Program Studi Teknik Kimia
Fakultas Teknik Universitas Sebelas Maret 2020



# PENGENDALIAN PROSES

## SIFAT DINAMIK SISTEM ORDER DUA

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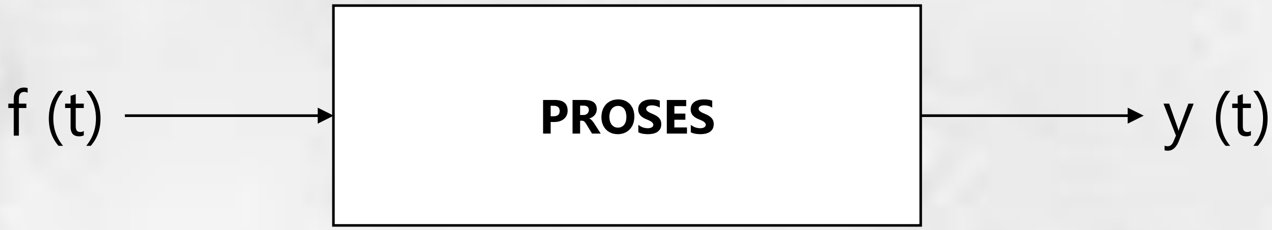
# Sistem Order Dua






Persamaan diferensial orde kedua:

$$a_2 \frac{d^2y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y = b\, f\,(t)$$

Jika $a_0 \neq 0$

$$\frac{a_2}{a_0}\frac{d^2y}{dt^2} + \frac{a_1}{a_0}\frac{dy}{dt} + y = \frac{b}{a_0} f\,(t)$$

$$\tau^2 = \frac{a_2}{a_0}, 2\zeta\tau = \frac{a_1}{a_0}, \text{dan } Kp = \frac{b}{a_0}$$

$$\tau^2 \frac{d^2y}{dt^2} + 2\zeta\tau \frac{dy}{dt} + y = Kp\, f\,(t)$$

$\tau$ : periode osilasi alamiah sistem

$\zeta$ : damping factor

$K_p$ : gain

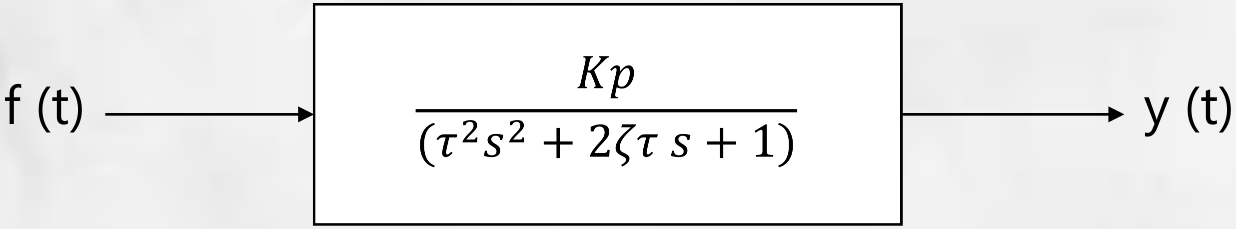
Transformasi laplace:

$$\tau^2 s^2 y\ (s) + 2\zeta\tau\ s\ y\ (s) + y(s) = Kp\ f\ (s)$$

$$y\ (s)\ (\tau^2 s^2 + 2\zeta\tau\ s + 1) = Kp\ f\ (s)$$

Fungsi transfer:

$$\frac{y(s)}{f(s)} = \frac{Kp}{(\tau^2 s^2 + 2\zeta\tau\ s + 1)}$$



# Tanggapan dinamik sistem order dua

Fungsi transfer:

$$\frac{y(s)}{f(s)} = \frac{Kp}{(\tau^2 s^2 + 2\zeta\tau\, s + 1)}$$

Jika ada perubahan variabel input sebesar 1 satuan:

$f(t) = 1$ maka $f(s) = 1/s$

$$y(s) = \frac{Kp}{s(\tau^2 s^2 + 2\zeta\tau\, s + 1)}$$

akar persamaan dari $\tau^2 s^2 + 2\zeta\tau\, s + 1$ adalah :

