PP KONDUKSI

STEADY STATE

ONE DIMENSIONAL

PERSAMAAN UMUM

- BIDANG DATAR

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q'}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Asumsi :

Gradien suhu hanya pada sumbu x

$$\frac{\partial^2 T}{\partial x^2} + \cancel{\frac{\partial^2 T}{\partial y^2}} + \cancel{\frac{\partial^2 T}{\partial z^2}} + \frac{q'}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

cont'd

Tidak ada panas dibangkitkan dalam slab

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q'}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Kondisi steady state

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q'}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

cont'd

Sehingga :

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q'}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$\partial^2 T / \partial^2 x = 0$

Boundary cond. :

X = 0	$T = T_1$
X = L	$T = T_2$

Peny. PD :

$d^2T / dx^2 = 0$

$dT / dx = C1$

T = C1 . x + C2, masukkan BC :

T1 = C2

T2 = C1.L + C2

Sehingga :

T2 = C1.L + T1

Jadi :

C1 = (T2 – T1) / L

C2 = T1

Persamaan akhir :

- $$T = \frac{(T2 - T1)}{L} \cdot X + T1$$

Atau :

$$\frac{T - T1}{T2 - T1} = \frac{x}{L}$$

Hukum Fourier

- Laju pp :

  $q = - kA \, dT/dx$ dengan $dT/dx = T_2 - T_1 / L$

Sehingga :

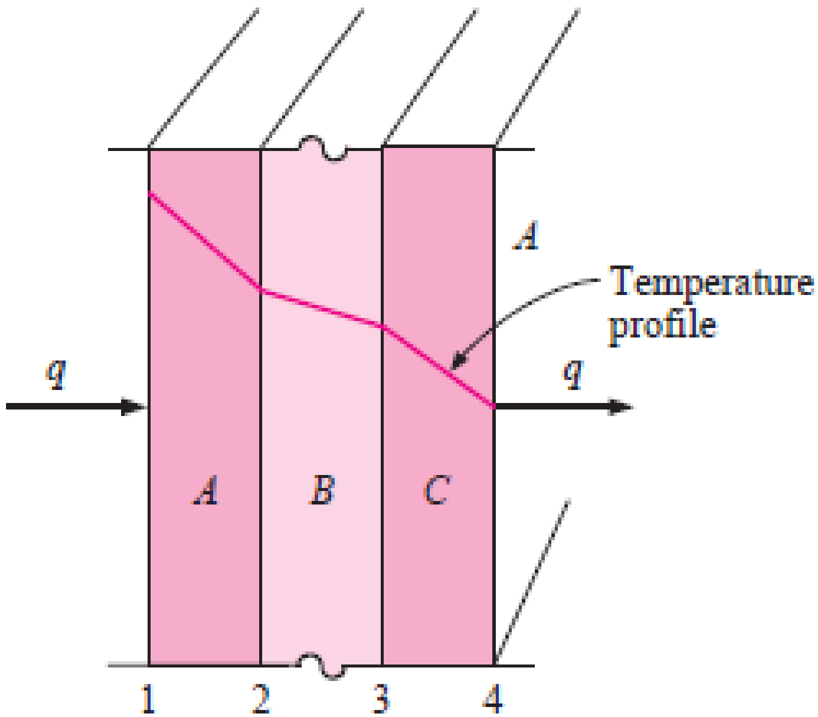
$q_x = - kA (T_2 - T_1) / L$

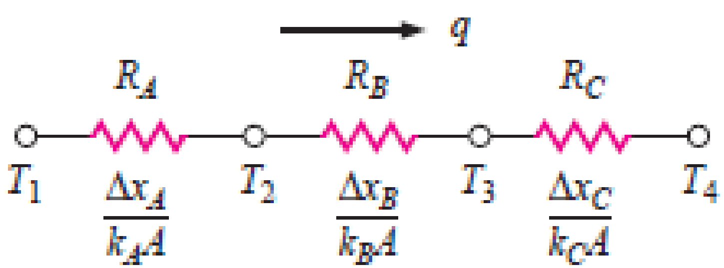
$= T_1 - T_2 / (L/kA)$

$q_x / A = (T_1 - T_2) / R$ ← hambatan

Laju pp = beda suhu antara 2 permukaan dibagi dengan tahanan bahan

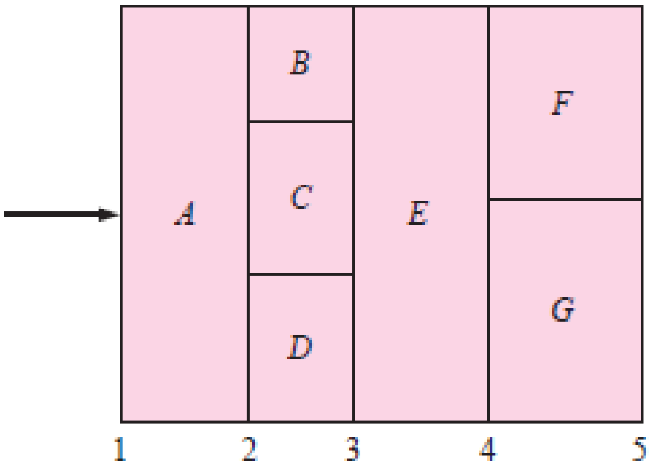
PP konduksi searah dengan hambatan rangkap-koordinat kartesian



