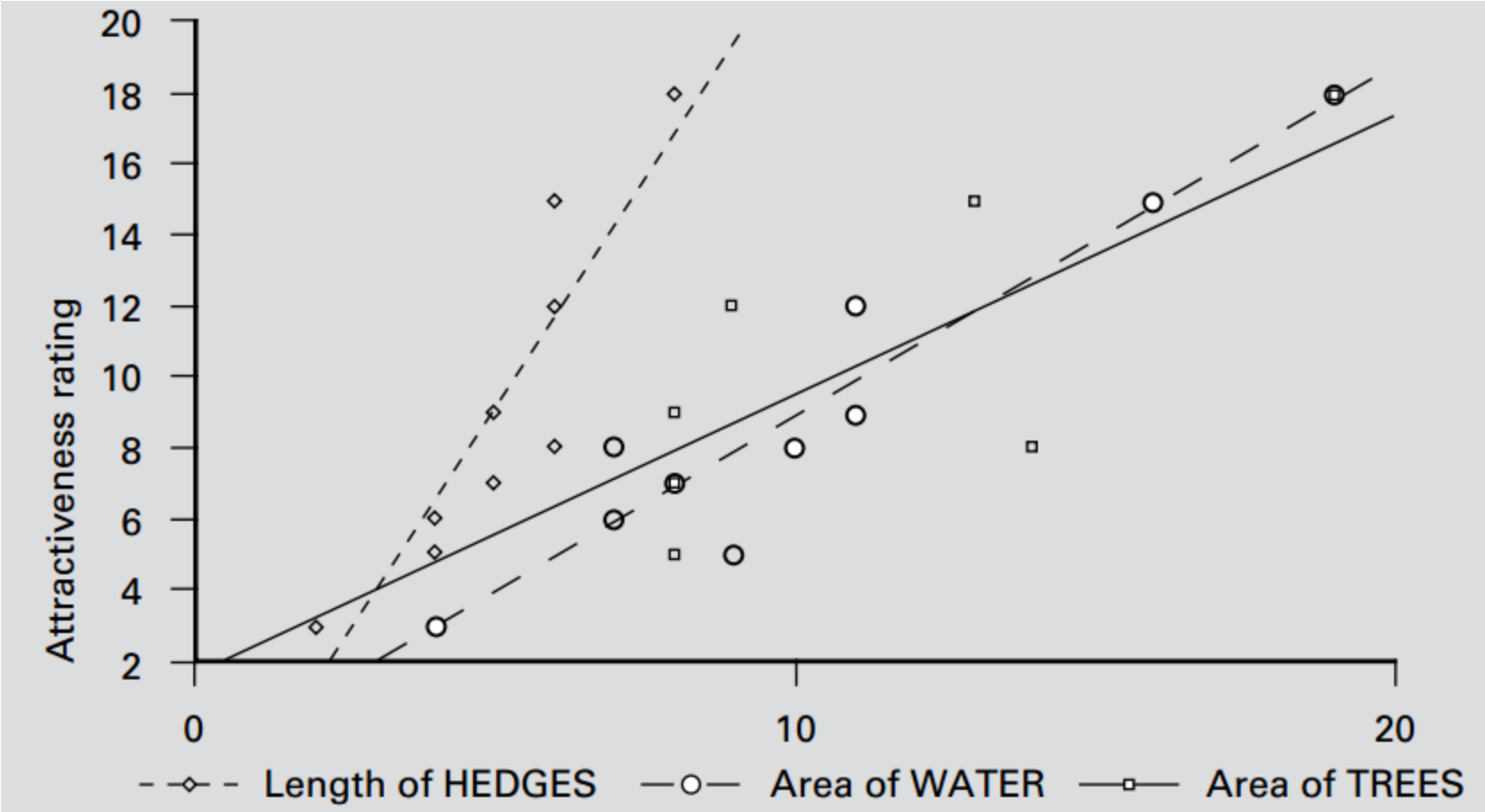
Analisis Regresi Berganda

- Bagian 3

- Adakah pengaruh antara rating tempat rekseasi dengan luas taman, danau dan pagar?

