$c_1 = \Omega$ shg $\boxed{V_\theta = \Omega r}$

$$\frac{\partial P}{\partial r} = \rho \frac{v_\theta^2}{r} = \rho \Omega^2 r$$

$$\frac{\partial P}{\partial z} = -\rho g$$

$$dP = \frac{\partial P}{\partial r} dr + \frac{\partial P}{\partial z} dz$$

$$\int_{P_0}^{P} dP = \int_0^r \rho \Omega^2 r \, dr - \int_{z_0}^{z} \rho g \, dz$$

$$P - P_0 = \rho \frac{\Omega^2}{2} r^2 - \rho g (z - z_0) \rightarrow \boxed{P = P_0 + \rho \frac{\Omega^2}{2} r^2 + \rho g z_0 - \rho g z}$$

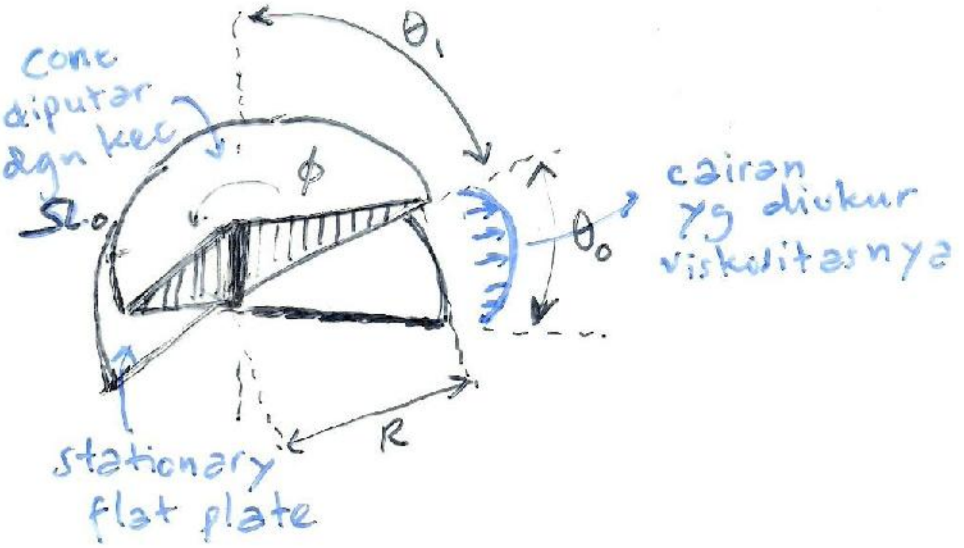
permukaan cairan / posisi $= z_s$

$P = P_0$ = tekanan udara

$$P_0 = P_0 + \rho \frac{\Omega^2}{2} r^2 + \rho g z_0 - \rho g z_s$$

$$\rightarrow \boxed{z_s = \frac{\Omega^2}{2g} r^2 + z_0} \quad (\text{parabola})$$

Contoh 3.5-3 TORQUE RELATIONSHIPS OF VELOCITY DISTRIBUTION IN THE PLATE & CONE VISCOMETER



$\theta_0 \ll$

shg $\theta_1 \approx 90°$

Perhit viskositas berdasarkan pd pengukuran torsi yg diperlukan utk memutar cone dgn kec sudut tetap $= \Omega_0$

asumsi : 1. $\tau_{\theta\phi}$ yg dominan
2. $\tau_{\theta\phi}$ hampir konstan utk seluruh bag fluida
3. End effect diabaikan

Dicari : 1. Hub antara torsi dan viskositas
2. $V_\phi = f(r, \theta)$

asumsi 2 utk menyelesaikan masalah :
1. Steady state
2. $V_r = V_\theta = 0$, $V_\phi = f(r,\theta)$ $\quad \tau_{r\phi}$ & $\tau_{\theta\phi}$ exist
3. Gravitasi diabaikan
4. Creeping flow (aliran lambat)