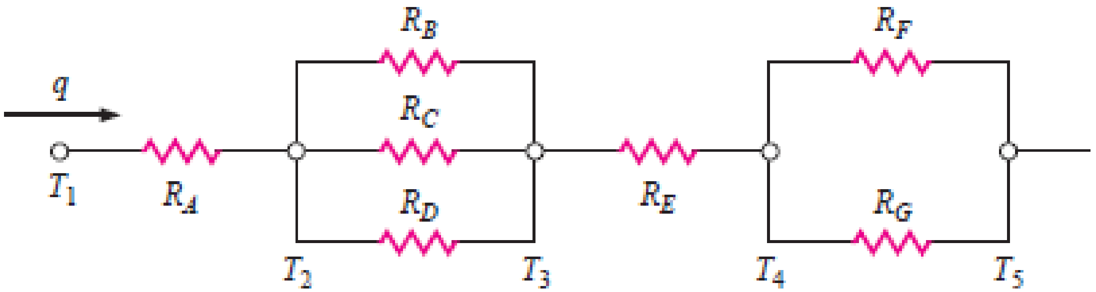


$$q = \frac{T_1 - T_4}{\Delta x_A / k_A A + \Delta x_B / k_B A + \Delta x_C / k_C A}$$

PP konduksi searah dengan hambatan rangkap



$$R = \frac{\Delta T}{q/A}$$



Multilayer Conduction

EXAMPLE 2-1

An exterior wall of a house may be approximated by a 4-in layer of common brick [$k = 0.7$ W/m · °C] followed by a 1.5-in layer of gypsum plaster [$k = 0.48$ W/m · °C]. What thickness of loosely packed rock-wool insulation [$k = 0.065$ W/m · °C] should be added to reduce the heat loss (or gain) through the wall by 80 percent?

■ **Solution**

The overall heat loss will be given by

$$q = \frac{\Delta T}{\sum R_{th}}$$

Because the heat loss with the rock-wool insulation will be only 20 percent (80 percent reduction) of that before insulation

$$\frac{q \text{ with insulation}}{q \text{ without insulation}} = 0.2 = \frac{\sum R_{th} \text{ without insulation}}{\sum R_{th} \text{ with insulation}}$$

We have for the brick and plaster, for unit area,

$$R_b = \frac{\Delta x}{k} = \frac{(4)(0.0254)}{0.7} = 0.145 \text{ m}^2 \cdot {}^\circ\text{C/W}$$
$$R_p = \frac{\Delta x}{k} = \frac{(1.5)(0.0254)}{0.48} = 0.079 \text{ m}^2 \cdot {}^\circ\text{C/W}$$

so that the thermal resistance without insulation is

$$R = 0.145 + 0.079 = 0.224 \text{ m}^2 \cdot {}^\circ\text{C/W}$$

Then

$$R \text{ with insulation} = \frac{0.224}{0.2} = 1.122 \text{ m}^2 \cdot {}^\circ\text{C/W}$$

and this represents the sum of our previous value and the resistance for the rock wool

$$1.122 = 0.224 + R_{rw}$$
$$R_{rw} = 0.898 = \frac{\Delta x}{k} = \frac{\Delta x}{0.065}$$

so that

$$\Delta x_{rw} = 0.0584 \text{ m} = 2.30 \text{ in}$$

- Lihat kasus-kasus transfer panas pada plat datar dan dinding komposit

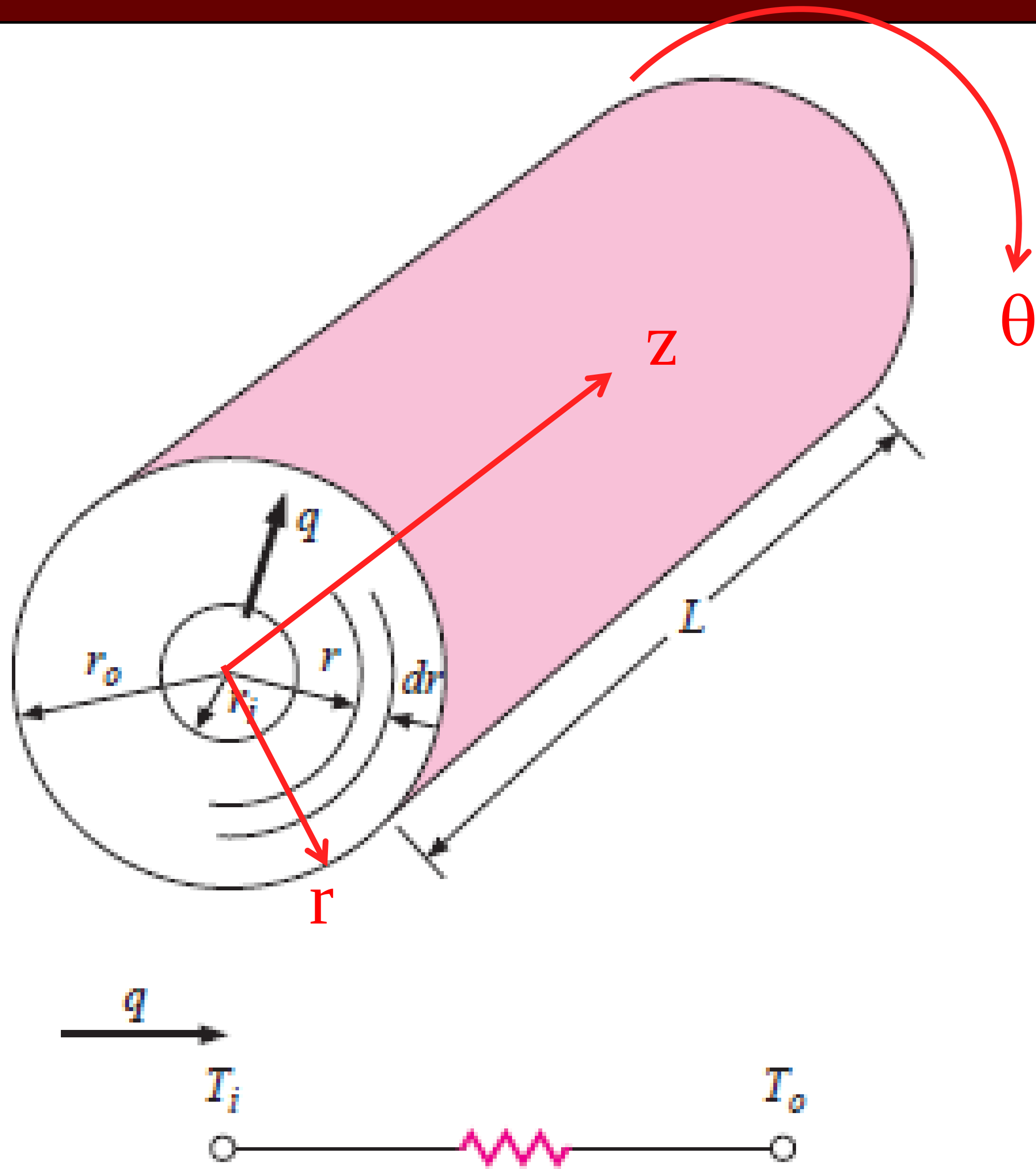
  (literatur : Holman, Serth dll)

