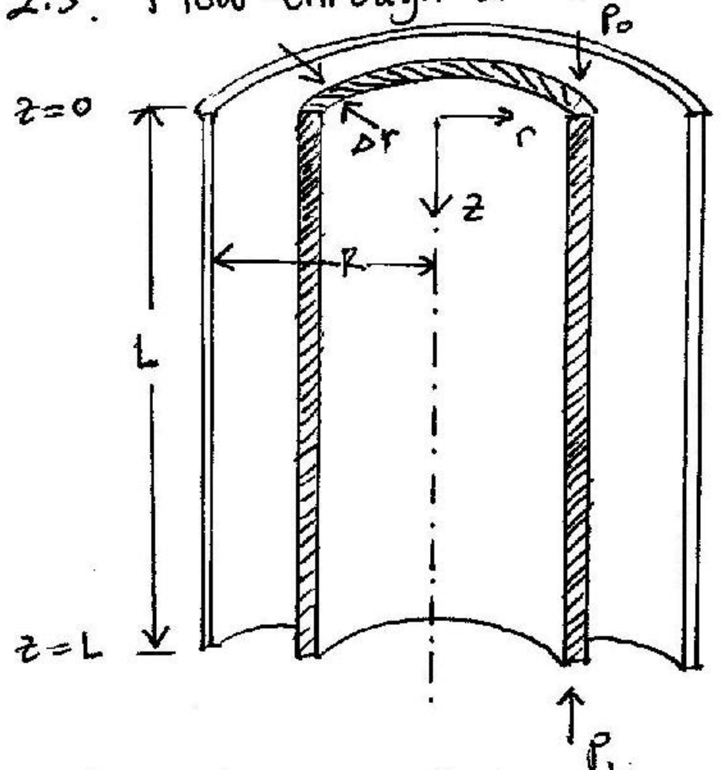
2.3. Flow through a circular tube



Cairan Newtonian mengalir laminer dlm pipa tegak

Pipa sangat panjang shg end effect diabaikan

elemen volume = $2\pi\bar{r}\Delta r L$

Asumsi :
1. Stedy state
2. Newtonian fluid, ρ, μ tetap
3. $V_z = f(r)$
   $V_r = 0$ (dinding rapat tidak bocor)
   $V_\theta = 0$ (tidak ada gerakan berputar)
4. Tidak ada slip pd dinding

Neraca momentum:

$$2\pi r L \tau_{rz}\Big|_{r=r} - 2\pi r L \tau_{rz}\Big|_{r=r+\Delta r}$$

$$+ V_z^2\, 2\pi\bar{r}\Delta r \rho\Big|_{z=0} - V_z^2\, 2\pi\bar{r}\Delta r\rho\Big|_{z=L}$$

$$+ P_0\, 2\pi\bar{r}\Delta r - P_L\, 2\pi\bar{r}\Delta r + \rho\, 2\pi\bar{r}\Delta r L\, g = 0$$

$$V_z = f(r) \quad \text{shg} \quad V_z\Big|_{z=0} = V_z\Big|_{z=L} \quad (V_z \neq f(z))$$

dibagi $2\pi L\Delta r$, diambil $\Delta r \to 0$

$$\lim_{\Delta r \to 0}\left[\frac{(r\,\tau_{rz}|_{r=r+\Delta r}) - (r\,\tau_{rz}|_{r=r})}{\Delta r}\right] = \left(\frac{P_0 - P_L}{L} + \rho g\right) r$$

$$\frac{d}{dr}(r\tau_{rz}) = \left[\frac{(p_0 - \rho g.0) - (p_L - \rho g L)}{L}\right] r$$

$$\mathcal{P} = p - \rho g z$$

$$\frac{d}{dr}(r\tau_{rz}) = \left[\frac{\mathcal{P}_0 - \mathcal{P}_L}{L}\right] r$$

$$r\tau_{rz} = \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2L}\right) r^2 + c_1$$

$$\tau_{rz} = \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2L}\right) r + \frac{c_1}{r}$$

Pd $r = 0$ $\quad \frac{c_1}{r} = \infty \rightarrow c_1 = 0$ $\quad$ (BC.1)

juga pd $r = 0$ tidak ada gradien kec. $\rightarrow \left.\frac{dV_z}{dr}\right|_{r=0} = 0$

$$\tau_{rz} = 0 \rightarrow c_1 = 0$$

$$\tau_{rz} = \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2L}\right) r$$

Hukum Newton utk viskositas pd keadaan ini :

$$\tau_{rz} = -\mu \frac{dV_z}{dr}$$

$$\frac{dV_z}{dr} = -\left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2\mu L}\right) r$$

$$V_z = -\left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{4\mu L}\right) r^2 + c_2$$

BC. 2. $\quad$ pd $r = R$ tidak ada slip $V_z = 0$

$$V_z = \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{4\mu L}\right)(R^2 - r^2)$$

atau

$$\boxed{V_z = \frac{(\mathcal{P}_0 - \mathcal{P}_L) R^2}{4\mu L}\left[1 - \left(\frac{r}{R}\right)^2\right]}$$

(i) V_{max} di $r = 0$

$$V_{max} = \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{4\mu L}\right) R^2$$

(ii) Debit $\quad dQ = V_z \, dA$

$$Q = \int_0^R V_z \, 2\pi r \, dr$$

$$= \int_0^R 2\pi \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{4\mu L}\right) R^2 \left(r - \frac{r^3}{R^2}\right) dr$$

$$= \pi \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2\mu L}\right) R^2 \left(\frac{R^2}{2} - \frac{R^4}{4R^2}\right)$$

$$= \pi \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{8\mu L}\right) R^4$$

(iii) $$\bar{V}_{av} = \langle V_z \rangle = \frac{Q}{A} = \frac{\pi \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{8\mu L}\right) R^4}{\pi R^2} = \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{8\mu L}\right) R^2$$

(iv) F = gaya gesek pd dinding $r = R$

$$= 2\pi R L \, \tau_{rz} \Big|_{r=R}$$

$$= 2\pi R L \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2L}\right) R = (\mathcal{P}_0 - \mathcal{P}_L) \pi R^2$$

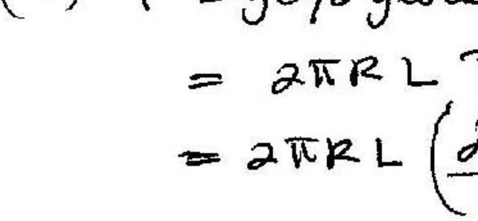
Ingat : berlaku hanya pd aliran laminar

$$Re < 2100$$

$$Re = \frac{\rho \langle V_z \rangle D}{\mu} \qquad D = \text{diameter}$$

panjang pipa ada efek entrance / exit (end effect)

$$L_e = 0{,}035 \, D \, Re$$



Laminer : $\bar{V}_{av} = \langle V_z \rangle = \frac{1}{2} V_{max}$

Turbulen : $V_{av} = e \, . \, V_{max}$

$$0{,}6 \leq e \leq 0{,}7$$