$$p_1 = -\frac{\zeta}{\tau} + \frac{\sqrt{\zeta^2 - 1}}{\tau}$$

$$p_2 = -\frac{\zeta}{\tau} - \frac{\sqrt{\zeta^2 - 1}}{\tau}$$

$\Rightarrow$

$$y(s) = \frac{Kp/\tau^2}{s(s - p_1)(s + p_2)}$$

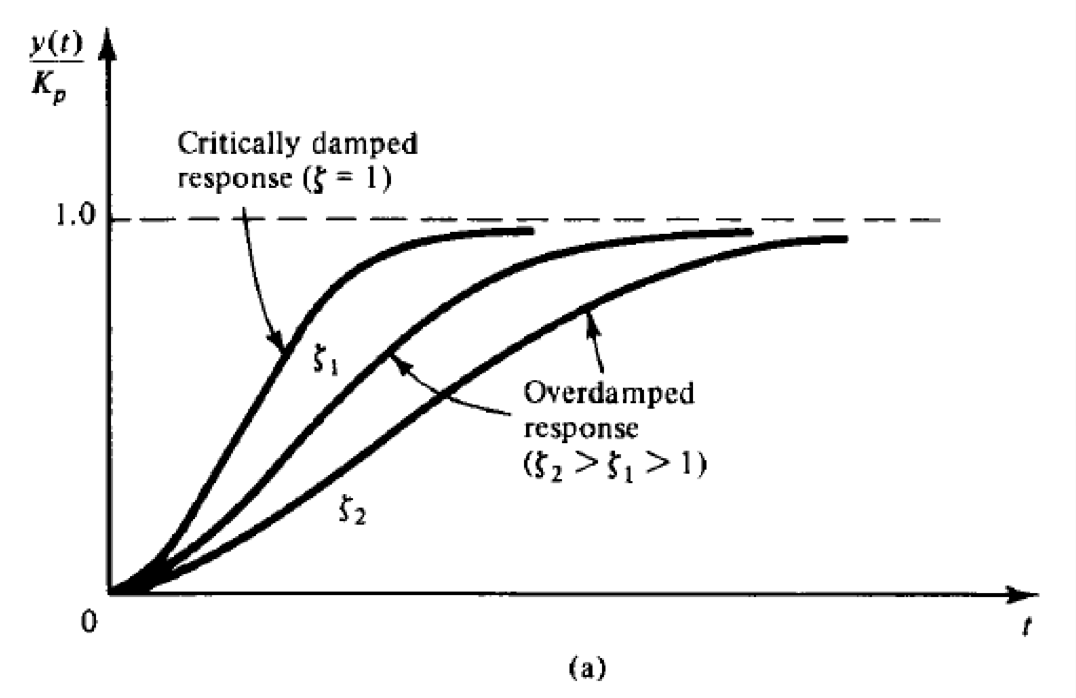
Bentuk tanggapan sistem order dua sangat tergantung nilai $p_1$ dan $p_2$.

# Tanggapan dinamik sistem order dua

Bentuk tanggapan sistem order dua dibedakan menjadi tiga kondisi:

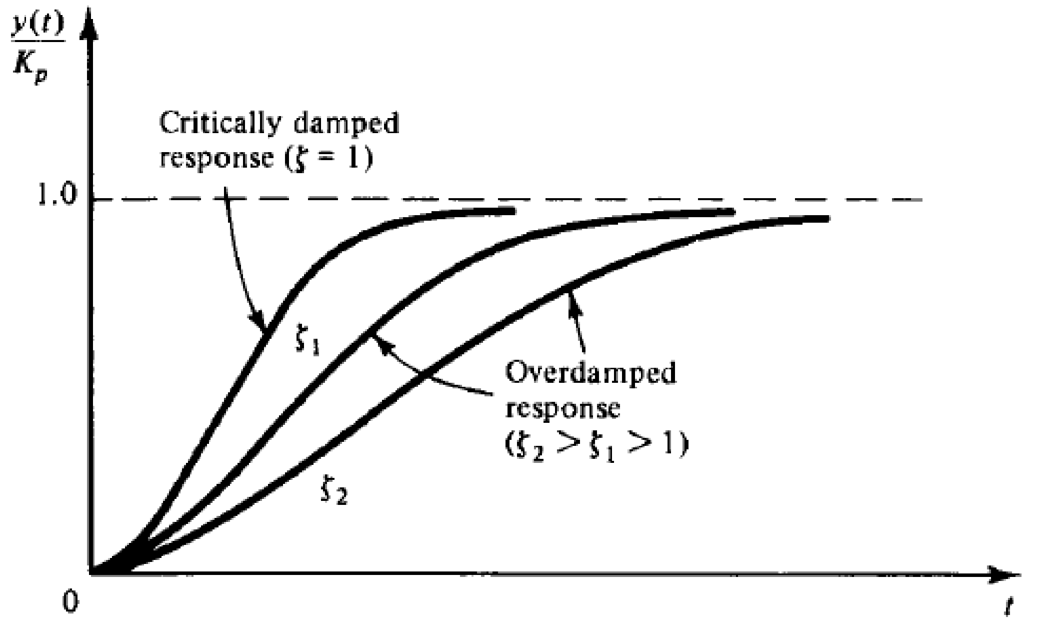
1. *Overdamped responses* terjadi ketika ζ > 1, memiliki dua akar yang berbeda dan nyata.