PERSAMAAN UMUM

- KOORDINAT SILINDER

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial T}{\partial r}\right]+\frac{1}{r}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}+\frac{q^{'}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

- Buat asumsi seperti pada plat datar



$$R_{th} = \frac{\ln(r_o/r_i)}{2\pi k L}$$

Cont'd

- Silinder tunggal :

$A = 2 \Pi r L$

$q_r = - kA\, dT/dr$

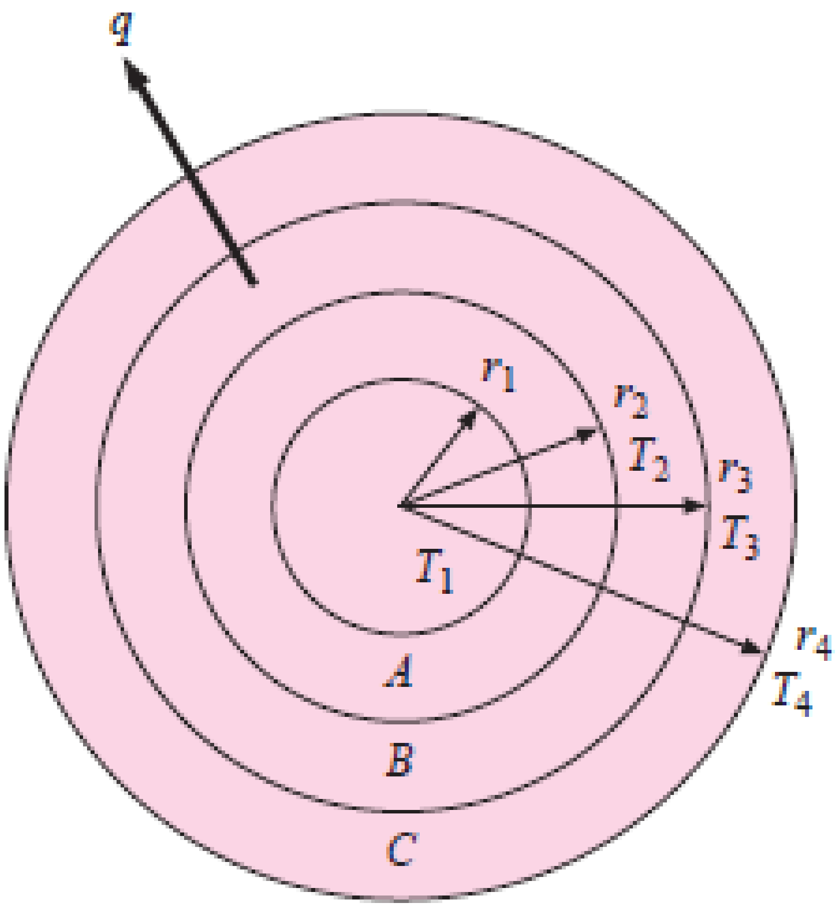
$q_r = (T1 – T2) / R$

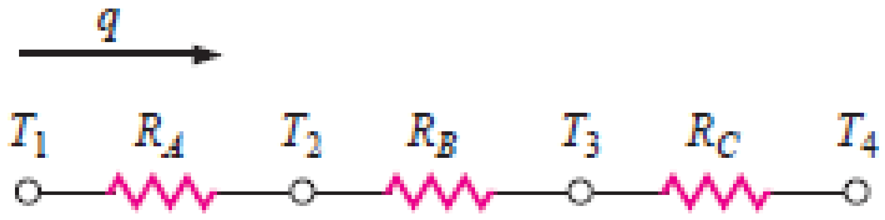
$q_r = [(T1 – T2).2 \Pi k L] / \ln (r_o / r_i)$

Silinder komposit :

$$q = \frac{2 \prod L (T_1 - T_4)}{\frac{\ln (r_2 / r_1)}{k_1} + \frac{\ln (r_3 / r_2)}{k_2} + \frac{\ln (r_4 / r_3)}{k_3}}$$

