5. PD Eksak

- Suatu PD dikatakan eksak jika :

$$M(x,y)\partial x + N(x,y)\partial y = 0$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

- Penyelesaian : F(x,y)=c

$$Cara\ .1$$

$$\frac{\partial F(x,y)}{\partial x} = M(x,y)$$

$$F(x,y) = \int M(x,y)\partial x + g(y)$$

$$\frac{\partial F(x,y)}{\partial y} = \frac{\partial}{\partial y}\int M(x,y) + g'(y) = N(x,y)$$

Cara.2

$$\frac{\partial F(x,y)}{\partial y} = N(x,y)$$

$$F(x,y) = \int N(x,y)\partial y + f(x)$$

$$\frac{\partial F(x,y)}{\partial x} = \frac{\partial}{\partial x}\int N(x,y) + f'(x) = M(x,y)$$

6. PD non Eksak

- PD non eksak jika : $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$
- Dibawa ke bentuk eksak dengan faktor integral
- Jika u (x,y) merupakan faktor integral maka :

  u(x,y).M(x,y)dx+u(x,y).N(x,y). dy=0

  Merupakan PD eksak

$$\frac{\partial}{\partial y}(u.M) = \frac{\partial}{\partial x}(u.N)$$

$$u\frac{\partial M}{\partial y} + M\frac{\partial u}{\partial y} = u\frac{\partial N}{\partial x} + N\frac{\partial u}{\partial x}$$

$$u\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = -\left(M\frac{\partial u}{\partial y} - N\frac{\partial u}{\partial x}\right)$$

$$u = -\frac{M\frac{\partial u}{\partial y} - N\frac{\partial u}{\partial x}}{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}} (rumus.umum)$$

Bentuk Khusus

- U=f(x)

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial x} \quad dan \quad \frac{\partial u}{\partial y} = 0$$

$$u = -\frac{-N\left(\frac{\partial u}{\partial x}\right)}{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}$$

$$\frac{\partial u}{u} = \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} \partial x$$

$$Sehingga:$$

$$U = \ell^{\int f(x)\partial x} \quad dengan:$$

$$f(x) = \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}$$

U=f(y)

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial y} \; dan \qquad \frac{\partial u}{\partial x} = 0$$

$$u = -\frac{-M\left(\frac{\partial u}{\partial y}\right)}{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}$$

$$\frac{\partial u}{u} = \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{-M} \partial y$$

Sehingga :

$$U = \ell^{\int g(y)\partial y} \; dengan :$$

$$g(y) = \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{-M}$$

U=f(x,y)

Dimisalkan bentuk peubah x,y=v, maka faktor integral u=u(v)

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x}\frac{\partial u}{\partial v} \; dan \qquad \frac{\partial u}{\partial y} = \frac{\partial v}{\partial y}\frac{\partial u}{\partial v}$$

$$u = -\frac{-M\left(\frac{\partial v}{\partial y}\frac{\partial u}{\partial v}\right) - N\left(\frac{\partial v}{\partial x}\frac{\partial u}{\partial v}\right)}{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}$$

$$\frac{\partial u}{u} = -\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M\frac{\partial v}{\partial y} - N\frac{\partial v}{\partial x}}\partial v$$

Sehingga :

$$U = \ell^{\int h(v)\partial v} \; dengan :$$

$$h(v) = -\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M\frac{\partial v}{\partial y} - N\frac{\partial v}{\partial x}}$$

PD linier tingkat satu

- Suatu PD, dimana bagian kiri persamaan mempunyai bentuk linier dalam y dan derivatifnya.

$$A(x)\frac{dy}{dx} + B(x)y = C(x)$$

- Bila dibagi dengan A(x) diperoleh :

$$\frac{dy}{dx} + P(x)y = Q(x)$$

dengan :

$$P(x) = \frac{B(x)}{A(x)} \; dan \; .Q(x) = \frac{C(x)}{A(x)}$$

Contoh :

$$\frac{dy}{dx} + (2x+1)y = x$$

$$\frac{dy}{dx} + 2xy^2 = 2x \; (salah)$$

Penyelesaian

- Ada 3 cara : metode faktor integral, metode lagrange dan metode substitusi
- Metode faktor integral :

$$R\frac{dy}{dx} + P(x).R.y = Q(x).R \quad ....(1)$$

$$\frac{d}{dx} R.Y = Q(x).R$$

$$R\frac{dy}{dx} + y\frac{dR}{dx} = Q(x).R \quad ....(2)$$

dari (1) *dan* (2) *maka* :

$$\frac{dR}{dx} = P(x)R$$

$$\frac{dR}{R} = P(x)dx$$

$$\ln R = \int P(x)dx$$

$$R = e^{\int P(x)dx}$$