Semakin besar nilai ζ, maka semakin lama mencapai *ultimate value.*



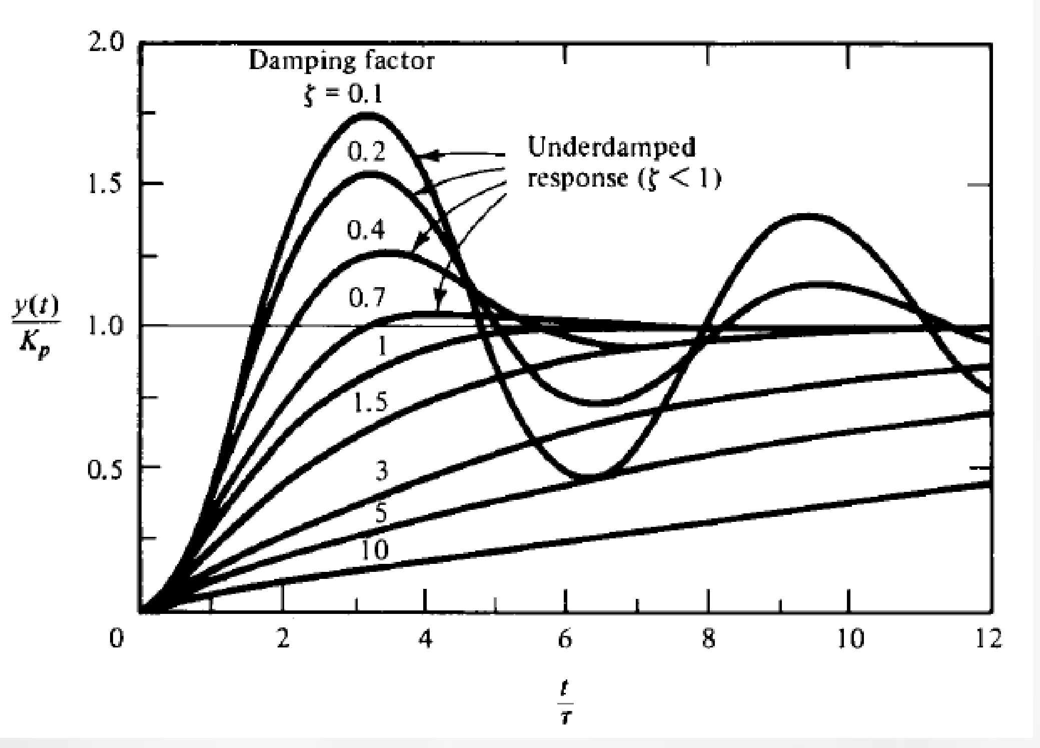
2. *Critically damped responses* terjadi ketika ζ = 1, memiliki dua akar yang sama dan nyata.

*Critically damped* lebih cepat mencapai *ultimate value* dibandingkan *overdamped system.*

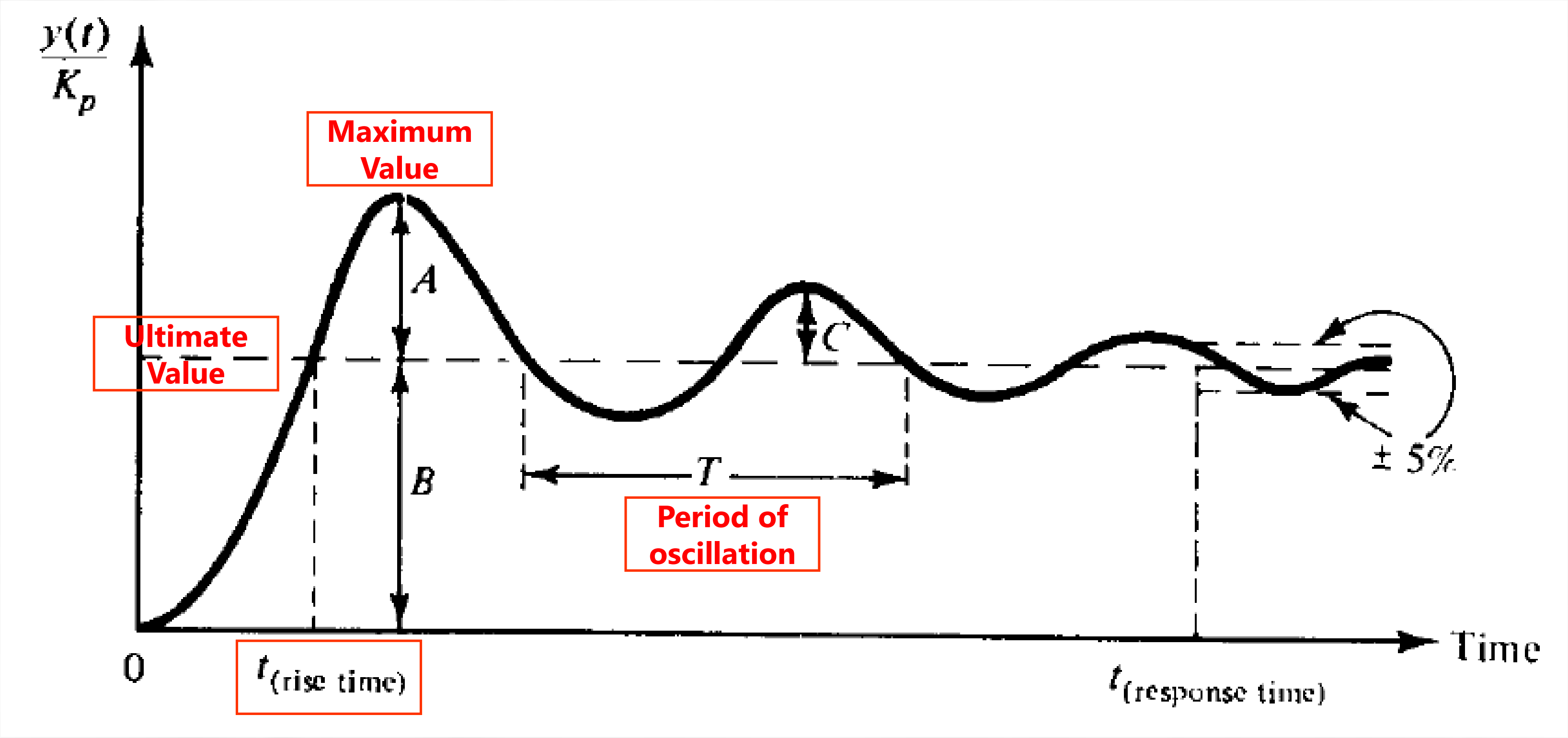


(a)

3. *Underdamped responses* terjadi ketika ζ < 1, memiliki dua akar imajiner. *Underdamped responses* lebih cepat mencapai *ultimate value* dibandingkan *critically damped* dan *overdamped system. Underdamped responses* tidak menetap pada posisi tersebut. Namun menimbulkan osilasi dengan penurunan amplitudo. Perilaku osilasi menjadi lebih terlihat dengan nilai ζ yang semakin kecil.



