Hub antara τ dan v pd Newtonian fluid: tabel 3.a.6.

$$\tau_{r\theta} = \tau_{\theta r} = -\mu \left[r \frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta} \right]$$

$$\tau_{r\theta} = -\mu \left[r \frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) \right]$$

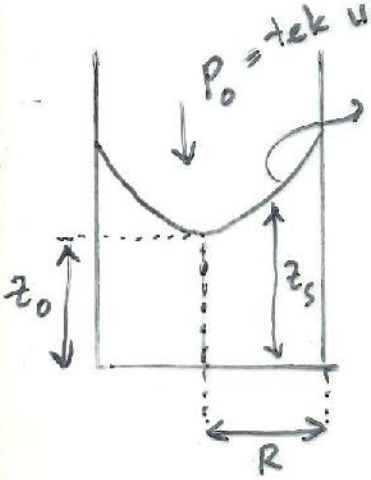
$$r \frac{d}{dr}\left(\frac{v_\theta}{r}\right) = 2\Omega_0 R^2 \left(\frac{1}{r^2}\right)\left(\frac{\kappa^2}{1-\kappa^2}\right)$$

$$\tau_{r\theta} = -\mu \cdot 2\Omega_0 R^2 \left(\frac{1}{r^2}\right)\left(\frac{\kappa^2}{1-\kappa^2}\right)$$

$$\text{Torsi} = 2\pi R L \, \tau_{r\theta}\Big|_R \cdot R$$

gaya — lengan gaya

Contoh 3.5-2 Shape of The Surface of a Rotating Fluid



asumsi:
- steady state
- Newtonian fluid
  ρ, μ tetap
- $v_r = v_z = 0$
  $v_\theta = f(r)$
- P = tekanan = f(r, z)
  - centrifugal force
  - gravitasi

Dicari:
$v_\theta = f(r)$
$P = f(r, z)$
$z_s = f(r)$ → profil permukaan cairan

Dari pers kontinyuitas: $\frac{dv_\theta}{d\theta} = 0$

Pers. gerak: arah r → $\rho \frac{v_\theta^2}{r} = \frac{\partial P}{\partial r}$

arah θ → $0 = \mu \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right)$

arah z → $0 = -\frac{\partial P}{\partial z} - \rho g_z$

BC: $r = R$; $v_\theta = \Omega R$

$r = 0$; $v_\theta = 0$

$$\frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}(r v_\theta)\right) = 0 \rightarrow \frac{d}{dr}(r v_\theta) = c_1 r$$

$$v_\theta = c_1 r + \frac{c_2}{r}$$

$r = 0,\ v_\theta = 0 : c_2 = 0$

shg. $v_\theta = c_1 r$

BC 2. $\Omega R = c_1 R \rightarrow c_1 = \Omega$ shg $\boxed{v_\theta = \Omega r}$

$$\frac{\partial p}{\partial r} = \rho \frac{v_\theta^2}{r} = \rho \Omega^2 r$$

$$\frac{\partial p}{\partial z} = -\rho g$$

$$dp = \frac{\partial p}{\partial r} dr + \frac{\partial p}{\partial z} dz$$

$$\int_{p_0}^{p} dp = \int_0^r \rho \Omega^2 r \, dr - \int_{z_0}^{z} \rho g \, dz$$

$$p - p_0 = \rho \frac{\Omega^2}{2} r^2 - \rho g (z - z_0) \rightarrow \boxed{p = p_0 + \rho \frac{\Omega^2}{2} r^2 + \rho g z_0 - \rho g z}$$

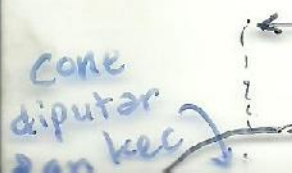
permukaan cairan / posisi $= z_s$

$p = p_0$ = tekanan udara

$$p_0 = p_0 + \rho \frac{\Omega^2}{2} r^2 + \rho g z_0 - \rho g z_s$$

$$\rightarrow \boxed{z_s = \frac{\Omega^2}{2g} r^2 + z_0} \quad \text{(parabola)}$$

Contoh 3.5-3 TORQUE RELATIONSHIPS OF VELOCITY DISTRIBUTION IN THE PLATE & CONE VISCOMETER



$\theta_0 \ll$

shg $\theta_1 \approx 90^\circ$