Penyelesaian :
Kontinyuitas : tabel 3.4.1.C

$$\cancel{\frac{\partial \rho}{\partial t}}^{ss} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 \cancel{V_r})^{=0} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\rho \cancel{V_\theta}^{=0} \sin\theta)$$
$$+ \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(\rho V_\phi) = 0 \quad \rightarrow \quad V_\phi \neq f(\phi)$$

Pers. gerak tabel 3.4.9 dlm bentuk τ
Komponen r :

$$\rho\left(\cancel{\frac{\partial V_r}{\partial t}} + \cancel{V_r}\frac{\partial V_r}{\partial r} + \cancel{\frac{V_\theta}{r}}\frac{\partial V_r}{\partial \theta} + \cancel{\frac{V_\theta}{r\sin\theta}}\frac{\partial V_r}{\partial \phi} - \frac{\cancel{V_\theta^2} + V_\phi^2}{r}\right)$$
$$= -\frac{\partial P}{\partial r} - \left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \cancel{\tau_{rr}}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\cancel{\tau_{r\theta}}\sin\theta)\right.$$
$$\left. + \frac{1}{r\sin\theta}\cancel{\frac{\partial \tau_{r\phi}}{\partial \phi}} - \frac{\cancel{\tau_{\theta\theta}} + \cancel{\tau_{\phi\phi}}}{r}\right) + \cancel{\rho g_r}$$

$$\rho\frac{V_\phi^2}{r} = \frac{\partial P}{\partial r} \quad \ldots\ldots (3.5.28a)$$

Komp. θ :
$$\rho\left(\cancel{\frac{\partial V_\theta}{\partial t}} + \cancel{V_r}\frac{\partial V_\theta}{\partial r} + \cancel{\frac{V_\theta}{r}}\frac{\partial V_\theta}{\partial \theta} + \cancel{\frac{V_\phi}{r\sin\theta}}\frac{\partial V_\theta}{\partial \phi} + \cancel{\frac{V_r V_\theta}{r}}\right.$$
$$\left. - \frac{V_\phi^2\cot\theta}{r}\right) = \frac{1}{r}\frac{\partial P}{\partial \theta} - \left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \cancel{\tau_{r\theta}}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\cancel{\tau_{\theta\theta}}\sin\theta)\right.$$
$$\left. + \frac{1}{r\sin\theta}\cancel{\frac{\partial \tau_{\theta\phi}}{\partial \phi}} + \cancel{\frac{\tau_{r\theta}}{r}} - \frac{\cot\theta}{r}\cancel{\tau_{\phi\phi}}\right) + \cancel{\rho g_\theta}$$

$$\rho\frac{V_\phi^2\cot\theta}{r} = \frac{1}{r}\frac{\partial P}{\partial \theta} \quad \ldots\ldots (3.5.28b)$$

Komp. ϕ :

$$\rho\left(\cancel{\frac{\partial v_\phi}{\partial t}} + \cancel{v_r}\frac{\partial v_\phi}{\partial r} + \cancel{\frac{v_\theta}{r}}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\cancel{\frac{\partial v_\phi}{\partial \phi}} + \cancel{\frac{v_\phi v_r}{r}} + \cancel{\frac{v_\theta v_\phi}{r}}\cot\theta\right)$$

$$= -\frac{1}{r\sin\theta}\cancel{\frac{\partial p}{\partial \phi}} - \left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\phi}) + \frac{1}{r}\frac{\partial \tau_{\theta\phi}}{\partial \theta} + \frac{1}{r\sin\theta}\cancel{\frac{\partial \tau_{\phi\phi}}{\partial \phi}}\right.$$

$$\left. + \frac{\tau_{r\phi}}{r} + \frac{2\cot\theta}{r}\tau_{\theta\phi}\right) + \cancel{\rho g_\phi}$$

$$0 = \left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\phi}) + \frac{1}{r}\frac{\partial \tau_{\theta\phi}}{\partial \theta} + \frac{\tau_{r\phi}}{r} + \frac{2\cot\theta}{r}\tau_{\theta\phi}\right) \quad (3.5.28\ c$$

creeping flow $\rightarrow v_\phi^2 \simeq 0 \rightarrow$ pers. a dan b tidak diperhatikan.

