Ch. 3. THE EQUATIONS OF CHANGE FOR ISOTHERMAL SYSTEMS

Kekekalan Momentum
Massa
Energi
} Unsteady state
lebih dari 1 dimensi

Elemen volume → persamaan → PD $\xrightarrow{BC}$ JAWABAN

Persamaan Umum → disederhanakan
(Table) sesuai problem

- Biasanya banyak sukunya
- Disusun dlm koordinat
  - Cartesian
  - Silindris
  - Spheris (Bola)
- PD Umum meliputi :
  - pers. Kontinyuitas
  - pers. Gerak
  - pers. Energi Mekanik
  - pers. Shear Stress

1. Partial time derivative : $\frac{\partial C}{\partial t}$

ilustrasi : perhitungan jumlah ikan dlm sungai pd berbagai saat, pengamat berdiri pd posisi yg tetap

2. Total time derivative :

$$\frac{dC}{dt} = \frac{\partial C}{\partial t} + \frac{\partial C}{\partial x}\frac{dx}{dt} + \frac{\partial C}{\partial y}\frac{dy}{dt} + \frac{\partial C}{\partial z}\frac{dz}{dt}$$

ilustrasi : perhit. jml ikan dlm sungai pd berbagai waktu dgn posisi pengamat berubah sembarang (pd motor boat)

$\frac{dx}{dt}$, $\frac{dy}{dt}$ dan $\frac{dz}{dt}$ → komp. kec. pengamat

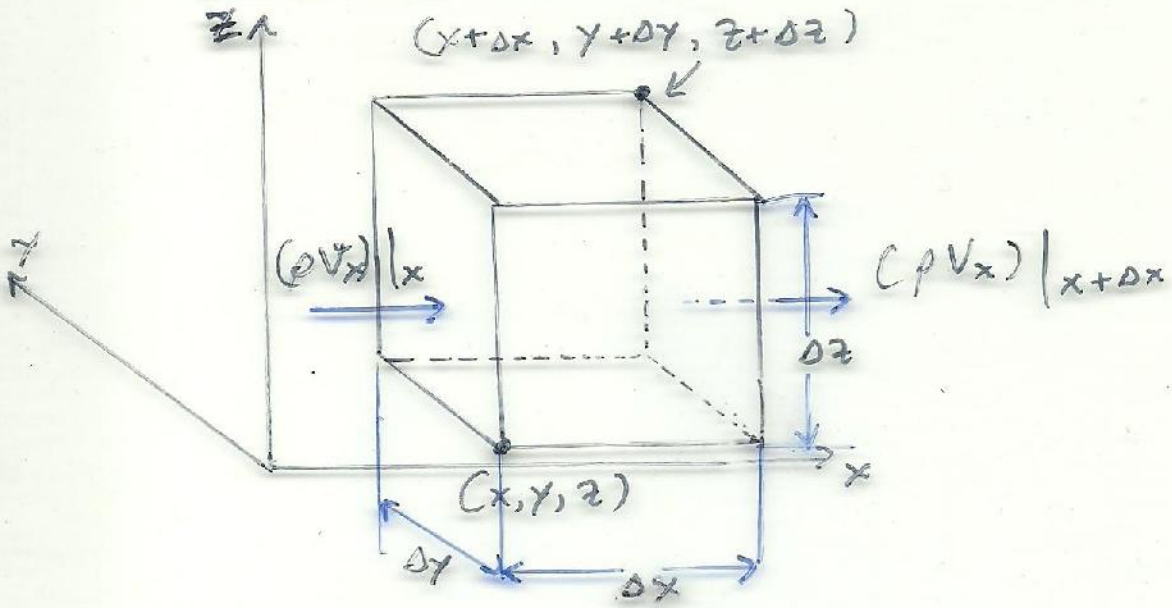
3. Substantial time derivative

$$\frac{D C}{D t} = \frac{\partial C}{\partial t} + v_x \frac{\partial C}{\partial x} + v_y \frac{\partial C}{\partial y} + v_z \frac{\partial C}{\partial z}$$

ilustrasi = pengamat bergerak mengikuti kec arus sungai (sampan / kano)

I. PERS. KONTINYUITAS

Dikembangkan dari Neraca Massa pd elemen volume $\Delta x\,\Delta y\,\Delta z$.



Neraca massa pd elemen volume;

$$\Delta x\Delta y\Delta z\frac{\partial \rho}{\partial t} = \Delta y\Delta z\left[(\rho v_x)|_x - (\rho v_x)|_{x+\Delta x}\right] + \Delta x\Delta z\left[(\rho v_y)|_y - (\rho v_y)|_{x+\Delta y}\right] + \Delta x\Delta y\left[(\rho v_z)|_z - (\rho v_z)|_{z+\Delta z}\right]$$

$$\lim_{\substack{\Delta x\\ \Delta y\\ \Delta z}\to 0} \longrightarrow \boxed{\frac{\partial \rho}{\partial t} = -\left(\frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z)\right)}$$

pers. Kontinyuitas

dgn simbol vektor: $\frac{\partial \rho}{\partial t} = -(\nabla \cdot \rho v)$

Untuk ρ tetap → incompressible fluid.:

