Program Studi Teknik Kimia

Fakultas Teknik Universitas Sebelas Maret 2020

PENGENDALIAN PROSES

TK6543

CH. 5 MODELLING CONSIDERATIONS FOR CONTROL PROCESS

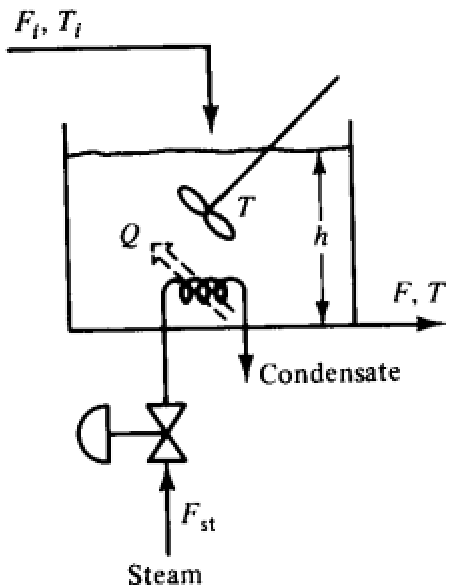
Tika Paramitha, S.T., M.T.

Model Input-Output

Presentasi model dalam bentuk diagram blok:

- Memudahkan dalam melihat keterkaitan input dan output
- Presentasi yang baik untuk sistem jamak (sistem yang terdiri dari beberapa proses)
- Digunakan lebih lanjut dalam perancangan sistem pengendalian

Example 5.1. : Model Input-Output untuk *Stirred Tank Heater*



NE :

$$V \frac{dT}{dt} = F_i (T_i - T) + \frac{Q}{\rho C_p}$$

$$\boxed{Q = U A_t (T_{st} - T)}$$

$$V \frac{dT}{dt} = F_i (T_i - T) + \frac{U A_t}{\rho C_p} (T_{st} - T)$$

$$V \frac{dT}{dt} + \left(F_i + \frac{U A_t}{\rho C_p} \right) T = F_i . T_i + \frac{U A_t}{\rho C_p} T_{st}$$

$$\frac{dT}{dt} + \left(\frac{F_i}{V} + \frac{U A_t}{V \rho C_p} \right) T = \frac{F_i}{V} . T_i + \frac{U A_t}{V \rho C_p} T_{st}$$

$\frac{F_i}{V} \rightarrow \frac{1}{\tau}$, $\frac{U A_t}{V \rho C_p} \rightarrow K$, $\left(\frac{1}{\tau} + K \right) \rightarrow a$

$$\frac{dT}{dt} + a T = \frac{1}{\tau} T_i + K T_{st}$$

pada saat steady state :

$$0 + a T_s = \frac{1}{\tau} T_{i,s} + K T_{st,s}$$

$$\frac{d (T - T_s)}{dt} + a (T - T_s) = \frac{1}{\tau} (T_i - T_{i,s}) + K (T_{st} - T_{st,s})$$

$$\frac{dT'}{dt} + a T' = \frac{1}{\tau} T_i' + K T_{st}'$$

Metode inverse operator :

$$\frac{dT'}{dt} + aT' = \frac{1}{\tau} T_i' + K T_{st}'$$

$$\boxed{y = y_c + y_p}$$

$$\frac{dT'}{dt} + aT' = \frac{1}{\tau} T_i' + K T_{st}'$$

$$\left(\frac{d}{dt} + a\right) T' = \frac{1}{\tau} T_i' + K T_{st}'$$

$$\downarrow$$

$$(D + a) T' = \frac{1}{\tau} T_i' + K T_{st}'$$

$Q(x)$

y_c : $m + a = 0$

$m_1 = -a$

$$\boxed{y_c = c_1 e^{-at}}$$

$$y_p = e^{m_1 x} \int e^{(m_2 - m_1)x} \int e^{(m_3 - m_2)x} \cdots \int Q(x)\, e^{-m_n x} (dx)^n$$