Rating	Taman	Danau	Pagar
18	19	19	8
9	8	11	5
8	14	10	4
8	10	7	6
5	8	9	4
12	9	11	6
15	13	16	6
3	2	4	2
6	4	7	4
7	8	8	5

Misal plot masing-masing variabel



Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.834[a]	.696	.658	2.707

a. Predictors: (Constant), Luas taman

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.950[a]	.902	.890	1.536

a. Predictors: (Constant), Luas area danau

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.897[a]	.804	.779	2.175

a. Predictors: (Constant), Luas pagar

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.974[a]	.950	.924	1.272

a. Predictors: (Constant), Luas pagar, Luas taman, Luas area danau

b. Dependent Variable: Rating

Model Regresi Linier Berganda?

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \Lambda\ \beta_k x_{ik} + \varepsilon_i$$

$$= \beta_0 + \sum_{j=1}^{k} \beta_j x_{ij} + \varepsilon_i, \qquad i = 1,2,\mathrm{K}\ ,n$$

Dengan MKT, bentuk L untuk mengestimasi parameter

$$\mathrm{L} = \sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} \left(y_i - \beta_0 - \sum_{j=1}^{k} \beta_j x_{ij} \right)^2 \Rightarrow\Rightarrow \frac{\partial \mathrm{L}}{\partial \beta_0}, \ldots, \frac{\partial \mathrm{L}}{\partial \beta_j}$$

Sehingga dapat diperoleh

$$\begin{aligned}
n\hat{\beta}_0 + \hat{\beta}_1 \sum_{i=1}^{n} x_{i1} + \hat{\beta}_2 \sum_{i=1}^{n} x_{i2} + \cdots + \hat{\beta}_k \sum_{i=1}^{n} x_{ik} &= \sum_{i=1}^{n} y_i \\
\hat{\beta}_0 \sum_{i=1}^{n} x_{i1} + \hat{\beta}_1 \sum_{i=1}^{n} x_{i1}^2 + \hat{\beta}_2 \sum_{i=1}^{n} x_{i1} x_{i2} + \cdots + \hat{\beta}_k \sum_{i=1}^{n} x_{i1} x_{ik} &= \sum_{i=1}^{n} x_{i1} y_i \\
\vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots & \\
\hat{\beta}_0 \sum_{i=1}^{n} x_{ik} + \hat{\beta}_1 \sum_{i=1}^{n} x_{ik} x_{i1} + \hat{\beta}_2 \sum_{i=1}^{n} x_{ik} x_{i2} + \cdots + \hat{\beta}_k \sum_{i=1}^{n} x_{ik}^2 &= \sum_{i=1}^{n} x_{ik} y_i
\end{aligned}$$

Dalam bentuk matriks …

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \Lambda\ \beta_k x_{ik} + \varepsilon_i$$

$$= \beta_0 + \sum_{j=1}^{k} \beta_j x_{ij} + \varepsilon_i, \qquad i = 1,2,\mathrm{K}\ ,n$$

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

Dengan

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \quad \mathbf{X} = \begin{bmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1k} \\ 1 & x_{21} & x_{22} & \cdots & x_{2k} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_{n1} & x_{n2} & \cdots & x_{nk} \end{bmatrix} \quad \boldsymbol{\beta} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{bmatrix} \quad \text{and} \quad \boldsymbol{\epsilon} = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_n \end{bmatrix}$$

$$L = \sum_{i=1}^{n} \epsilon_i^2 = \boldsymbol{\epsilon}'\boldsymbol{\epsilon} = (\mathbf{y} - \mathbf{X}\boldsymbol{\beta})'(\mathbf{y} - \mathbf{X}\boldsymbol{\beta}) \Rightarrow \frac{\partial L}{\partial \boldsymbol{\beta}} = 0 \Rightarrow \hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$$