PP konduksi searah dengan hambatan rangkap-koordinat silinder





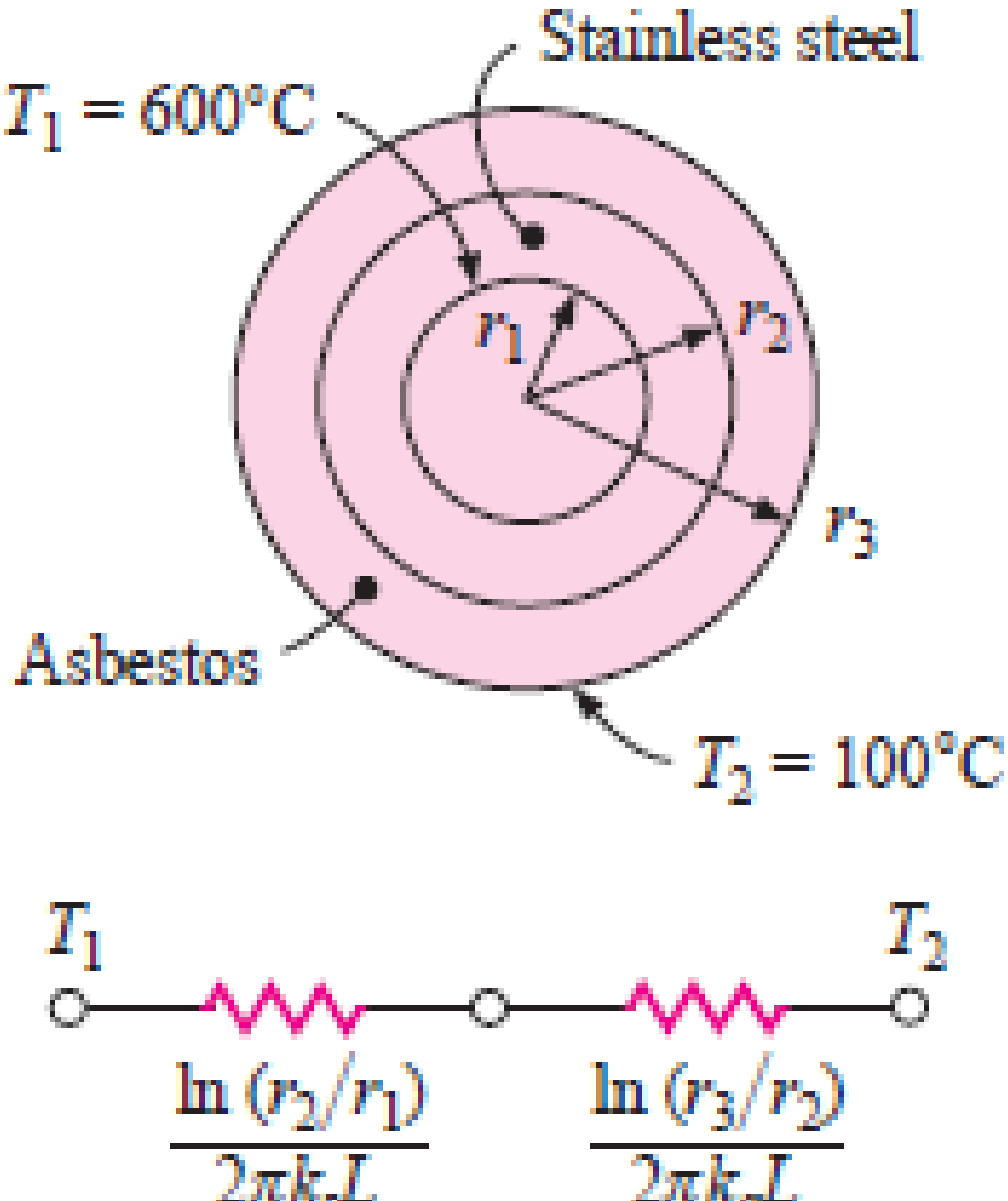
$$\frac{\ln(r_2/r_1)}{2\pi k_A L} \qquad \frac{\ln(r_3/r_2)}{2\pi k_B L} \qquad \frac{\ln(r_4/r_3)}{2\pi k_C L}$$

- Tebal isolasi pada tangki dapat ditentukan bila q dihitung dari neraca panas dan T masing-masing diketahui / ditentukan.

Lihat contoh soal 2-2 Holman

EXAMPLE 2-2 Multilayer Cylindrical System

A thick-walled tube of stainless steel [18% Cr, 8% Ni, $k = 19$ W/m · °C] with 2-cm inner diameter (ID) and 4-cm outer diameter (OD) is covered with a 3-cm layer of asbestos insulation [$k = 0.2$ W/m · °C]. If the inside wall temperature of the pipe is maintained at 600°C, calculate the heat loss per meter of length. Also calculate the tube–insulation interface temperature.


Stainless steel
$T_1 = 600°C$
r_1
r_2
r_3
Asbestos
$T_2 = 100°C$
T_1
T_2
$\frac{\ln(r_2/r_1)}{2\pi k_s L}$
$\frac{\ln(r_3/r_2)}{2\pi k_a L}$

Solution

Figure Example 2-2 shows the thermal network for this problem. The heat flow is given by

$$\frac{q}{L} = \frac{2\pi(T_1 - T_2)}{\ln(r_2/r_1)/k_s + \ln(r_3/r_2)/k_a} = \frac{2\pi(600 - 100)}{(\ln 2)/19 + (\ln \frac{5}{2})/0.2} = 680 \text{ W/m}$$

This heat flow may be used to calculate the interface temperature between the outside tube wall and the insulation. We have

$$\frac{q}{L} = \frac{T_a - T_2}{\ln(r_3/r_2)/2\pi k_a} = 680 \text{ W/m}$$

where T_a is the interface temperature, which may be obtained as

$$T_a = 595.8^\circ\text{C}$$

The largest thermal resistance clearly results from the insulation, and thus the major portion of the temperature drop is through that material.