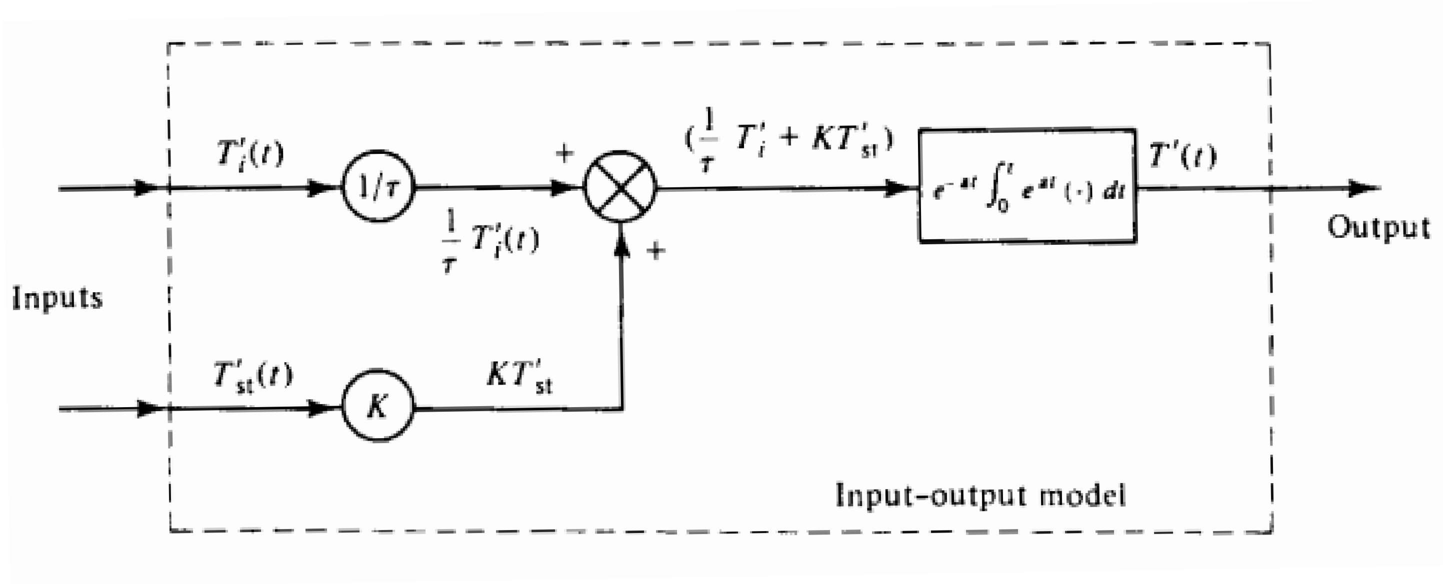
shg,
$$y_p = e^{-at} \int \left(\frac{1}{\tau} T_i' + K\, T_{st}' \right) e^{at}\, dt$$

$$y = y_c + y_p$$

$$T'(t) = C_1 e^{-at} + e^{-at} \int e^{at} \left(\frac{1}{\tau} T_i' + K\, T_{st}' \right) dt$$

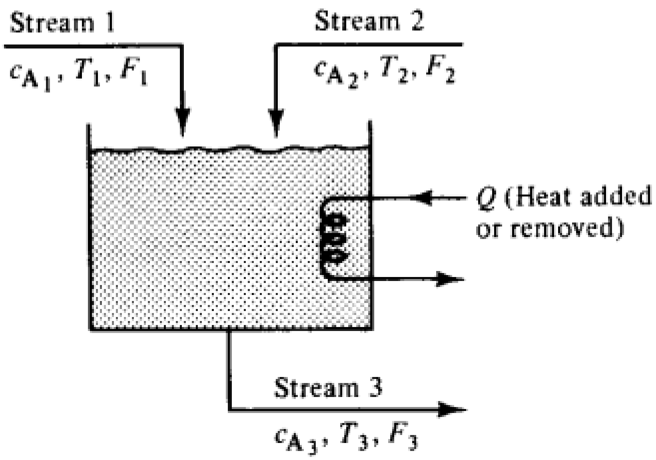
$t = 0$, $T' = 0$, shg $C_1 = 0$

$$T'(t) = e^{-at} \int e^{at} \left(\frac{1}{\tau} T_i' + K\, T_{st}' \right) dt$$



$$T'(t) = e^{-at} \int_0^t e^{at} \left[\frac{1}{\tau} T'_i + KT'_{st} \right] dt$$

Example 5.2. : Model Input-Output untuk *Mixing Process*



NM komponen A

$$V \frac{dC_{A3}}{dt} = (C_{A1} - C_{A3}) F_1 + (C_{A2} - C_{A3}) F_2$$

$\frac{dC_{A3}}{dt} = 0$ saat steady state

$$\frac{dC_{A3}}{dt} + \left(\frac{F_1}{V} + \frac{F_2}{V}\right) C_{A3} = \frac{F_1}{V} C_{A1} + \frac{F_2}{V} C_{A2}$$

