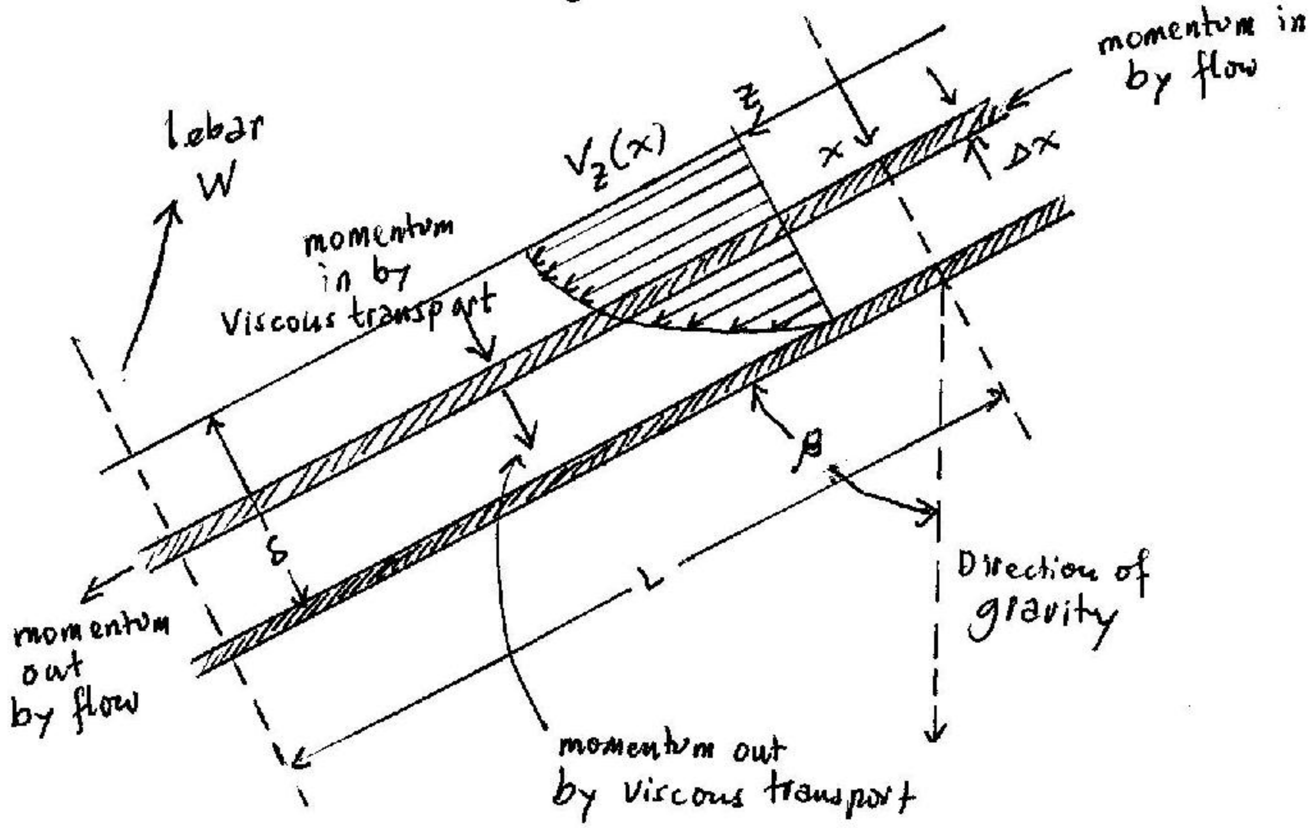
Chapter 2

2.2. Flow of a falling film



Asumsi :
1. Steady state
2. $V_x = V_y = 0$
3. $V_z = f(x)$
4. Fluida Newtonian
5. End effect diabaikan

elemen volume : $W L \Delta x$

$m = L W \Delta x \rho$

transport molekuler (viscous) : $A \tau_{xz}$

transport sec. bulk : $A \rho V_z^2$

Neraca momentum :

$$LW\tau_{xz}\big|_x + \rho W \Delta x V_z^2\big|_{z=0} - \left[LW\tau_{xz}\big|_{x+\Delta x} + \rho W \Delta x V_z^2\big|_{z=L}\right] + mg\cos\beta = 0$$

$m = LW\Delta x \rho$

$V_z \neq f(z)$ shg $V_z\big|_{z=0} = V_z\big|_{z=L}$

Diperoleh:

$$\tau_{xz}\Big|_x - \tau_{xz}\Big|_{x+\Delta x} + \Delta x \rho g \cos\beta = 0$$

$$\frac{d\tau_{xz}}{dx} = \rho g \cos\beta \quad \ldots\ldots \quad (\text{PD dlm } \tau)$$

$$\tau_{xz} = \rho g \cos\beta \, x + C_1$$

BC 1) Kontak cairan-udara $x = 0 \quad \tau_{xz} = 0$

$$C_1 = 0$$

$$\tau_{xz} = \rho g \cos\beta \, x$$

Cairan Newtonian : $\tau_{xz} = -\mu \dfrac{dV_z}{dx}$

$$-\mu \frac{dV_z}{dx} = \rho g \cos\beta \, x$$

$$V_z = -\frac{\rho g \cos\beta}{2\mu} x^2 + C_2$$

BC. 2) Kontak cairan-padatan

kec. cairan di perm padat = kec. padatan

$V_z = 0$ di $x = \delta$ (tetap)

$$\boxed{V_z = \frac{\rho g \delta^2 \cos\beta}{2\mu}\left(1 - \frac{x^2}{\delta^2}\right)}$$

* Debit: $dQ = V_z \, dA$

$$= V_z . W \, dx$$

$$Q = \int_0^\delta \frac{\rho g \delta^2}{2\mu} (\cos\beta) . W \left(1 - \frac{x^2}{\delta^2}\right) dx$$

$$Q = \int_0^\delta \frac{\rho g W \delta^2}{2\mu} \cos\beta \left(1 - \frac{x^2}{\delta^2}\right) dx$$

$$Q = \frac{\rho g W \delta^3 \cos\beta}{3\mu}$$

* Kecepatan maksimum: $V_{z\,max}$ pd $x = 0$

$$V_{z\,max} = \frac{\rho g \delta^2 \cos\beta}{2\mu}$$

*. Kecepatan rerata $= \dfrac{\text{Debit}}{\text{luas tampang}} = \dfrac{\int_0^\delta V_z\, dA}{\int_0^\delta dA}$

$$\langle V_z \rangle = \frac{\dfrac{\rho g \delta^3 W \cos\beta}{3\mu}}{W\delta} = \frac{\rho g \delta^2 \cos\beta}{3\mu}$$

* Gaya gesek di permukaan $x = \delta$

$$F = A\, \tau_{xz}\Big|_{x=\delta}$$

$$= LW.\left(\rho g \delta.\cos\beta\right) = \rho g \delta\, LW \cos\beta$$