**Karakteristik *underdamped responses*:**



**Problem III.51**

Consider a second-order system with the following transfer function:

$$G(s) = \frac{y(s)}{m(s)} = \frac{1}{s^2 + s + 1}$$

Introduce a step change into the system and find :

(a) Percent overshoot (amplitudo maksimum dari variabel keluaran sistem dihitung dari nilai akhirnya. Besaran ini menunjukkan kestabilan relatif dari sistem. Semakin besar *overshoot*, sistem semakin tak stabil)

(b) Decay ratio (perbandingan antara amplitudo kedua dan pertama)

(c) Maximum value of y(t)

(d) Ultimate value of y(t)

(e) Rise time (waktu yang dibutuhkan oleh variabel keluaran sistem untuk naik dari 0% ke 100%)

(f) Period of oscilation

**Jawaban Problem III.51**

$$y(s) = \frac{1}{s^2 + s + 1}\frac{1}{s} \qquad \tau = 1, \qquad 2\tau\zeta = 1, \qquad \zeta = 0{,}5$$

a. Overshoot $= \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right) = \exp\left(\frac{-\pi \times 0{,}5}{\sqrt{1-0{,}5^2}}\right) = 0{,}16$

b. Decay ratio $= (\text{Overshoot})^2 = 0{,}03$

c. Ultimate value $= y\ (t \to \infty) = \lim_{s\to 0}[s\ y(s)] = 1$

d. Maximum value = ultimate value + A

Overshoot $= \frac{A}{B} = \frac{A}{y\ (t \to \infty)} = \frac{A}{1} = 0{,}16$

$A = 0{,}16$

Maximum value $= 1 + 0{,}16 = 1{,}16$

**Jawaban Problem III.51**

$$y(s) = \frac{1}{s^2 + s + 1}\frac{1}{s} \qquad \tau = 1, \qquad 2\tau\zeta = 1, \qquad \zeta = 0{,}5$$

$$\text{e. rise time} = y\,(t_r) = 1 - \frac{2}{\sqrt{3}} e^{-\frac{t_r}{2}} \sin\left(\frac{\sqrt{3}}{2} t_r + \tan^{-1}\sqrt{3}\right) = 1$$

$$t_r =$$

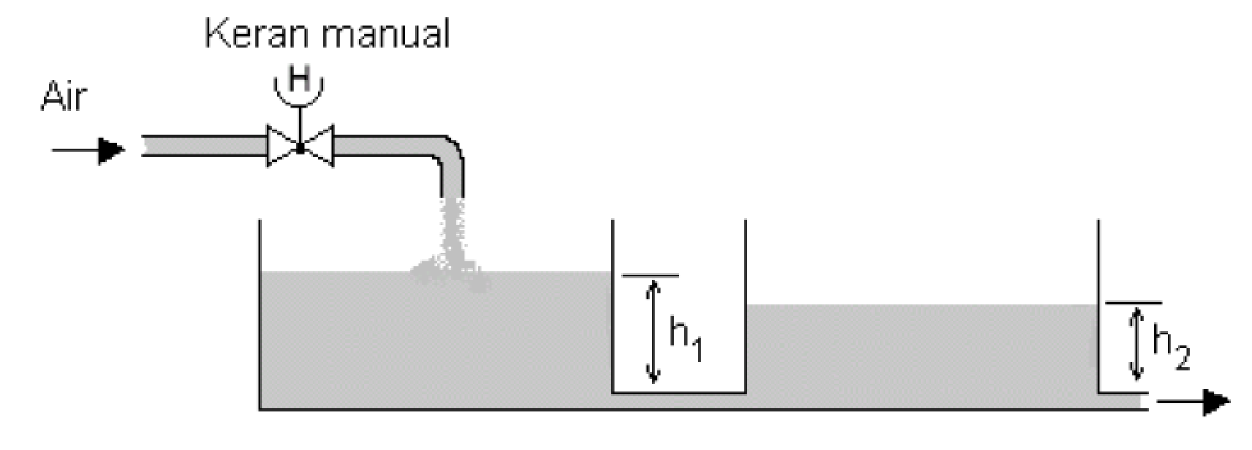
$$\text{f. period of oscillation} = T = \frac{2\pi\tau}{\sqrt{1-\zeta^2}} = \frac{2\pi}{\sqrt{1-0{,}5^2}} = \frac{4\pi}{\sqrt{3}}$$