NE

$$\rho c_p V \frac{dT_3}{dt} = \rho F_1 c_p (T_1 - T_3) + \rho F_2 c_p (T_2 - T_3) \pm Q \qquad ; \rho c_p . V$$

$\frac{dT_3}{dt} = 0$ saat steady state

$$\frac{dT_3}{dt} + \left(\frac{F_1}{V} + \frac{F_2}{V}\right) T_3 = \frac{F_1}{V} T_1 + \frac{F_2}{V} T_2 + \frac{Q}{\rho c_p V}$$

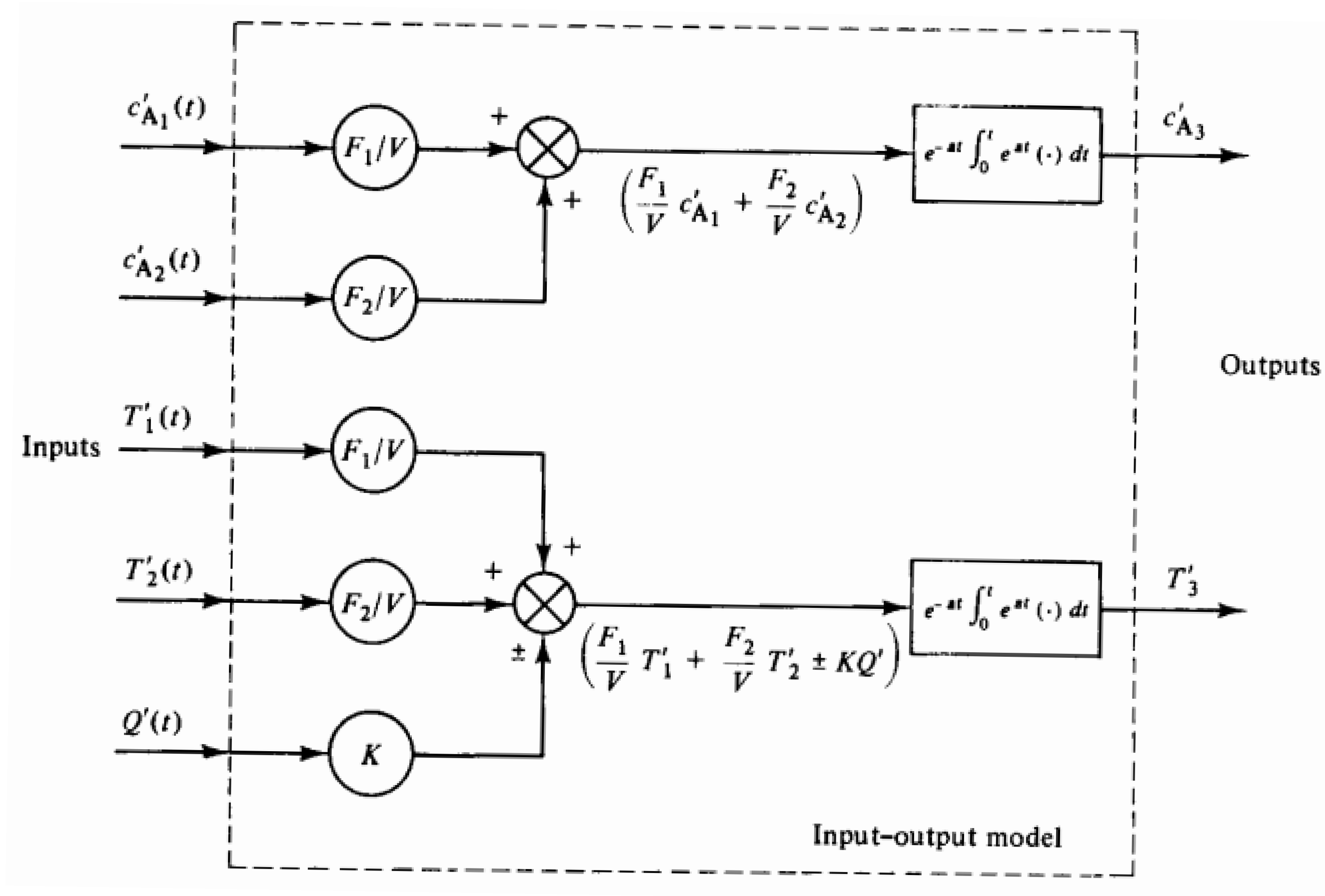
$$\frac{dC_{A3}'}{dt} + a\, C_{A3}' = \frac{F_1}{V} C_{A1}' + \frac{F_2}{V} C_{A2}'$$

$$\frac{dT_3'}{dt} + a\, T_3' = \frac{F_1}{V} T_1' + \frac{F_2}{V} T_2' \pm K Q'$$

