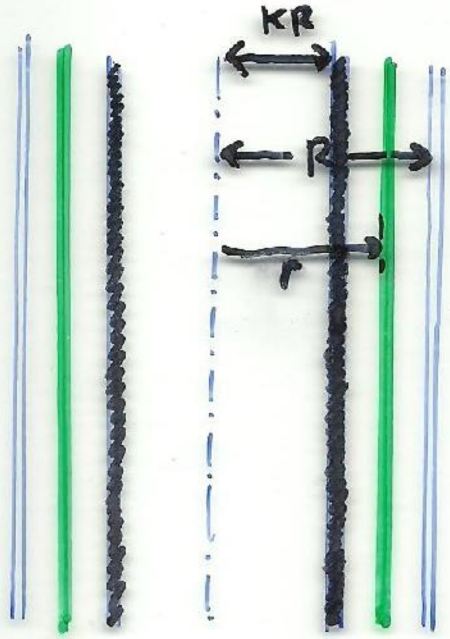
2.4. Flow Through an annulus



Fluida Newtonian, mengalir laminer dlm annulus
(jari² luar pipa bag. dlm = KR
jari² dlm pipa bag. luar = R
panjang L

asumsi² :
1. Steady state, aliran laminer
2. Newtonian fluid, μ tetap
3. $V_z = f(r)$
$V_r = V_\theta = 0$

Dari neraca momentum pd elemen volume $2\pi r \Delta r L$ diperoleh :

$$\frac{d}{dr}(r\tau_{rz}) = \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{L}\right) r$$

BC : (asumsi tak ada slip)

1) $r = KR \qquad V_z = 0$

2) $r = R \qquad V_z = 0$

$$\tau_{rz} = \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2L}\right) r + \frac{C_1}{r}$$

Disuatu tempat tertentu (λR), $\frac{dV_z}{dr} = 0$ or $\tau_{rz} = 0$; λ akan dicari

$$C_1 = -\left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2L}\right)\lambda^2 R^2$$

$$\tau_{rz} = \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2L}\right)\left(r - \frac{\lambda^2 R^2}{r}\right)$$

$$\tau_{rz} = -\mu \frac{dv_z}{dr}$$

$$\frac{dv_z}{dr} = \left(\frac{\wp_0 - \wp_L}{2\mu L}\right)\left(\frac{\lambda^2 R^2}{r} - r\right)$$

$$v_z = \frac{\wp_0 - \wp_L}{2\mu L}\left(\lambda^2 R^2 \ln r - \frac{1}{2} r^2\right) + C_2$$

masukkan BC

$$0 = \frac{\wp_0 - \wp_L}{2\mu L}\left(\lambda^2 R^2 \ln KR - \frac{1}{2} K^2 R^2\right) + C_2 \quad \ldots (1)$$

$$0 = \frac{\wp_0 - \wp_L}{2\mu L}\left(\lambda^2 R^2 \ln R - \frac{1}{2} R^2\right) + C_2 \quad \ldots (2)$$

dari 2 pers diatas diperoleh :

$$\lambda = \left(\frac{K^2 - 1}{2 \ln K}\right)^{\frac{1}{2}}$$

$$C_2 = -\left(\frac{\wp_0 - \wp_L}{2\mu L}\right)\left[\frac{K^2 - 1}{2 \ln K} R^2 \ln R - \frac{1}{2} R^2\right]$$

$$= \left(\frac{\wp_0 - \wp_L}{2\mu L}\right) R^2 \left[1 - \left(\frac{K^2 - 1}{\ln K}\right) \ln R\right]$$

shg :

$$v_z = \left(\frac{\wp_0 - \wp_L}{4\mu L}\right) R^2 \left[1 + \frac{K^2 - 1}{\ln K} \ln\left(\frac{r}{R}\right) - \left(\frac{r}{R}\right)^2\right]$$

(i) $v_{z\,max}$ pd $r = \lambda R = \left(\frac{K^2 - 1}{2 \ln K}\right)^{\frac{1}{2}} R$

$$v_{z\,max} = \left(\frac{\wp_0 - \wp_L}{4\mu L}\right) R^2 \left[1 + \frac{K^2 - 1}{\ln K} \ln \lambda - \lambda^2\right]$$

$$= \left(\frac{\wp_0 - \wp_L}{4\mu L}\right) R^2 \left[1 - \lambda^2 + 2\lambda^2 \ln \lambda\right]$$

(ii) $$Q = \int_{KR}^{R} v_z \, 2\pi r \, dr$$

$$= \frac{\pi (\mathcal{P}_0 - \mathcal{P}_L) R^4}{8\mu L} \left((1-K^4) - \frac{(1-K^2)^2}{\ln \frac{1}{K}} \right)$$

(iii) $$\langle v_z \rangle = \frac{Q}{A} = \frac{Q}{\pi R^2 (1-K^2)}$$

$$= \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{8\mu L} \right) R^2 \left(\frac{1-K^4}{1-K^2} - \frac{1-K^2}{\ln \frac{1}{K}} \right)$$

(iv) Gaya fluid pd dinding

$$F_z = -\tau_{rz}\Big|_{r=KR} 2\pi K R L + \tau_{rz}\Big|_{r=R} 2\pi R L$$

$$= \pi R^2 (1-K^2) (\mathcal{P}_0 - \mathcal{P}_L)$$