# situasi fisikal sistem order dua

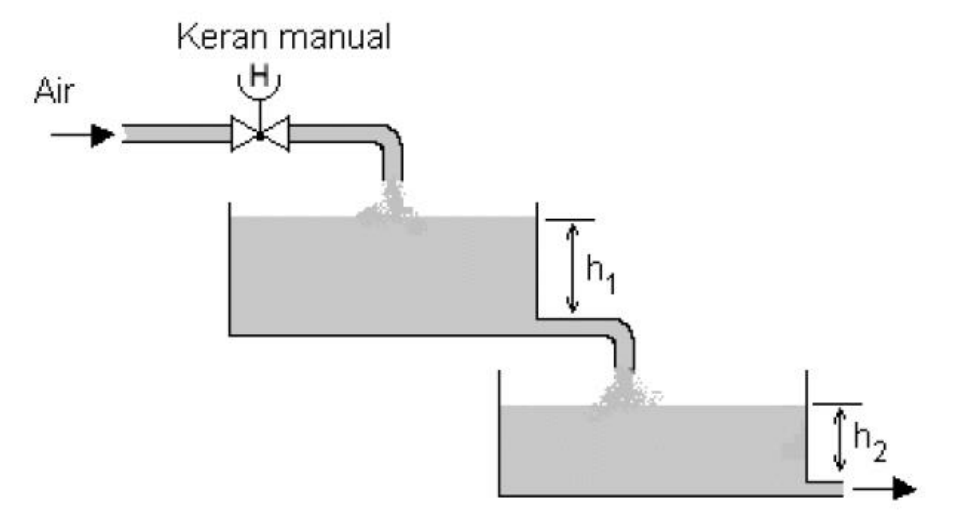
1. Proses multikapasitas, yaitu proses yang terdiri dari dua atau lebih kapasitas (sistem order satu) yang tersusun seri.
2. Proses yang merupakan order dua sesungguhnya.
3. Sistem proses dengan pengendali.

# Contoh Sistem Order Dua (Proses Multikapasitas)

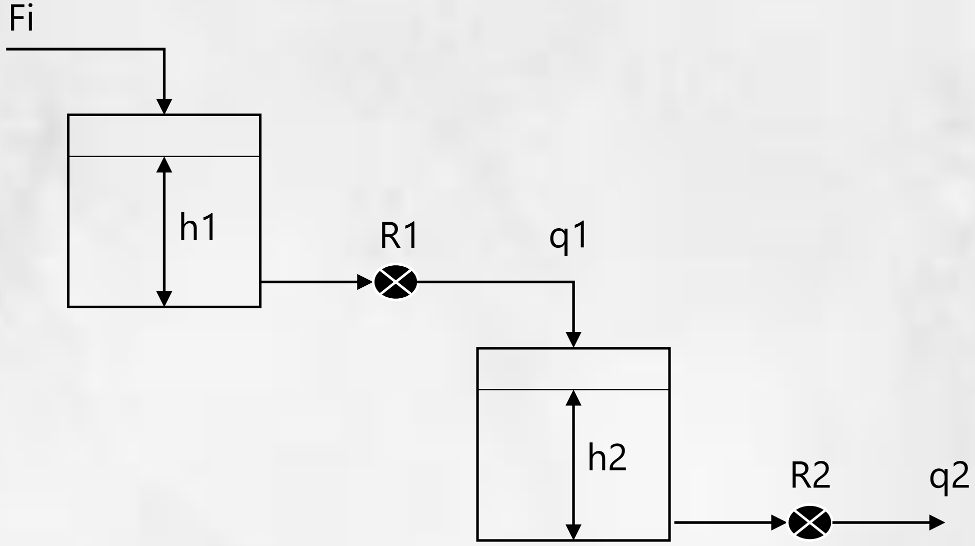
Dua sistem orde satu seri dengan interaksi




Dua sistem orde satu seri tanpa interaksi



# Dua sistem orde satu seri tanpa interaksi




$$A_1 = 1$$

$$A_2 = \frac{1}{2}$$

$$R_1 = \frac{1}{2}$$

$$R_2 = \frac{2}{3}$$

Dua tangki non interaksi dihubungkan seri seperti pada gambar di atas. Tentukanlah $h_1$ dan $h_2$ sebagai fungsi waktu (respon tinggi cairan tangki), jika $q_1 = \frac{h_1}{R_1}$ dan $q_2 = \frac{h_2}{R_2}$ serta laju umpan mengalami perubahan sebesar 3 satuan.

## Neraca massa tangki 1

$$A_1 \frac{dh_1}{dt} = F_i - q_1$$

$$A_1 \frac{dh_1}{dt} = F_i - \frac{h_1}{R_1}$$

$$A_1 \frac{dh_1}{dt} + \frac{h_1}{R_1} = F_i$$

$$\frac{dh_1}{dt} + \frac{h_1}{A_1 R_1} = \frac{F_i}{A_1}$$

$$\frac{dh_1}{dt} + \frac{h_1}{1\left(\frac{1}{2}\right)} = \frac{F_i}{1}$$

$$\frac{dh_1}{dt} + 2h_1 = F_i$$

## Variabel deviasi

$$\frac{dh_1'}{dt} + 2h_1' = F_i'$$

## Transformasi laplace

$$s\,h_1'(s) + 2\,h_1'(s) = F_i'(s)$$

$$h_1'(s)\,(s+2) = F_i'(s)$$

$$h_1'(s) = \frac{F_i'(s)}{(s+2)}$$

## Neraca massa tangki 2

$$A_2 \frac{dh_2}{dt} = q_1 - q_2$$

$$A_2 \frac{dh_2}{dt} = \frac{h_1}{R_1} - \frac{h_2}{R_2}$$

$$\frac{dh_2}{dt} = \frac{h_1}{A_2 R_1} - \frac{h_2}{A_2 R_2}$$

$$\frac{dh_2}{dt} = \frac{h_1}{\frac{1}{2}\cdot\frac{1}{2}} - \frac{h_2}{\frac{1}{2}\cdot\frac{2}{3}}$$