So…

$$\begin{bmatrix} n & \sum_{i=1}^{n} x_{i1} & \sum_{i=1}^{n} x_{i2} & \cdots & \sum_{i=1}^{n} x_{ik} \\ \sum_{i=1}^{n} x_{i1} & \sum_{i=1}^{n} x_{i1}^2 & \sum_{i=1}^{n} x_{i1}x_{i2} & \cdots & \sum_{i=1}^{n} x_{i1}x_{ik} \\ \vdots & \vdots & \vdots & & \vdots \\ \sum_{i=1}^{n} x_{ik} & \sum_{i=1}^{n} x_{ik}x_{i1} & \sum_{i=1}^{n} x_{ik}x_{i2} & \cdots & \sum_{i=1}^{n} x_{ik}^2 \end{bmatrix} \begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \vdots \\ \hat{\beta}_k \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_{i1}y_i \\ \vdots \\ \sum_{i=1}^{n} x_{ik}y_i \end{bmatrix}$$

Contoh 2

- Dalam jurnal pendidikan *"Self regulated Learning Strategies and Achievement in an Introduction to Information Systems Course"* (Catherine S Chen, 2002, *Information Technology, Learning and Performance journal* Vol 20 No 1, *College of Bussiness Ball State University*,) **meneliti tentang pembelajaran mandiri yang mungkin dipengaruhi oleh regulasi metakognisi, waktu dan lingkungan pembelajaran, upaya regulasi, pembelajaran *peer*, pengalaman IT, penggunaan software.**

Kasus di atas kira-kira pake alat statistika apa ya?

Meta kognisi

+

Upaya regulasi

→

Pembelajaran mandiri

Kembali ke Contoh 2

No	Meta kognisi	Upaya Reg	Pemb. Mandiri	No	Meta kognisi	Upaya Reg	Pemb. Mandiri
1	13	11	9	21	12	12	12
2	14	13	11	22	14	12	12
3	14	13	13	23	15	14	15
4	14	13	11	24	12	11	10
5	13	10	8	25	13	12	10
6	14	12	11	26	12	14	13
7	14	13	11	27	19	14	14
8	15	10	11	28	15	11	12
9	14	12	11	29	14	10	11
10	11	7	9	30	16	11	11
11	16	13	11	31	14	12	13
12	15	13	11	32	12	8	10
13	15	12	12	33	13	10	11
14	16	14	12	34	16	12	12
15	11	9	6	35	16	10	14
16	10	14	11	36	16	12	11
17	16	15	15	37	12	13	11
18	14	13	12	38	11	9	7
19	16	12	10	39	12	13	12
20	12	12	11	40	16	14	12

$$\mathbf{X} = \begin{bmatrix} 1 & 13 & 11 \\ 1 & 14 & 13 \\ 1 & 14 & 13 \\ 1 & 14 & 13 \\ 1 & 13 & 10 \\ 1 & 14 & 12 \\ 1 & 14 & 13 \\ 1 & 15 & 10 \\ & & \\ 1 & 16 & 14 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} 9 \\ 11 \\ 13 \\ 11 \\ 8 \\ 11 \\ 11 \\ 11 \\ \\ 12 \end{bmatrix}$$

$$\mathbf{X'X} = \begin{bmatrix} 1 & 1 & & 1 \\ 13 & 14 & & 16 \\ 11 & 13 & & 14 \end{bmatrix},$$

$$\mathbf{X'y} = \begin{bmatrix} 1 & 1 & & 1 \\ 13 & 14 & & 16 \\ 11 & 13 & & 14 \end{bmatrix} \begin{bmatrix} 12 \\ 12 \\ \\ 12 \end{bmatrix}$$

tentukan $\hat{\boldsymbol{\beta}} = (\mathbf{X'X})^{-1}\mathbf{X'y}$

Model regreli linier ganda contoh 2

Model		Unstandardized Coeffici B	Std. Er
1	(Constant)	.325	1
	Meta Kognisi	.340	
	Upaya Regulasi	.519	

a. Dependent Variable: Pembelajaran Mandiri

- $y = 0.325 + 0.34\ X_1 + 0.519\ X_2$

1. Uji signifikansi regresi linier ganda

- Uji untuk menentukan apakah ada hubungan linier antara variabel respon y dengan prediktor x1,x2,x3,…,xk