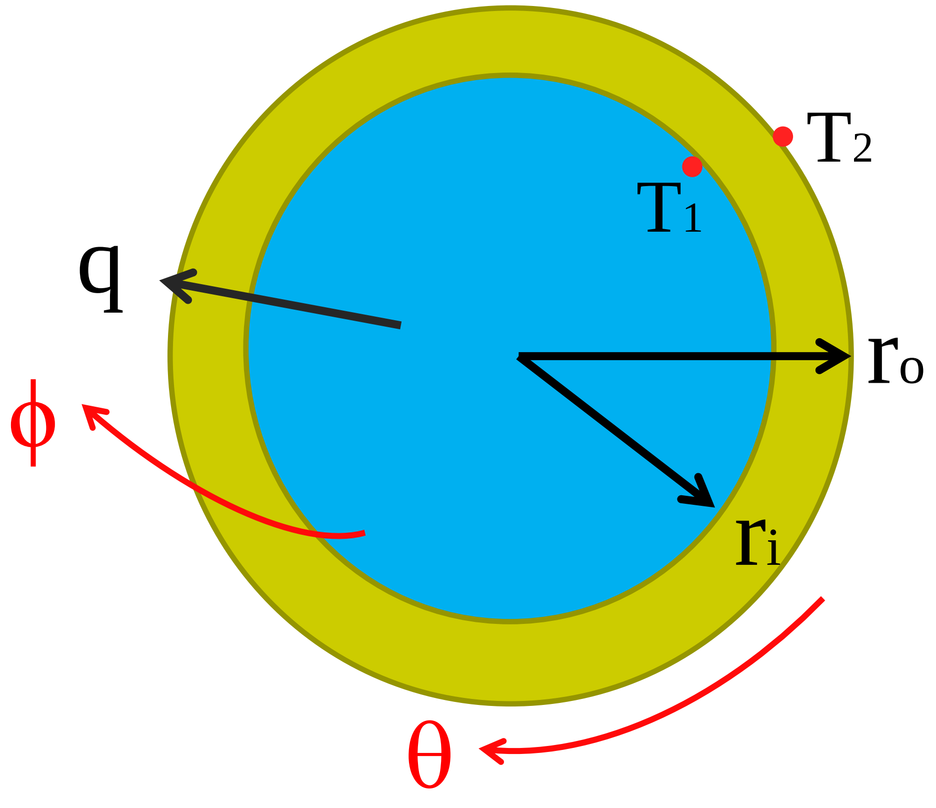
PERSAMAAN UMUM

- KOORDINAT BOLA

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial T}{\partial r}\right]+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial T}{\partial\theta}\right]+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\phi^2}+\frac{q'}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

- Buat asumsi seperti pada plat datar

PP konduksi searah koordinat bola / spherical



Cont'd

- bola :

  $A = 4 \Pi r^2$

  $q_r = - kA \, dT/dr$

  $q_r = (T_1 – T_2) / R$

  $= 4 \Pi r_i r_o k (T_1 – T_2) / (r_o – r_i)$

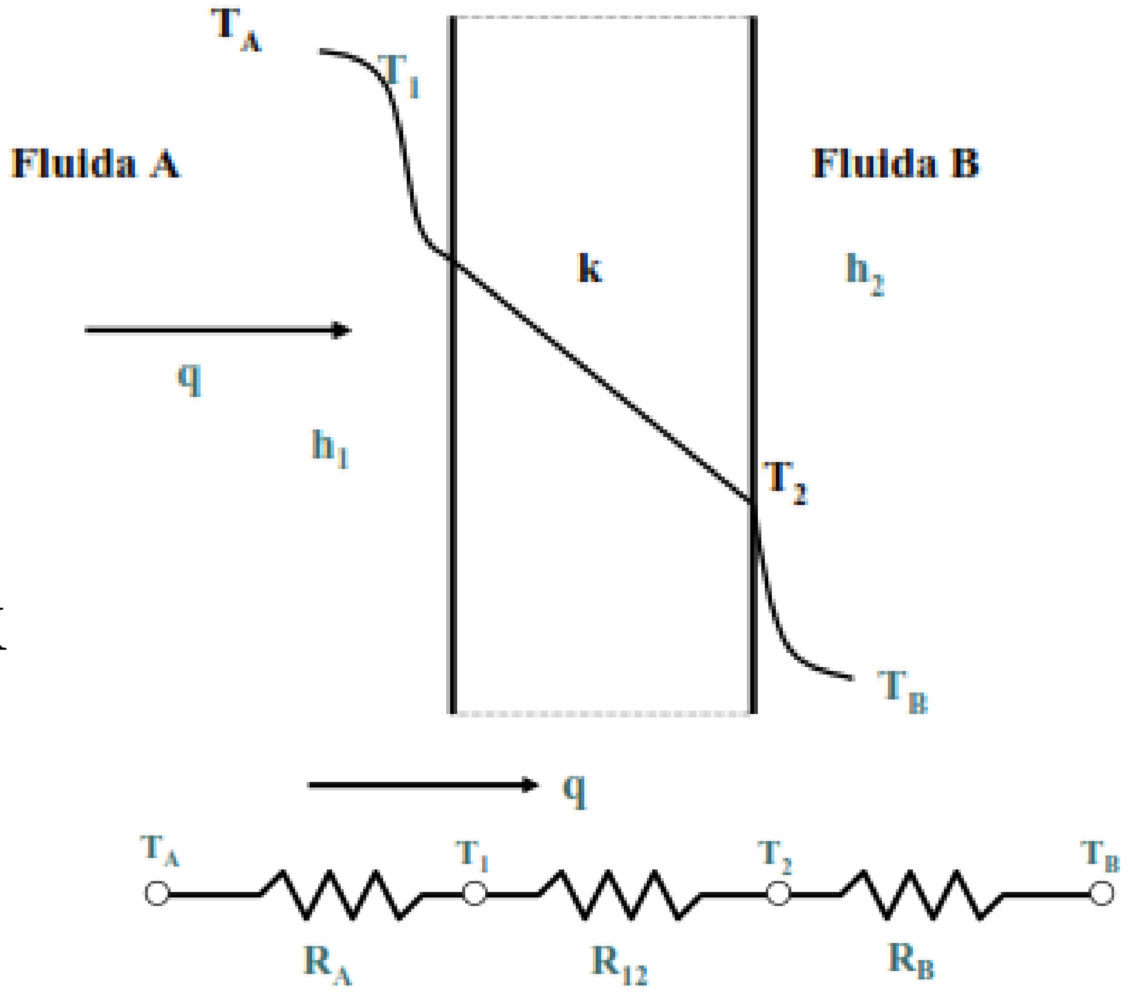
$$q = \frac{4\pi k \left(T_i - T_o\right)}{1/r_i - 1/r_o}$$