$C_{A1}' = C_{A1} - C_{A1,s}$ $\qquad$ $C_{A2}' = C_{A2} - C_{A2,s}$ $\qquad$ $C_{A3}' = C_{A3} - C_{A3,s}$

$T_1' = T_1 - T_{1,s}$ $\qquad$ $T_2' = T_2 - T_{2,s}$ $\qquad$ $T_3' = T_3 - T_{3,s}$

$Q' = Q - Q_s$ $\qquad$ $a = \frac{F_1}{V} + \frac{F_2}{V}$ $\qquad$ $K = \frac{1}{\rho c_p V}$



$$c'_{A_3}(t) = c_1 e^{-at} + e^{-at}\int_0^t e^{at}\left[\frac{F_1}{V}c'_{A_1} + \frac{F_2}{V}c'_{A_2}\right]dt$$

$$T'_3(t) = c_2 e^{-at} + e^{-at}\int_0^t e^{at}\left[\frac{F_1}{V}T'_1 + \frac{F_2}{V}T'_2 \pm KQ'\right]dt$$

Degrees of Freedom (DOF) / Derajat Kebebasan

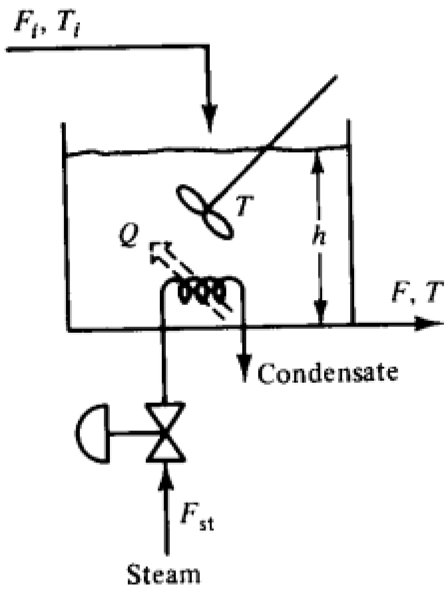
Ada 3 kemungkinan:

- DOF > 0 → sistem *underspecified*
- DOF < 0 → sistem *overspecified*
- DOF = 0 → sistem *perfectly specified*

Untuk mendapatkan DOF=0:

- Spesifikasi gangguan (input)
- Spesifikasi tujuan pengendalian (*control loop*)

Example 5.3. : Degree of Freedom untuk *Stirred Tank Heater*



$$A\frac{dh}{dt} = F_i - F$$

$$Ah\frac{dT}{dt} = F_i(T_i - T) + \frac{Q}{\rho c_p}$$

Jumlah persamaan : 2 persamaan

Jumlah variabel : 6 variabel (h, Fi, F, T, Ti, Q)

A, ρ, dan cp adalah parameter (sudah ditentukan nilainya).

DOF = jumlah variabel – jumlah persamaan = 6 – 2 = 4

Untuk mendapatkan DOF = 0,

- Spesifikasi gangguan (input) : Fi, Ti
- Spesifikasi tujuan pengendalian (*control loop*) : T, h