Langkah-langkah :

i. $H_0 : \beta_1 = \beta_2 = \cdots = \beta_k = 0$

$H_1 : \beta_j \neq 0$, setidaknya satu j

ii. Pilih α

iii. Susun tabel ANAVA

SV	JK	db	RK	F0
Regresi	JKR	k	RKR	RKR/RKS
Sesatan	JKS	n-p	RKS	
Total	JKT	n-1		

iv. Tolak H0 jika F0>Ftabel=Fα,k,n-p

Dengan $JKS = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2 = \sum_{i=1}^{n} e_i^2$ =e'e

Substitusi e=y-ŷ=y-Xβ̂

jadi

$JKS = \mathbf{y'y} - \hat{\boldsymbol{\beta}}' \mathbf{X'y}$

$$JK_T = \sum_{i=1}^{n} y_i^2 - \frac{\left(\sum_{i=1}^{n} y_i\right)^2}{n} = \mathbf{y'y} - \frac{\left(\sum_{i=1}^{n} y_i\right)^2}{n}$$

$$JK_S = \mathbf{y'y} - \frac{\left(\sum_{i=1}^{n} y_i\right)^2}{n} - \left[\hat{\boldsymbol{\beta}}'\mathbf{X'y} - \frac{\left(\sum_{i=1}^{n} y_i\right)^2}{n}\right]$$

$$JK_R = \hat{\boldsymbol{\beta}}'\mathbf{X'y} - \frac{\left(\sum_{i=1}^{n} y_i\right)^2}{n}$$

2. R^2 dan adjusted R^2

R-squared disebut juga dengan koefisien determinasi sebagai ukuran statistika kecocokan dengan model, dirumuskan :

$$R^2 = \frac{JK_R}{JK_T} = 1 - \frac{JK_S}{JK_T}$$

Contoh sebelumnya;

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.721[a]	.519	.493	1.304

a. Predictors: (Constant), upaya_reg, metakognisi

R-squared=0.519 menunjukkan sebesar 51.9%

Model menerangkan variabilitas variabel respon sebesar 51.9%

Problem.

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Durbin-Watson
1	.721[a]	.519	.493	1.304	1.889

a. Predictors: (Constant), Upaya Regulasi, Meta Kognisi

b. Dependent Variable: Pembelajaran Mandiri

Masalah → R-suared bertambah ketika regresor bertambah, sulit untuk menentukan kenaikan tsb karena penambahan regresor

→ Alternatif : menggunakan adjusted R-squared

$$R^2_{\text{adj}} = 1 - \frac{JK_S/(n-p)}{JK_T/(n-1)}$$

3. Koefisien Korelasi Karl Pearson

Analisis korelasi → untuk mengetahui kekuatan relasi linier yang diperoleh dari model regresi

Kekuatan linier ini diukur dengan koefisien korelasi r :

$$r_{xy} = \frac{n\left(\sum_{i=1}^{n} x_i y_i\right) - \left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)}{\sqrt{\left[n\sum_{i=1}^{n} x_i^{\,2} - \left(\sum_{i=1}^{n} x_i\right)^2\right]\left[n\sum_{i=1}^{n} y_i^{\,2} - \left(\sum_{i=1}^{n} y_i\right)^2\right]}}$$

$$-1 \le r \le 1$$

Contoh 3

X_1	X_2	Y	X_1X_2	X_1Y	X_2Y	X_1^2	X_2^2	Y^2
5	5	6	25	30	30	25	25	36
4	5	5	20	20	25	16	25	25
7	8	8	56	56	64	49	64	64
6	6	6	36	36	36	36	36	36
4	5	5	20	20	25	16	25	25
6	5	6	30	36	30	36	25	36
7	5	6	35	42	30	49	25	36
5	4	5	20	25	20	25	16	25
6	7	7	42	42	49	36	49	49
8	7	8	56	64	56	64	49	64
6	5	6	30	36	30	36	25	36
4	7	5	28	20	35	16	49	25
68	69	73	398	427	430	404	413	457