dengan $R = \dfrac{(r_o - r_i)}{4\pi \, k \, r_o r_i}$

Koefisien perpindahan panas konduksi dan konveksi simultan

KOEFISIEN PERPINDAHAN PANAS MENYELURUH, U

Koefisien PP overall pada bidang datar



k [=] W/m.K

h [=] W/m².K

$$q = \frac{T_A - T_B}{1/h_1 A + \Delta x/kA + 1/h_2 A} = \frac{A\left(T_A - T_B\right)}{1/h_1 + \Delta x/k + 1/h_2}$$

$$q = UA\ \Delta T_{menyeluruh}$$

$$U = \frac{1}{1/h_1 + \Delta x/k + 1/h_2}$$

[=] $W/m^2.°C$

[=] $Btu/h.ft^2.°F$

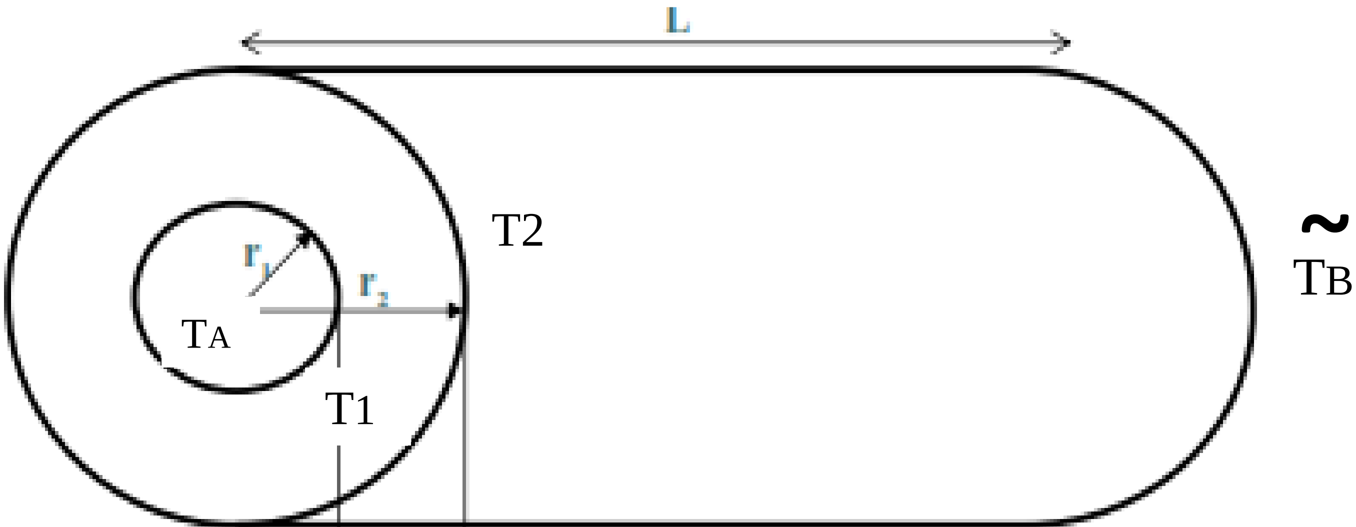
Bidang datar disusun seri

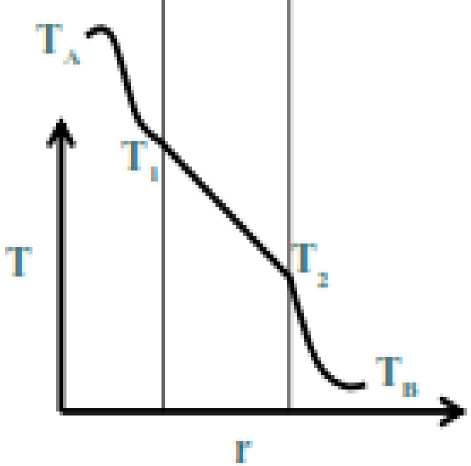
$$q = \frac{T_A - T_B}{1/h_1 A + \Sigma\left(\Delta x/kA\right) + 1/h_2 A} = \frac{A\left(T_A - T_B\right)}{1/h_1 + \Sigma\left(\Delta x/k\right) + 1/h_2}$$

$$U = \frac{1}{1/h_1 + \Sigma\left(\Delta x/k\right) + 1/h_2}$$

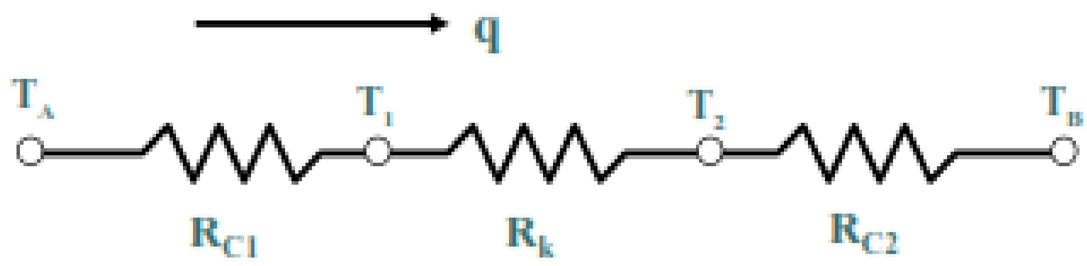
$$U = \frac{1}{A\left(R_{C_1} + \Sigma R_k + R_{C_2}\right)}$$