Sehingga, DOF = 4 – variabel gangguan – variabel yang dikontrol

DOF = 4 – 2 – 2 = 0

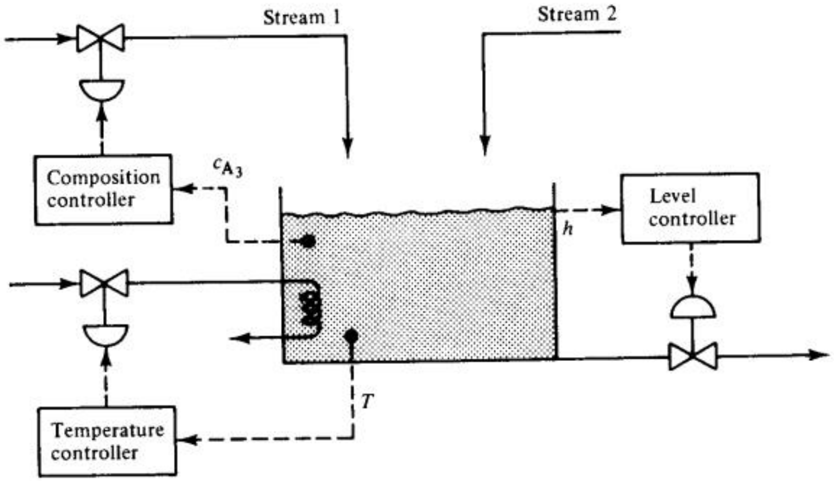
Example 5.8. : Degree of Freedom untuk *Mixing Process*

$$\frac{dV}{dt} = (F_1 + F_2) - F_3$$

$$V\frac{dc_{A_3}}{dt} = (c_{A_1} - c_{A_3})F_1 + (c_{A_2} - c_{A_3})F_2$$

$$\rho c_p V\frac{dT_3}{dt} = c_{A_1}F_1[\Delta\tilde{H}_{S_1} - \Delta\tilde{H}_{S_3}] + c_{A_2}F_2[\Delta\tilde{H}_{S_2} - \Delta\tilde{H}_{S_3}]$$

$$+ \rho F_1 c_p(T_1 - T_3) + \rho F_2 c_p(T_2 - T_3) \pm Q$$



Jumlah persamaan : 3 persamaan

Jumlah variabel : 11 variabel (V, F_1, F_2, F_3, c_{A1}, c_{A2}, c_{A3}, T_1, T_2, T_3, Q)

ρ, cp, ΔH_{S1}, ΔH_{S2} , dan ΔH_{S3} adalah parameter (sudah ditentukan nilainya).

DOF = jumlah variabel – jumlah persamaan = 11 – 3 = 8

Untuk mendapatkan DOF = 0,

- Spesifikasi gangguan (input) = F_1, T_1, F_2, T_2, c_{A2}.
- Spesifikasi tujuan pengendalian (*control loop*) = c_{A3}, T, dan h.

Sehingga, DOF = 4 – variabel gangguan – variabel yang dikontrol

DOF = 8 – 5 – 3 = 0

Program Studi Teknik Kimia
Fakultas Teknik Universitas Sebelas Maret 2020

PENGENDALIAN PROSES
TK6543

CH. 6 LINEARIZATION OF NONLINEAR SYSTEMS

Tika Paramitha, S.T., M.T.

Linierisasi Sistem Satu Variabel

Linearization is the process by which we approximate nonlinear system with linear ones. It is widely used in the study of process dynamics and design of control systems for the following reasons:

1. We can have closed-form, analytic solutions for linear systems. Thus we can have a complete and general picture of a process's behaviour independently of the particular values of the parameters and input variables. This is not possible for nonlinear systems, and computer simulation provides us only with the behaviour of the systems at specified values of inputs and parameters.
2. All the significant developments toward the design of effective control systems have been limited to linear processes.

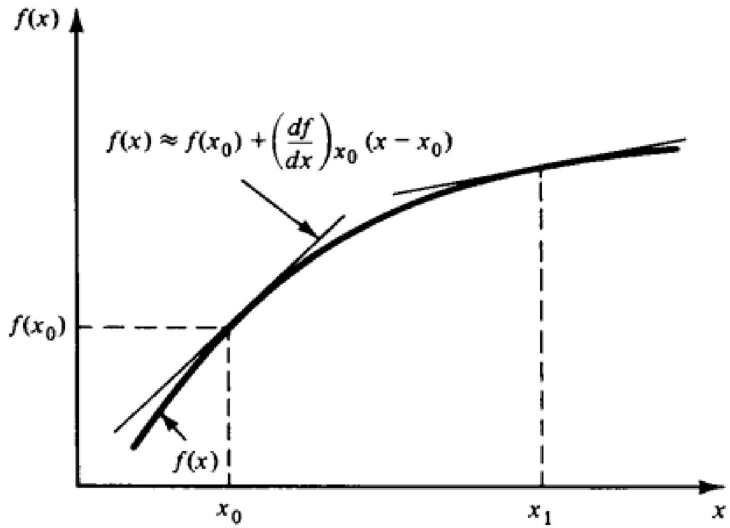
Linierisasi Sistem Satu Variabel

Ekspansi Deret Taylor :

$$f(x) = f(x_0) + \left(\frac{df}{dx}\right)_{x_0} \frac{x - x_0}{1!} + \left(\frac{d^2f}{dx^2}\right)_{x_0} \frac{(x - x_0)^2}{2!} + \cdots + \left(\frac{d^nf}{dx^n}\right)_{x_0} \frac{(x - x_0)^n}{n!} + \cdots$$

Orde 2 dan lebih tinggi bernilai kecil, sehingga dapat diabaikan.