		Y	X1	X2
Y	Pearson Correlation	1	.859**	.707*
	Sig. (2-tailed)		.000	.010
	N	12	12	12
X1	Pearson Correlation	.859**	1	.402
	Sig. (2-tailed)	.000		.195
	N	12	12	12
X2	Pearson Correlation	.707*	.402	1
	Sig. (2-tailed)	.010	.195	
	N	12	12	12

**. Correlation is significant at the 0.01 level (2-tailed).

*. Correlation is significant at the 0.05 level (2-tailed).

$$r_{x_1x_2} = \frac{n\left(\sum x_1x_2\right) - \left(\sum x_1\right)\left(\sum x_2\right)}{\sqrt{\left(n\sum x_1^2 - \left(\sum x_1\right)^2\left(n\sum x_2^2 - \left(\sum x_2\right)^2\right)\right)}}$$

$$= \frac{12(398) - (68.69)}{\sqrt{\left(12.404 - (68)^2\right)\left(12.413 - (69)^2\right)}}$$

$$= 0.401918$$

4. Uji sig. Koef. Korelasi
misal : reg.Sederhana ant X1 dg y

i. $H_0 : \rho = 0$

$H_1 : \rho > 0$

ii. $\alpha = 0.05$

$$r_{x_1 y} = \frac{n\left(\sum x_1 y\right) - \left(\sum x_1\right)\left(\sum y\right)}{\sqrt{\left(n\sum x_1^2 - \left(\sum x_1\right)^2\left(n\sum y^2 - \left(\sum y\right)^2\right)\right)}}$$

$$= \frac{12(427) - (68.73)}{\sqrt{\left(12.404 - (68)^2\right)\left(12.457 - (73)^2\right)}} = 0.858678$$

X_1	X_2	Y	X_1X_2	X_1Y	X_2Y	X_1^2	X_2^2	Y^2
5	5	6	25	30	30	25	25	36
4	5	5	20	20	25	16	25	25
7	8	8	56	56	64	49	64	64
6	6	6	36	36	36	36	36	36
4	5	5	20	20	25	16	25	25
6	5	6	30	36	30	36	25	36
7	5	6	35	42	30	49	25	36
5	4	5	20	25	20	25	16	25
6	7	7	42	42	49	36	49	49
8	7	8	56	64	56	64	49	64
6	5	6	30	36	30	36	25	36
4	7	5	28	20	35	16	49	25
68	69	73	398	427	430	404	413	457

$$t = \frac{r_{x_1 y}\sqrt{n-2}}{\sqrt{1-r_{x_1 y}^2}}$$

$$= \frac{0.858678\sqrt{12-2}}{\sqrt{1-(0.858678)^2}}$$

$$= 5.298129$$

Bandingkan dengan t tabel $t(\alpha, n-2) = t(0.05,10) = 1.812$

Karena t>1.812 maka Ho ditolak, jadi ada korelasi positif antara X1 dengan y

5. Standard Error (Kesalahan Baku)

Prinsip OLS: meminimalkan error. Oleh karena itu, ketepatan dari nilai dugaan sangat ditentukan oleh *standard error* dari masing-masing penduga. Adapun standard error dirumuskan sebagai berikut:

$$Se = \sqrt{\frac{\sum (Y - \hat{Y})^2}{n-2}} = \sqrt{\frac{SST - SSR}{n-2}} = \sqrt{\frac{\sum Y^2 - b\sum XY}{n-2}} = \sqrt{RK_S}$$

$$\text{cth.} S_e = \sqrt{RK_S} = \sqrt{0.152} = 0.390$$

Kembali ke contoh 3

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.945[a]	.894	.870	.390

a. Predictors: (Constant), X2, X1

6. Standard error coef

Kembali ke contoh 3..