$$\frac{dh_2}{dt} = 4h_1 - 3h_2$$

$$\frac{dh_2}{dt} + 3h_2 = 4h_1$$

## Variabel deviasi

$$\frac{dh_2'}{dt} + 3h_2' = 4h_1'$$

## Transformasi laplace

$$s\,h_2'(s) + 3\,h_2'(s) = 4\,h_1(s)$$

$$h_2'(s)\,(s+3) = 4\,h_1'(s)$$

$$h_2'(s) = \frac{4\,h_1'(s)}{(s+3)}$$

**Fungsi transfer tangki 1**

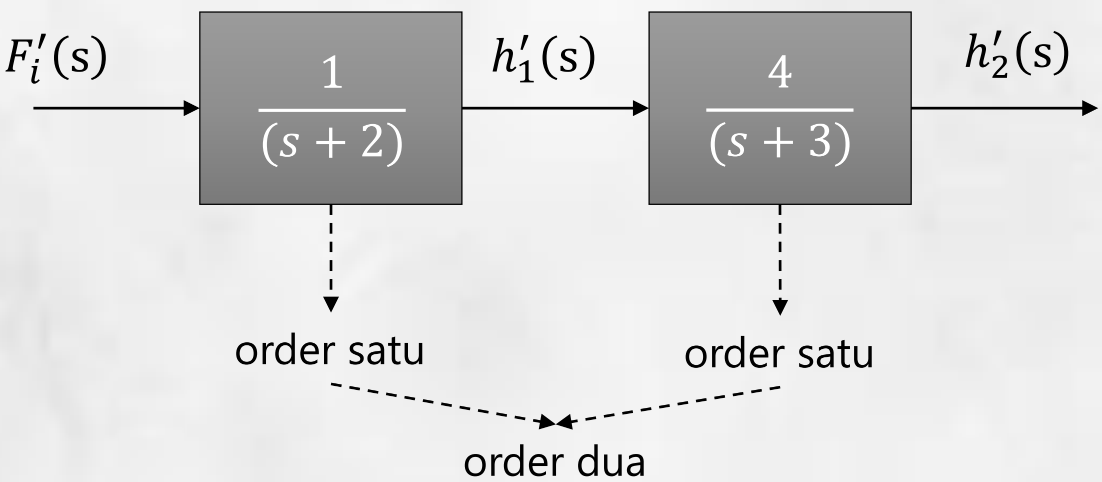
$$h_1'(\mathrm{s}) = \frac{F_i'(\mathrm{s})}{(s+2)} \text{ (transformasi } laplace\text{)}$$

$$\frac{h_1'(\mathrm{s})}{F_i'(\mathrm{s})} = \frac{1}{(s+2)}$$

**Fungsi transfer tangki 2**

$$h_2'(\mathrm{s}) = \frac{4h_1'(\mathrm{s})}{(s+3)} \text{(transformasi } laplace\text{)}$$

$$\frac{h_2'(\mathrm{s})}{h_1'(\mathrm{s})} = \frac{4}{(s+3)}$$



hasil transformasi Laplace :

$$h_1'(s) = \frac{Fi'(s)}{(s+2)} \qquad h_2'(s) = \frac{4}{(s+3)} h_1'(s)$$

sehingga :

$$h_2'(s) = \frac{4}{(s+3)} \frac{Fi'(s)}{(s+2)}$$

Jika laju umpan mengalami perubahan sebesar 3 satuan :

$$Fi'(t) = 3 \rightarrow Fi'(s) = 3/s$$

$$h_2'(s) = \frac{4}{(s+3)} \frac{3}{s} \frac{1}{(s+2)}$$

$$h_2'(s) = \frac{C_1}{(s+3)} + \frac{C_2}{(s)} + \frac{C_3}{(s+2)} = \frac{12}{s(s+2)(s+3)}$$

$C_1$ : dikali $(s+3)$

$$C_1 + \frac{C_2(s+3)}{s} + \frac{C_3(s+3)}{(s+2)} = \frac{12}{s(s+2)}$$

dimasukkan nilai $s = -3$

$$C_1 = \frac{12}{-3(-3+2)}$$

$$C_1 = 4$$

$C_2$ : dikali (s)

$$\frac{C_1\,(s)}{(s+3)} + C_2 + \frac{C_3\,(s)}{(s+2)} = \frac{12}{(s+2)(s+3)}$$

dimasukkan nilai s = 0

$$C_2 = \frac{12}{(0+2)(0+3)} = 2$$

$C_3$ : dikali (s+2)

$$\frac{C_1\,(s+2)}{(s+3)} + \frac{C_2\,(s+2)}{s} + C_3 = \frac{12}{s\,(s+3)}$$

dimasukkan nilai s = -2

$$C_3 = \frac{12}{-2(-2+3)} = -6$$

sehingga : $h_2'(s) = \frac{4}{s+3} + \frac{2}{s} - \frac{6}{s+2}$

Inverse transformasi laplace :