Koefisien PP overall pada silinder





Analogi listrik :



Luas permukaan untuk perpindahan panas zat alir :

- di dalam pipa, $A_1 = 2\pi r_1 L$
- di luar pipa, $A_2 = 2\pi r_2 L$

sehingga,

$$q = \frac{T_A - T_B}{\dfrac{1}{h_1 2\pi r_1 L} + \dfrac{\ln\left(r_2/r_1\right)}{2\pi k L} + \dfrac{1}{h_2 2\pi r_2 L}} = \frac{2\pi L\left(T_A - T_B\right)}{\dfrac{1}{h_1 r_1} + \dfrac{\ln\left(r_2/r_1\right)}{k} + \dfrac{1}{h_2 r_2}}$$

U dapat didasarkan atas bidang dalam atau bidang luar

- Bidang dalam

$$q = \frac{A_1 (T_A - T_B)}{\frac{1}{h_1} + \frac{A_1 \ln(r_2/r_1)}{2\pi k L} + \frac{A_1}{h_2 A_2}} = \frac{2\pi r_1 L (T_A - T_B)}{\frac{1}{h_1} + \frac{r_1 \ln(r_2/r_1)}{k} + \frac{r_1}{h_2 r_2}}$$

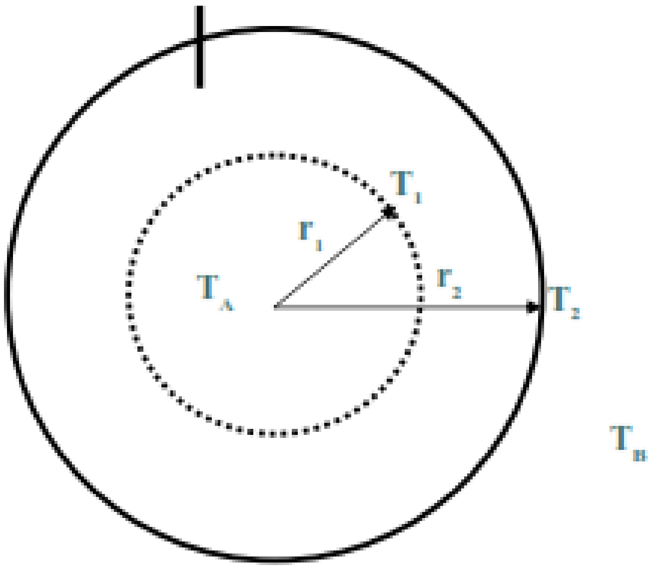
$$U_1 = \frac{1}{\frac{1}{h_1} + \frac{r_1 \ln(r_2/r_1)}{k} + \frac{r_1}{h_2 r_2}}$$

- Bidang luar

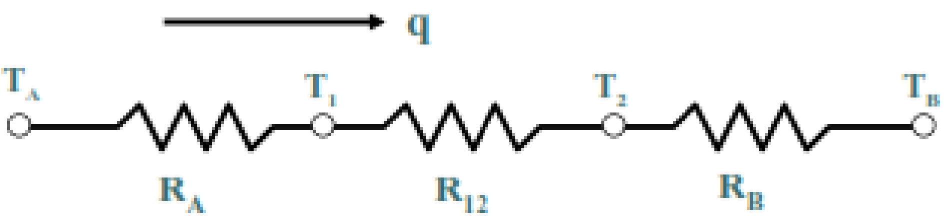
$$q = \frac{A_2 (T_A - T_B)}{\frac{A_2}{h_1 A_1} + \frac{A_2 \ln(r_2/r_1)}{2\pi k L} + \frac{1}{h_2}} = \frac{2\pi r_2 L (T_A - T_B)}{\frac{r_2}{h_1 r_1} + \frac{r_2 \ln(r_2/r_1)}{k} + \frac{1}{h_2}}$$

$$U_2 = \frac{1}{\frac{r_2}{h_1 r_1} + \frac{r_2 \ln(r_2/r_1)}{k} + \frac{1}{h_2}}$$

Koefisien PP overall pada bola



Analogi listrik :



Perpindahan panas menyeluruh dari zat alir di dalam pipa ke zat alir di luar pipa adalah

$$q = \frac{T_A - T_B}{\frac{1}{h_1 A_1} + \frac{1/r_1 - 1/r_2}{4\pi k} + \frac{1}{h_2 A_2}}$$

- Bidang dalam

$$q = \frac{A_1 (T_A - T_B)}{\frac{1}{h_1} + \frac{A_1 \left(\frac{1}{r_1} - \frac{1}{r_2} \right)}{4\pi k} + \frac{A_1}{h_2 A_2}} = \frac{4\pi r_1^2 (T_A - T_B)}{\frac{1}{h_1} + \frac{r_1^2 \left(\frac{1}{r_1} - \frac{1}{r_2} \right)}{k} + \frac{r_1^2}{h_2 r_2^2}}$$

$$U_1 = \frac{1}{\frac{1}{h_1} + \frac{r_1^2 \left(\frac{1}{r_1} - \frac{1}{r_2} \right)}{k} + \frac{r_1^2}{h_2 r_2^2}}$$