diasumsikan : $\rightarrow v_\phi(r,\theta) = r\, f(\theta)$

tabel 3.4.7 F $\quad \tau_{r\phi} = -\mu\left[\frac{1}{r\sin\theta}\cancel{\frac{\partial v_r}{\partial \phi}} + r\frac{\partial}{\partial r}\left(\cancel{\frac{v_\phi}{r}}\right)\right]$

$$\tau_{r\phi} \simeq 0$$

Pers. 3.5 28 c menjadi : $\frac{1}{r}\frac{\partial \tau_{\theta\phi}}{\partial \theta} = -\frac{2\cot\theta}{r}\tau_{\theta\phi}$

$$\rightarrow \frac{\partial \tau_{\theta\phi}}{\tau_{\theta\phi}} = -\frac{2\cos\theta}{\sin\theta}d\theta = -2\frac{d(\sin\theta)}{\sin\theta}$$

$$\tau_{\theta\phi} = \frac{c_1}{\sin^2\theta} \rightarrow c_1 \text{ dpt dievaluasi di } \theta = \frac{\pi}{2}$$

yg dpt diukur adalah $\mathcal{T}$ yg digunakan utk memutar cone dgn kec. Ω. $\mathcal{T}$ akan sama dgn $\mathcal{T}$ yg digunakan utk menahan plate, tetap diam.

$$\mathcal{T} = (\text{gaya})(\text{lengan gaya}) = \tau . A . (\text{lengan gaya})$$

$$\mathcal{T} = \int_0^R \tau_{\theta\phi}\Big|_{\theta=\frac{\pi}{2}} . 2\pi r . r . dr$$

$$= \int_0^R \frac{c_1}{\sin^2\frac{\pi}{2}} 2\pi r^2 dr = c_1\frac{2}{3}\pi R^3 \rightarrow c_1 = \frac{3\mathcal{T}}{2\pi R^3}$$

maka : $\left\| \tau_{\theta\phi} = \frac{3\mathcal{T}}{2\pi R^3\sin^2\theta} \right\|$

diperhatikan.

diasumsikan: $\rightarrow V_\phi(r,\theta) = r\, f(\theta)$

tabel 3.4.7 F $\quad \tau_{r\phi} = -\mu \left[\frac{1}{r\sin\theta} \cancel{\frac{\partial V_r}{\partial \phi}} + r \frac{\partial}{\partial r} \cancel{\left(\frac{V_\phi}{r}\right)} \right]$

$$\tau_{r\phi} \simeq 0$$

Pers. 3.5 28 c menjadi : $\frac{1}{r}\frac{\partial \tau_{\theta\phi}}{\partial\theta} = -\frac{2\cot\theta}{r}\tau_{\theta\phi}$

$$\rightarrow \frac{\partial \tau_{\theta\phi}}{\tau_{\theta\phi}} = -\frac{2\cos\theta}{\sin\theta}\, d\theta = -2\frac{d(\sin\theta)}{\sin\theta}$$

$$\tau_{\theta\phi} = \frac{c_1}{\sin^2\theta} \rightarrow c_1 \text{ dpt dievaluasi di } \theta = \frac{\pi}{2}$$

yg dpt diukur adalah $\mathcal{T}$ yg digunakan utk memutar cone dgn kec. Ω. $\mathcal{T}$ akan sama dgn $\mathcal{T}$ yg digunakan utk menahan plate, tetap diam.