$$h_2'(t) = 4e^{-3t} + 2 - 6e^{-2t}$$

※ $h_1'(s) = \frac{F_1'(s)}{s+2}$

$$h_1'(s) = \frac{3}{s(s+2)} = \frac{C_1}{s} + \frac{C_2}{(s+2)}$$

$C_1$ : dikalikan $s$

$$\frac{3}{(s+2)} = C_1 + \frac{C_2(s)}{(s+2)}$$

dimasukkan nilai $s = 0$

$$\frac{3}{(0+2)} = C_1$$

$$C_1 = 3/2$$

$C_2$ : dikalikan $(s+2)$

$$\frac{3}{s} = \frac{C_1(s+2)}{s} + C_2$$

dimasukkan nilai $s = -2$

$$\frac{3}{-2} = C_2$$

$$C_2 = -3/2$$

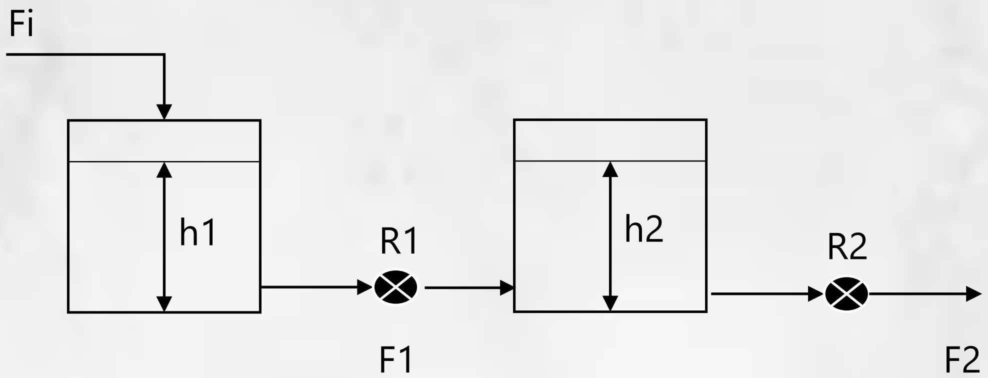
sehingga :

$$h_1'(s) = \frac{3/2}{s} - \frac{3/2}{s+2}$$

inversi transformasi laplace :

$$h_1'(t) = 3/2 - 3/2\, e^{-2t}$$

## Dua sistem orde satu seri dengan interaksi




$$A_1 = 1$$

$$A_2 = \frac{1}{2}$$

$$R_1 = \frac{1}{2}$$

$$R_2 = \frac{2}{3}$$

Tentukanlah $h_1$ dan $h_2$ sebagai fungsi waktu, jika $F_1 = \frac{h_1 - h_2}{R_1}$ dan $F_2 = \frac{h_2}{R_2}$. Laju umpan Fi mengalami perubahan sebesar 1 satuan.

NM tangki 1

$$A_1 \frac{dh_1}{dt} = F_i - F_1$$

$$A_1 \frac{dh_1}{dt} = F_i - \frac{h_1 - h_2}{R_1}$$

$$\frac{dh_1}{dt} = \frac{F_i}{A_1} - \frac{h_1}{R_1 A_1} + \frac{h_2}{R_1 A_1}$$

$$\frac{dh_1}{dt} = \frac{F_i}{1} - \frac{h_1}{1/2 \cdot 1} + \frac{h_2}{1/2 \cdot 1}$$

$$\frac{dh_1}{dt} = F_i - 2h_1 + 2h_2$$

variabel deviasi :

$$\frac{dh_1'}{dt} = F_i' - 2h_1' + 2h_2'$$

transformasi laplace :

$$s\, h_1'(s) = F_i'(s) - 2h_1'(s) + 2h_2'(s)$$

$$h_1'(s)\,(s+2) + h_2'(s)\,(-2) = F_i'(s)$$

$$h_1'(s)\,(s+2) + h_2'(s)\,(-2) = 1/s$$

Jika perubahan input sebesar satu satuan :

$$F_i'(t) = 1$$

$$F_i'(s) = 1/s$$

NM tangki 2

$$A_2 \frac{dh_2}{dt} = F_1 - F_2$$

$$A_2 \frac{dh_2}{dt} = \frac{h_1 - h_2}{R_1} - \frac{h_2}{R_2}$$

$$\frac{dh_2}{dt} = \frac{h_1}{R_1 A_2} - \frac{h_2}{R_1 A_2} - \frac{h_2}{R_2 A_2}$$

$$\frac{dh_2}{dt} = \frac{h_1}{1/2 \cdot 1/2} - \frac{h_2}{1/2 \cdot 1/2} - \frac{h_2}{2/3 \cdot 1/2}$$

$$\frac{dh_2}{dt} = 4h_1 - 4h_2 - 3h_2$$

$$\frac{dh_2}{dt} = 4h_1 - 7h_2$$

variabel deviasi :

$$\frac{dh_2'}{dt} = 4h_1' - 7h_2$$

transformasi laplace

$$s\, h_2'(s) = 4h_1(s) - 7h_2(s)$$

$$h_1(s)\,(-4) + h_2(s)\,(s+7) = 0$$

**Fungsi transfer tangki 2**

$$h_1'(\mathrm{s})(-4) + h_2'(\mathrm{s})(s+7) = 0 \text{ (transformasi } \textit{laplace}\text{)}$$