- Bidang luar

$$q = \frac{A_2\left(T_A - T_B\right)}{\frac{A_2}{h_1 A_1} + \frac{A_2\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{4\pi k} + \frac{1}{h_2}} = \frac{4\pi r_2^{\,2}\left(T_A - T_B\right)}{\frac{r_2^{\,2}}{h_1 r_1^{\,2}} + \frac{r_2^{\,2}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{k} + \frac{1}{h_2}}$$

$$U_2 = \frac{1}{\frac{r_2^{\,2}}{h_1 r_1^{\,2}} + \frac{r_2^{\,2}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{k} + \frac{1}{h_2}}$$

Konduksi lapis rangkap

- Contoh aplikasi :

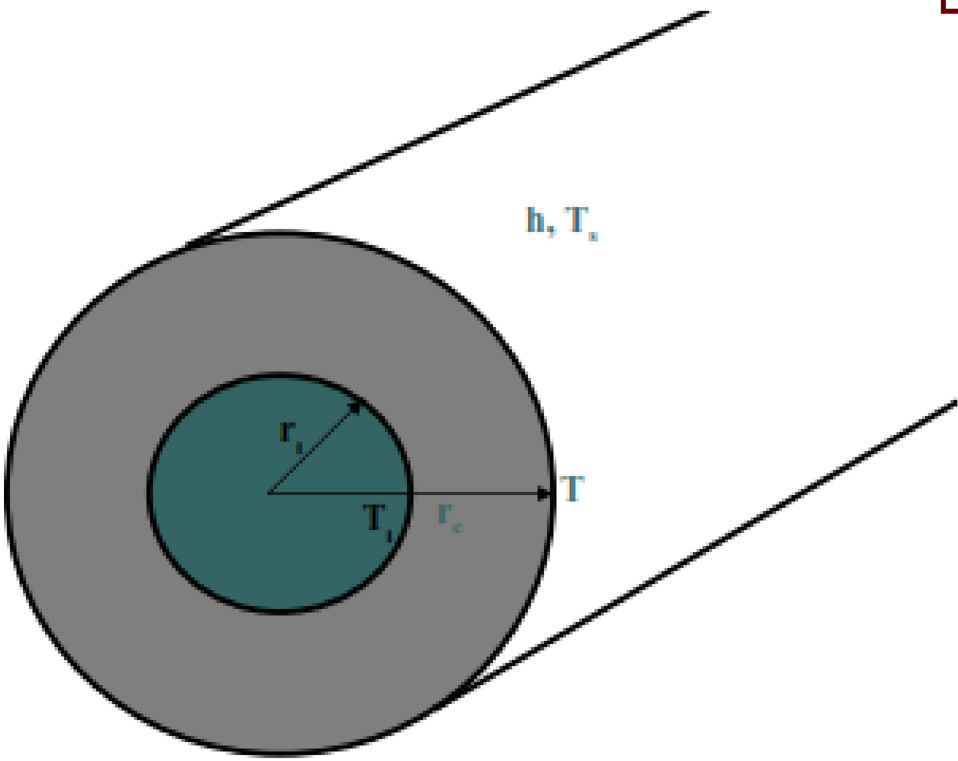
  - penggunaan isolasi pada pipa / tangki

  - bentuk geometris bisa plat datar, silinder atau bola

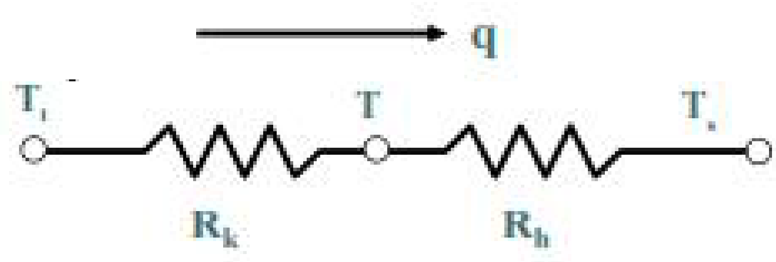
Tujuan pemasangan isolasi :

- Menjaga kondisi suhu fluida dalam pipa / tangki atau suhu di luarnya
- Menentukan jumlah rugi kalor yang dilepas

Tebal isolasi kritis



- Analogi listrik untuk pipa yang dilapisi isolasi :



$$R_k = \frac{\ln\left(r_c / r_i\right)}{2\pi k L} \qquad R_h = \frac{1}{2\pi r_c L h}$$

Persamaan perpindahan panas untuk pipa terisolasi

$$q = \frac{\Delta T_{menyeluruh}}{\Sigma R_{th}} = \frac{T_i - T_s}{\frac{\ln\left(r_c / r_i\right)}{2\pi k L} + \frac{1}{2\pi r_c L h}}$$

$$q = \frac{2\pi L\left(T_i - T_s\right)}{\frac{\ln\left(r_c / r_i\right)}{k} + \frac{1}{r_c h}}$$

Jari-jari kritis

- $r_c = k/h$
- Perpindahan panas maksimm apabila jari-jari kritis = rasio konduktivitas termal isolasi dengan koefisien perpindahan panas permukaan
- $r_c < k/h$ → pp meningkat dengan penambahan tebal isolasi
- $r_c > k/h$ → pp menurun dengan penambahan tebal isolasi

Contoh soal

- Sebuah benda berbentuk pipa berdiameter 5 cm dan bersuhu 200 ˚C diisolasi dengan menggunakan asbes (k = 0,17 W/m.˚C). Benda tsb terkena udara kamar bersuhu 20 C dengan h udara = 3,0 W/ m^2.°C.

a. Turunkan persamaan utk jari-jari kritis isolasi tsb.

b. Hitung jari-jari kritis isolasi asbes.

c. Hitung panas yang hilang pada jari-jari kritis.

d. Hitung panas yang hilang jika tanpa isolasi!