$$2\ \text{var}, s_{bi} = \sqrt{\frac{Se}{\left(\sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n}\right)\left(1 - r_{12}^2\right)}}$$

$$s_{b1} = \sqrt{\frac{0.390}{\left(404 - \frac{(68)^2}{12}\right)\left(1 - 0.402^2\right)}} = 0.099$$

dengan cara yang sama, diperoleh

$s_{b2} = 0.106$

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error
1	(Constant)	.639	.649
	X1	.570	.099
	X2	.385	.106

a. Dependent Variable: Y

Langkah-langkah Analisis Regresi Berganda

Akan dibentuk model Regresi X1, X2 dan y (contoh 3)

→ Secara komputerisasi :

1. Bentuk model

$$\hat{y} = 0.639 + 0.570X_1 + 0.2385X_2$$

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error
1	(Constant)	.639	.649
	X1	.570	.099
	X2	.385	.106

a. Dependent Variable: Y

2. Uji signifikansi koefisien regresi–Uji F

Diperuntukkan guna melakukan uji hipotesis koefisien (slop) regresi secara bersamaan.

i. $H_0 : \beta_2 = \beta_3 = \beta_4 = \ldots\ldots\ldots = \beta_k = 0$, k=1,2
   H_1 : Tidak demikian (paling tidak ada satu slop yang $\neq 0$)
   Dimana: k adalah banyaknya variabel bebas.

ii. Misal dipilih tingkat signifikansi 5%=0.05

iii. Tabel ANOVA

Tabel ANOVA

SV	JK	df	RK	F Hit
Regresi	JKR	k	RKR = JKR/k	$F = \frac{RKR}{RKS}$
Sesatan	JKS	n-k-1	RKS= JKS/(n-k-1)	
Total	JKT	n-1		

Bandingkan F_{Hit} dengan $F_{\alpha(k,n-k-1)}$

n pasangan data (Xi,Yi)

ANOVA[a]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	11.547	2	5.773	37.929	.000[b]
	Residual	1.370	9	.152		
	Total	12.917	11			

a. Dependent Variable: Y

b. Predictors: (Constant), X2, X1

Ho ditolak karena

→ F=37.929>Ftabel=4.26 atau α=0.05>0

→ $F_{\alpha(k,n-k-1)}$ =Fo.05;2;12-2-1)

Artinya H1 diterima, dkl hubungan antara X1, X2 dengan Y berarti

3. Uji signifikansi koefisien regresi secara individu – Uji t

Pengujian koefisien regresi secara individu.

i. H_{0bj} : $\beta_j = 0$

H_{1bj} : $\beta_j \neq 0; j = 0, 1, 2.....,$ k k adalah koefisien slop.

ii. $\alpha = 5\%$

iii. Statistika uji

$$t_1 = \frac{b_1}{s_{b_1}} = \frac{0.570}{0.099} = 5.758 \Rightarrow 5.758 > t_{\frac{\alpha}{2},n-p} = t_{\frac{0.05}{2},9} = 2.262 \Rightarrow H_{0b1} \text{ditolak}$$

$$t_2 = \frac{b_2}{s_{b_2}} = 3.632 \Rightarrow 3.632 > t_{\frac{0.05}{2},9} = 2.262 \Rightarrow H_{0b2} \text{ditolak}$$

Model		Unstandardized Coefficients		Standardized Coefficients		
		B	Std. Error	Beta	t	Sig.
1	(Constant)	.639	.649		.984	.351
	X1	[redacted]	[redacted]	[redacted]	5.778	.000
	X2	[redacted]	[redacted]	[redacted]	3.646	.005

a. Dependent Variable: Y

4.

- Hitung koefisien determinasi/ adjusted

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.945[a]	.894	.870	.390

a. Predictors: (Constant), X2, X1

Hitung korelasi

Correlations

		Y	X1	X2
Y	Pearson Correlation	1	.859**	.707*
	Sig. (2-tailed)		.000	.010
	N	12	12	12
X1	Pearson Correlation	.859**	1	.402
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