$$\mathcal{T} = (\text{gaya})(\text{lengan gaya}) = \tau . A . (\text{lengan gaya})$$

$$\mathcal{T} = \int_0^R \tau_{\theta\phi}\Big|_{\theta=\frac{\pi}{2}} . 2\pi r . r . dr$$

$$= \int_0^R \frac{c_1}{\sin^2\frac{\pi}{2}} 2\pi r^2 dr = c_1 \frac{2}{3}\pi R^3 \rightarrow c_1 = \frac{3\mathcal{T}}{2\pi R^3}$$

maka : $\quad \boxed{\tau_{\theta\phi} = \frac{3\mathcal{T}}{2\pi R^3 \sin^2\theta}}$

tabel 3.4.7 hub. $\tau_{\theta\phi}$ dgn V_ϕ

$$\tau_{\theta\phi} = \tau_{\phi\theta} = -\mu\left[\frac{\sin}{r}\frac{\partial}{\partial\theta}\left(\frac{V_\phi}{\sin\theta}\right) + \overbrace{\frac{1}{r\sin\theta}\frac{\partial V_\theta}{\partial\phi}}^{=0}\right]$$

$$\frac{3\mathcal{T}}{2\pi R^3\sin^2\theta} = -\mu \sin\theta \frac{d}{d\theta}\left(\frac{V_\phi/r}{\sin\theta}\right)$$

$$\frac{V_\phi / r}{\sin\theta} = -\int \frac{3\,\mathcal{T}}{2\pi\mu R^3 \sin^3\theta}\, d\theta$$

$$\frac{V_\phi / r}{\sin\theta} = -\int \frac{3\,\mathcal{T}}{2\pi\mu R^3} \operatorname{cosec}^3\theta \, d\theta \quad \text{(jabarkan)}$$

$$V_\phi / r = \frac{3\,\mathcal{T}}{4\pi R^3 \mu}\left[\cot\theta + \frac{1}{2}\left(\ln\frac{1+\cos\theta}{1-\cos\theta}\right)\sin\theta\right] + C_2$$

$$\theta = \frac{\pi}{2}, \; V_\phi = 0 \longrightarrow C_2 = 0$$

shg: $$\frac{V_\phi}{r} = \frac{3\,\mathcal{T}}{4\pi R^3 \mu}\left[\cot\theta + \frac{1}{2}\left(\ln\frac{1+\cos\theta}{1-\cos\theta}\right)\sin\theta\right] \quad \ldots\ldots (D)$$

$\mathcal{T}$ akan dinyatakan dlm variabel yg diketahui dlm r & θ_1

pd $\theta = \theta_1$, $V_\phi = \Omega\, r \sin\theta_1$

$$\Omega \sin\theta_1 = \frac{3\,\mathcal{T}}{4\pi\mu R^3}\left[\cot\theta_1 + \frac{1}{2}\left(\ln\frac{1+\cos\theta_1}{1-\cos\theta_1}\right)\sin\theta_1\right] \quad \ldots (E)$$

(D/E)

$$\frac{V_\phi}{r} = \Omega \sin\theta_1 \left[\frac{\cot\theta + \frac{1}{2}\left(\ln\frac{1+\cos\theta}{1-\cos\theta}\right)\sin\theta}{\cot\theta_1 + \frac{1}{2}\left(\ln\frac{1+\cos\theta_1}{1-\cos\theta_1}\right)\sin\theta_1}\right]$$

$$\theta_0 \approx 0 \rightarrow \theta_1 \cong \frac{\pi}{2}$$

$$\frac{V_\phi}{r} = \Omega \frac{\cos\theta}{\cos\theta_1} = \Omega \frac{\sin\left(\frac{\pi}{2} - \theta\right)}{\sin\left(\frac{\pi}{2} - \theta_1\right)} = \Omega\left(\frac{\frac{\pi}{2} - \theta}{\frac{\pi}{2} - \theta_1}\right)$$

$\sin x = x - \frac{1}{3}x^3 + \ldots$ ← deret Mc-Laurine