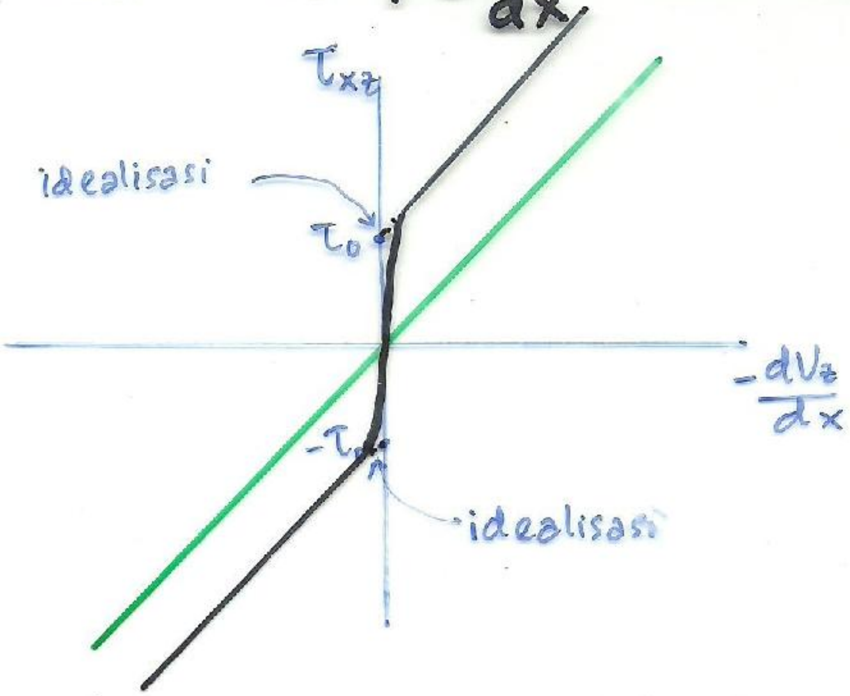
2.3.2. Bingham Flow in a Circular Tube

$$\tau_{xz} = \tau_0 - \mu_0 \frac{dV_z}{dx} \quad \text{utk} \quad \tau_{xz} > \tau_0$$

$$-\tau_0 \leq \tau_{xz} \leq \tau_0 \rightarrow \frac{dV_z}{dx} = 0$$

$$\tau_{xz} = -\tau_0 - \mu_0 \frac{dV_z}{dx} \quad \text{utk} \quad \tau_{xz} < -\tau_0$$



PD dlm τ sama dgn 'Newtonian Fluid'

$$\tau_{rz} = \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2L}\right) r$$

$$\tau_0 - \mu_0 \frac{dV_z}{dr} = \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2L}\right) r$$

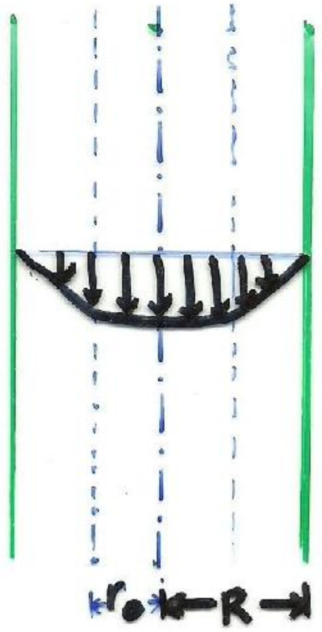
$$\frac{dV_z}{dr} = \frac{\tau_0}{\mu_0} - \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2\mu_0 L}\right) r$$

untuk $\tau_{rz} < \tau_0 \longrightarrow \frac{dV_z}{dr} = 0$

$r \leq r_0 \longrightarrow \frac{dV_z}{dr} = 0$

$$0 = \frac{\tau_0}{\mu_0} - \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2\mu_0 L}\right) r_0$$

$$\boxed{r_0 = \frac{2\tau_0 L}{(\mathcal{P}_0 - \mathcal{P}_L)}}$$



utk

$r_0 < r \leq R$ ada gradien kecepatan

$$\frac{dv_z}{dr} = \frac{\tau_0}{\mu_0} - \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2\mu_0 L}\right) r$$

$$v_z = \frac{\tau_0}{\mu_0} r - \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{4\mu_0 L}\right) r^2 + C_2$$

BC: $r = R$ $\quad v_z = 0$ (no slip at wall)

$$v_z^{>} = \frac{\tau_0}{\mu_0}(r - R) + \frac{\mathcal{P}_0 - \mathcal{P}_L}{4\mu_0 L}(R^2 - r^2)$$

utk $r \leq r_0$

$$v_z^{<} = \frac{\tau_0}{\mu_0}(r_0 - R) + \frac{\mathcal{P}_0 - \mathcal{P}_L}{4\mu_0 L}(R^2 - r_0^2) = \frac{(\mathcal{P}_0 - \mathcal{P}_L)R^2}{4\mu_0 L}\left(1 - \frac{r_0}{R}\right)^2$$

Debit

$$Q = \int_0^R v_z \, dA = \int_0^R v_z \, d(\pi r^2)$$

$$= v_z \pi r^2 \Big|_0^R - \int_0^R \pi r^2 \, dv_z$$

$$= \cancel{0.R - v_z.0} - \int_0^R \pi r^2 \frac{dv_z}{dr} dr$$

$$= -\int_0^{r_0} \pi r^2 \cancel{\frac{dv_z^{<}}{dr}}^{=0} dr - \int_{r_0}^R \pi r^2 \frac{dv_z^{>}}{dr} dr$$

$$= -\int_{r_0}^R \pi r^2 \frac{dv_z^{>}}{dr} dr = \pi \int_{r_0}^R \left[\left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2\mu_0 L}\right) r - \frac{\tau_0}{\mu_0}\right] r^2 dr$$

dgn $\tau_R = \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2L}\right) R$

$$Q = \frac{\pi(\mathcal{P}_0 - \mathcal{P}_L)R^4}{8\mu_0 L}\left[1 - \frac{4}{3}\left(\frac{\tau_0}{\tau_R}\right) + \frac{1}{3}\left(\frac{\tau_0}{\tau_R}\right)^4\right]$$