$$h_2'(\mathrm{s}) = h_1'(\mathrm{s})(4)$$

$$h_2'(\mathrm{s}) = \frac{h_1'(\mathrm{s})(4)}{(s+7)}$$

$$\frac{h_2'(\mathrm{s})}{h_1'(\mathrm{s})} = \frac{(4)}{(s+7)}$$

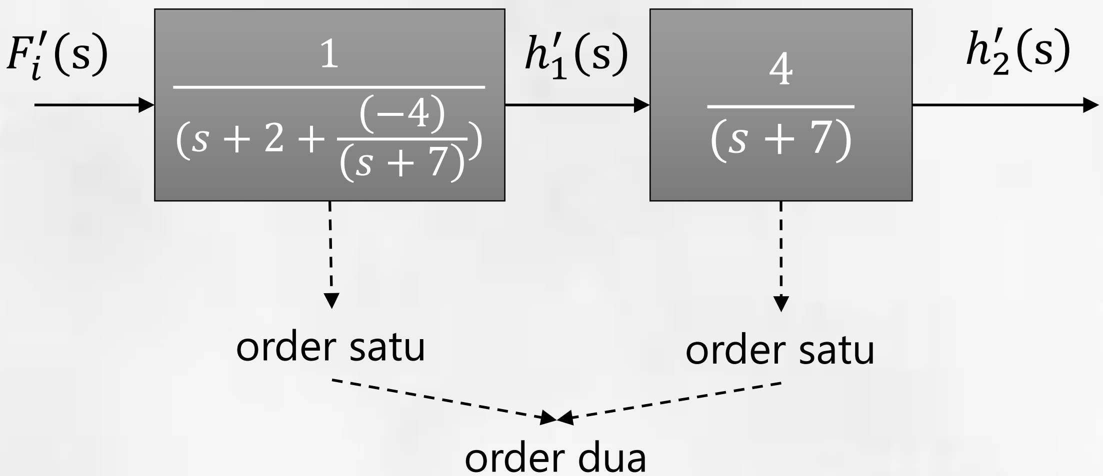
**Fungsi transfer tangki 1**

$$h_1'(\mathrm{s})(s+2) + h_2'(\mathrm{s})(-2) = F_i'(\mathrm{s}) \text{ (transformasi } \textit{laplace}\text{)}$$

$$h_1'(\mathrm{s})(s+2) + \frac{h_1'(\mathrm{s})(4)}{(s+7)}(-2) = F_i'(\mathrm{s})$$

$$h_1'(\mathrm{s})\left(s + 2 + \frac{(-4)}{(s+7)}\right) = F_i'(\mathrm{s})$$

$$\frac{h_1'(\mathrm{s})}{F_i'(\mathrm{s})} = \frac{1}{\left(s + 2 + \frac{(-4)}{(s+7)}\right)}$$



**Persamaan transformasi laplace tangki 1**

$$h_1'(s)(s+2) + h_2'(s)(-2) = F_i'(s)$$

Jika input sebesar 1 satuan, maka $F_i'(s) = {}^{1}/_{s}$

$$h_1'(s)(s+2) + h_2'(s)(-2) = {}^{1}/_{s}$$

**Persamaan transformasi laplace tangki 2**

$$h_1'(s)(-4) + h_2'(s)(s+7) = 0$$

dari kedua persamaan transformasi laplace bentuk matriksnya yaitu:

$$\begin{bmatrix} (s+2) & (-2) \\ (-4) & (s+7) \end{bmatrix} \begin{bmatrix} h_1'(s) \\ h_2'(s) \end{bmatrix} = \begin{bmatrix} 1/s \\ 0 \end{bmatrix}$$

Satuan $h_1'(s)$

Satuan $h_2'(s)$

hasil

$$\begin{bmatrix} (s+2) & (-2) \\ (-4) & (s+7) \end{bmatrix} \begin{bmatrix} h_1'(s) \\ h_2'(s) \end{bmatrix} = \begin{bmatrix} 1/s \\ 0 \end{bmatrix}$$

Sehingga nilai $h_1'(s)$ dan $h_2'(s)$ dapat ditentukan dengan persamaan berikut:

$$h_1'(s) = \frac{\begin{bmatrix} 1/s & (-2) \\ 0 & (s+7) \end{bmatrix}}{\begin{bmatrix} (s+2) & (-2) \\ (-4) & (s+7) \end{bmatrix}} = \frac{\frac{1}{s}(s+7)}{(s+2)(s+7)-(-4)(-2)} = \frac{(s+7)}{s(s^2+9s+6)}$$

$$h_2'(s) = \frac{\begin{bmatrix} (s+2) & 1/s \\ (-4) & 0 \end{bmatrix}}{\begin{bmatrix} (s+2) & (-2) \\ (-4) & (s+7) \end{bmatrix}} = \frac{-\frac{1}{s}(-4)}{(s+2)(s+7)-(-4)(-2)} = \frac{4}{s(s^2+9s+6)}$$

akar dari $s^2 + 9s + 6$

$b = 9; \quad a = 1 \; ; c = 6$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{1,2} = \frac{-9 \pm \sqrt{(-9)^2 - 4(1)(6)}}{2(1)}$$

$$x_{1,2} = \frac{-9 \pm \sqrt{57}}{2}$$

$x_1 = -0,725 \qquad x_2 = -8,275$

$(s + 0,725) \qquad (s + 8,275)$

sehingga :

$$h_1'(s) = \frac{(s+7)}{s(s+0,725)(s+8,275)} = \frac{C_1}{s} + \frac{C_2}{(s+0,725)} + \frac{C_3}{(s+8,275)}$$

$$h_2'(s) = \frac{4}{s(s+0,725)(s+8,275)} = \frac{D_1}{s} + \frac{D_2}{(s+0,725)} + \frac{D_3}{(s+8,275)}$$

diperoleh nilai :

$C_1 = 0,7778 \qquad D_1 = 0,4444$

$C_2 = -1,1464 \qquad D_2 = -0,7308$

$C_3 = -0,0204 \qquad D_3 = 0,0640$

inversi laplace :

$$h_1'(t) = 0,7778 - 1,1464\, e^{-0,725t} - 0,0204\, e^{-8,275t}$$

$$h_2'(t) = 0,4444 - 0,7308\, e^{-0,725t} + 0,0640\, e^{-8,275t}$$