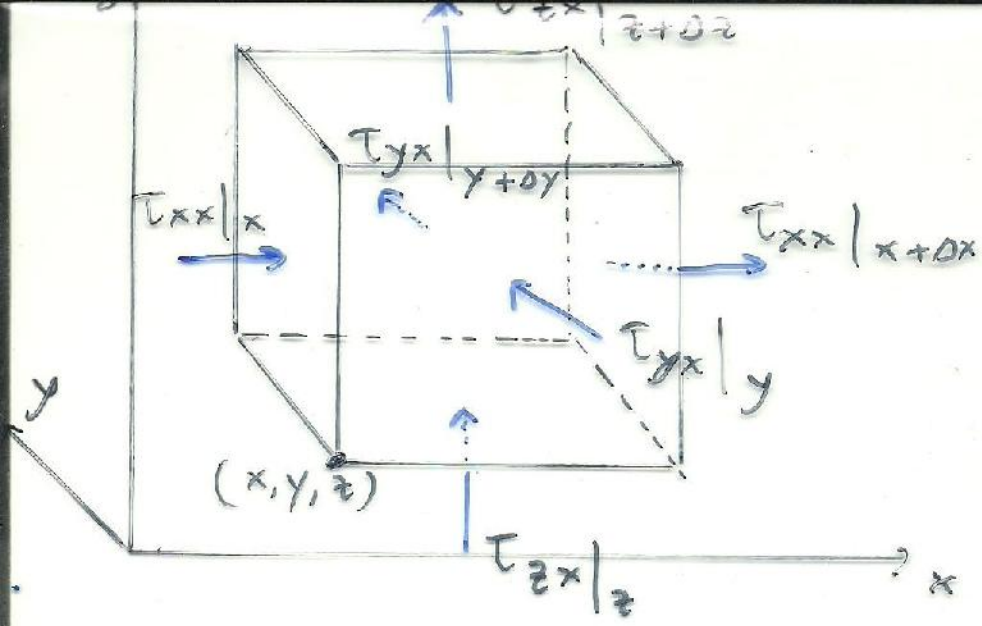
$$0 = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Sistem koordinat lain lihat tabel 3.4.1

II. PERS. GERAK

Dikembangkan dari Neraca Momentum pd elemen volume : $\Delta x, \Delta y, \Delta z$

$$\left\{\begin{array}{c}\text{rate of}\\ \text{momentum}\\ \text{accumulation}\end{array}\right\} = \left\{\begin{array}{c}\text{rate of}\\ \text{momentum}\\ \text{in}\end{array}\right\} - \left\{\begin{array}{c}\text{rate of}\\ \text{momentum}\\ \text{out}\end{array}\right\} + \left\{\begin{array}{c}\text{sum of}\\ \text{forces}\\ \text{acting on}\\ \text{system}\end{array}\right\}$$



the Net convective x-momentum flow into the el. volume is

$$\Delta y \Delta z \left(\rho v_x v_x|_x - \rho v_x v_x|_{x+\Delta x}\right) + \Delta x \Delta z \left(\rho v_y v_x|_y - \rho v_y v_x|_{y+\Delta y}\right)$$
$$+ \Delta x \Delta y \left(\rho v_z v_x|_z - \rho v_z v_x|_{z+\Delta z}\right)$$

The flux of x-momentum through a face perpendicular to the y. axis = τ_{yx} ... dst.

Total utk transport molekulernya (shear stress):

$$\Delta y \Delta z \left(\tau_{xx}|_x - \tau_{xx}|_{x+\Delta x}\right) + \Delta x \Delta z \left(\tau_{yx}|_y - \tau_{yx}|_{y+\Delta y}\right)$$
$$+ \Delta x \Delta y \left(\tau_{zx}|_z - \tau_{zx}|_{z+\Delta z}\right)$$

Total gaya yg bekerja arah x (graritasi & tekanan)

$$\Delta y \Delta x \Delta z \cdot \rho g_x + \Delta y \Delta z \left(p|_x - p|_{x+\Delta x}\right)$$

accumulation: $\Delta x \Delta y \Delta z \left(\partial(\rho v_x)/\partial t\right)$

Dgn membagi seluruh suku dgn $\Delta x \Delta y \Delta z$ dan ambil limit $\Delta x, \Delta y, \Delta z \to 0$ maka:

$$\frac{\partial}{\partial t}\rho v_x = -\left(\frac{\partial}{\partial x}\rho v_x v_x + \frac{\partial}{\partial y}\rho v_y v_x + \frac{\partial}{\partial z}\rho v_z v_x\right)$$
$$-\frac{\partial p}{\partial x} + \rho g_x - \left(\frac{\partial}{\partial x}\tau_{xx} + \frac{\partial}{\partial y}\tau_{yx} + \frac{\partial}{\partial z}\tau_{zx}\right)$$

Untuk komponen y:

$$\frac{\partial}{\partial t}\rho v_y = -\left(\frac{\partial}{\partial x}\rho v_x v_y + \frac{\partial}{\partial y}\rho v_y v_y + \frac{\partial}{\partial z}\rho v_z v_y\right)$$
$$-\left(\frac{\partial}{\partial x}\tau_{xy} + \frac{\partial}{\partial y}\tau_{yy} + \frac{\partial}{\partial z}\tau_{zy}\right) - \frac{\partial p}{\partial y} + \rho g_y$$