[III]** The pulleys shown in Fig. 4-15 have negligible weight and friction. The long rope has one section that is at 40°; assume its other segments are vertical. What is the value of F_W if the system is at equilibrium?

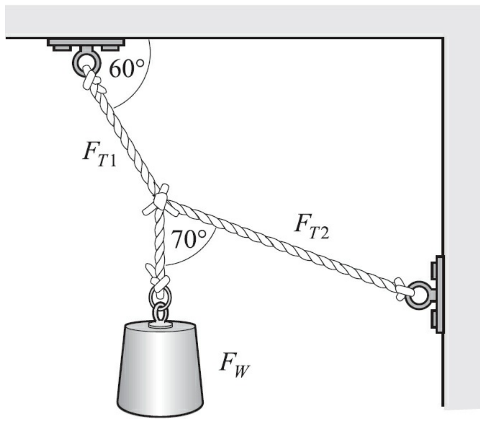

Fig. 4-13

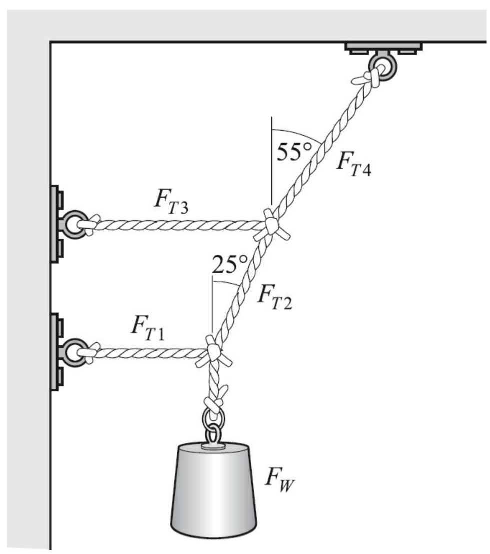

Fig. 4-14

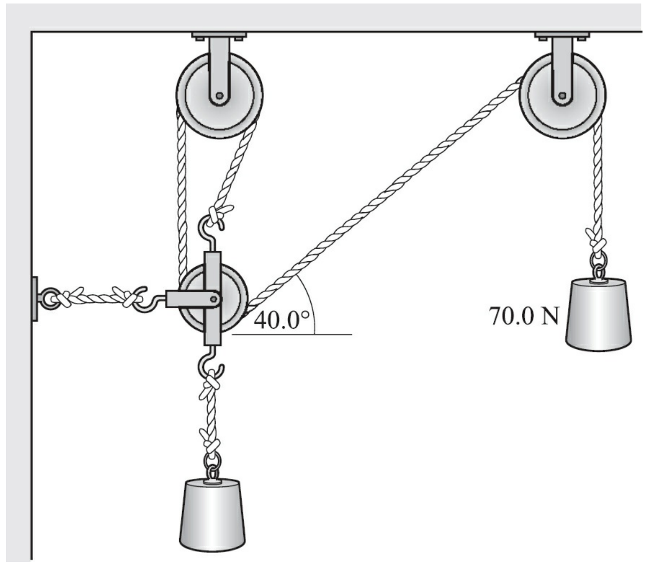

Fig. 4-15

4.32 **[III]** In Fig. 4-16, the system is at rest. (*a*) What is the maximum value that F_W can have if the friction force on the 40-N block cannot exceed 12.0 N? (*b*) What is the coefficient of static friction between the block and the tabletop?

4.33 **[III]** The block in Fig. 4-16 is just on the verge of slipping. If $F_W = 8.0$ N, what is the coefficient of static friction between the block and tabletop?

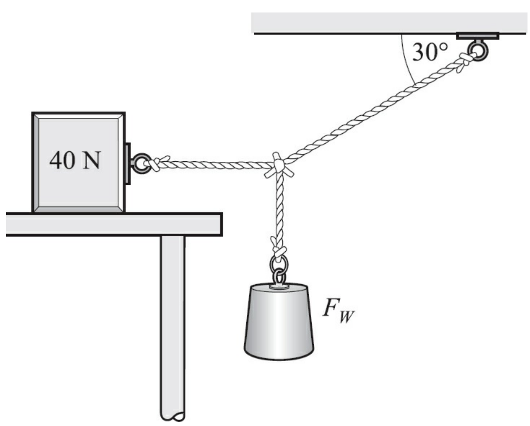

Fig. 4-16