Komponen z :

$$\frac{\partial}{\partial t}\rho v_z = -\left(\frac{\partial}{\partial x}\rho v_x v_z + \frac{\partial}{\partial y}\rho v_y v_z + \frac{\partial}{\partial z}\rho v_z v_z\right)$$
$$-\left(\frac{\partial}{\partial x}\tau_{xz} + \frac{\partial}{\partial y}\tau_{yz} + \frac{\partial}{\partial z}\tau_{zz}\right) - \frac{\partial p}{\partial z} + \rho g_z$$

Untuk sistem koordinat spheris & silindris lihat tabel

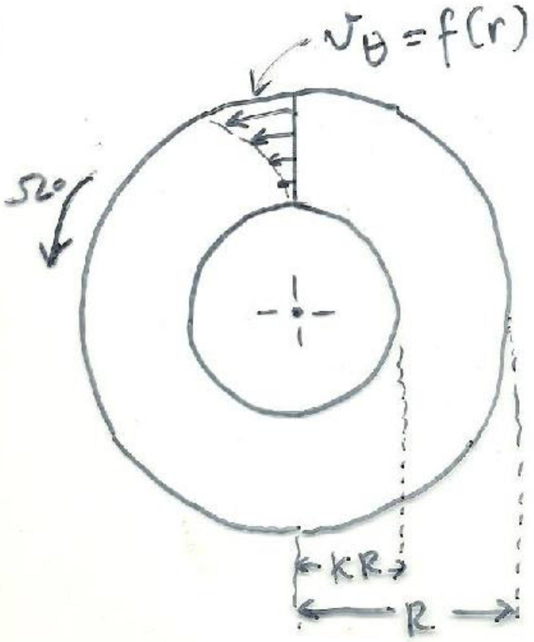
Untuk shearstress Newtonian fluids ; koord. Cartesian

$$\tau_{yx} = \tau_{xy} = -\mu\left[\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right]$$
$$\tau_{yz} = \tau_{zy} = -\mu\left[\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right]$$
$$\tau_{zx} = \tau_{xz} = -\mu\left[\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right]$$

Koordinat yg lain lihat tabel 3.4-6 , 3.4-7

PENERAPAN

Contoh 3.5-1 Tangential Annular Flow of a Newtonian Fluid.



Fluida Newton (ρ, μ tetap) berada antara 2 silinder koaksial; silinder luar bergerak dgn kecep. sudut tetap, Ω_0 dan silinder dlm diam. Kondisi steady state fluida bergerak laminer.

Cari $v_\theta = f(r)$

End effek diabaikan.

asumsi 2 : (summary)

1. Steady state
2. laminer, ρ tetap
3. Newtonian fluid, μ tetap
4. $v_r = v_z = 0$; $\frac{\partial p}{\partial \theta} = 0$

Gunakan Tabel 3.4.1 → pers. Kontinyuitas

Koord. silinder | $\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$

Plm v. komp. r :

$$\rho\left(\cancelto{ss}{\frac{\partial v_r}{\partial t}} + \cancelto{0}{v_r \frac{\partial v_r}{\partial r}} + \frac{v_\theta}{r}\cancelto{0}{\frac{\partial v_r}{\partial \theta}} - \frac{v_\theta^2}{r} + \cancelto{0}{v_z \frac{\partial v_r}{\partial z}}\right) = -\frac{\partial p}{\partial r}$$

$$+ \mu\left[\cancelto{0}{\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right)} + \frac{1}{r^2}\cancelto{0}{\frac{\partial^2 v_r}{\partial \theta^2}} - \frac{2}{r^2}\cancelto{0}{\frac{\partial v_\theta}{\partial \theta}} + \cancelto{0}{\frac{\partial^2 v_r}{\partial z^2}}\right]$$

$$+ \cancelto{0}{\rho g_r} \quad \dashrightarrow \quad \rho \frac{v_\theta^2}{r} = \frac{\partial p}{\partial r} \quad \ldots\ldots (1)$$

komp. θ = :

$$\rho\left[\cancelto{ss}{\frac{\partial v_\theta}{\partial t}} + \cancel{v_r}\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\cancelto{\neq f(\theta)}{\frac{\partial v_\theta}{\partial \theta}} + \cancelto{0}{\frac{v_r v_\theta}{r}} + \cancelto{0}{v_z \frac{\partial v_\theta}{\partial z}}\right] = -\frac{1}{r}\cancelto{\neq f(\theta)}{\frac{\partial p}{\partial \theta}}$$

$$+ \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{1}{r^2}\cancelto{\neq f(\theta)}{\frac{\partial^2 v_\theta}{\partial \theta^2}} + \frac{2}{r^2}\cancelto{0}{\frac{\partial v_r}{\partial \theta}} + \cancelto{0}{\frac{\partial^2 v_\theta}{\partial z^2}}\right]$$

$$+ \cancelto{0}{\rho g_\theta}$$

$$\mu\left[\frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}(r v_\theta)\right)\right] = 0 \quad \ldots (2)$$

komp. z :

$$\rho\left[\cancelto{ss}{\frac{\partial v_z}{\partial t}} + v_r \cancelto{0}{\frac{\partial v_z}{\partial r}} + \frac{v_\theta}{r}\cancelto{0}{\frac{\partial v_z}{\partial \theta}} + v_z \cancelto{0}{\frac{\partial v_z}{\partial z}}\right] = -\frac{\partial p}{\partial z}$$

$$+ \mu\left[\cancelto{0}{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)} + \frac{1}{r^2}\cancelto{0}{\frac{\partial^2 v_z}{\partial \theta^2}} + \cancelto{0}{\frac{\partial^2 v_z}{\partial z^2}}\right] + \rho g_z$$

$$\frac{\partial p}{\partial z} = \rho g_z \quad \ldots\ldots (3)$$

pers (2)

$$\frac{1}{r}\frac{d}{dr}(r v_\theta) = c_1$$

$$r v_\theta = c_1 r^2 + c_2$$

$$v_\theta = c_1 r^2 + \frac{c_2}{r}$$

BC :

$r = KR \qquad v_\theta = 0$

$r = R \qquad v_\theta = \Omega_0 R$

$$v_\theta = \Omega_0 R \frac{\left(\frac{KR}{r} - \frac{r}{KR}\right)}{\left(K - \frac{1}{K}\right)}$$