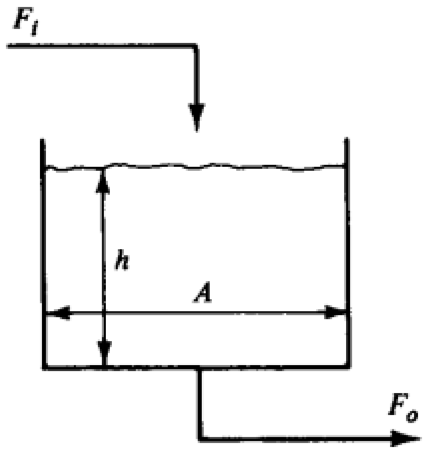
$$f(x) \approx f(x_0) + \left(\frac{df}{dx}\right)_{x_0} (x - x_0)$$



Example 6.1. : Linierisasi Sistem pada Tangki

Neraca Massa Total :

NM Input – NM Output + ~~NM Generasi~~ – ~~NM Konsumsi~~ = NM Akumulasi



$$F_i \cdot \rho - F_o \cdot \rho = \frac{d}{dt}(V\rho)$$

$$F_i \cdot \rho - F_o \cdot \rho = \frac{d(A \cdot h\,\rho)}{dt}$$

$$F_i - F_o = A\frac{dh}{dt}$$

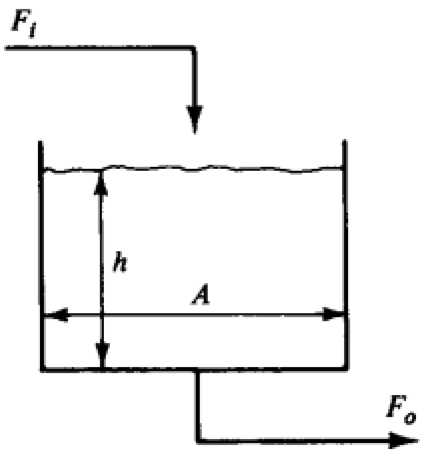
Jika diketahui Fo =α h, maka:

$$F_i = A\frac{dh}{dt} + \alpha h$$

Diperoleh **sistem dinamik linier.**

Neraca Massa Total :

NM Input – NM Output + ~~NM Generasi~~ – ~~NM Konsumsi~~ = NM Akumulasi



$$F_i \cdot \rho - F_o \cdot \rho = \frac{d}{dt}(V\rho)$$

$$F_i \cdot \rho - F_o \cdot \rho = \frac{d(A \cdot h\,\rho)}{dt}$$

$$F_i - F_o = A\frac{dh}{dt}$$

Jika diketahui $F_o = \beta\sqrt{h}$, maka:

$$A\frac{dh}{dt} + \beta\sqrt{h} = F_i$$

Diperoleh **sistem dinamik non-linier.**

Linierisasi:

$$\beta\sqrt{h} = \beta\sqrt{h_o} + \left[\frac{d}{dh}(\beta\sqrt{h})\right]_{h_o}(h-h_o)$$

$$\beta\sqrt{h} = \beta\sqrt{h_o} + \left[\frac{\beta}{2\sqrt{h}}\right]_{h_o}(h-h_o)$$

$$\beta\sqrt{h} = \beta\sqrt{h_o} + \frac{\beta}{2\sqrt{h_o}}(h-h_o)$$

$$\beta\sqrt{h} = \beta\sqrt{h_o} + \frac{\beta}{2\sqrt{h_o}}h - \frac{\beta}{2}\sqrt{h_o}$$

$$\beta\sqrt{h} = \frac{\beta}{2}\sqrt{h_o} + \frac{\beta}{2\sqrt{h_o}}h$$

sehingga :

$$A\frac{dh}{dt} + \frac{\beta}{2}\sqrt{h_o} + \frac{\beta}{2\sqrt{h_o}}h = F_i$$

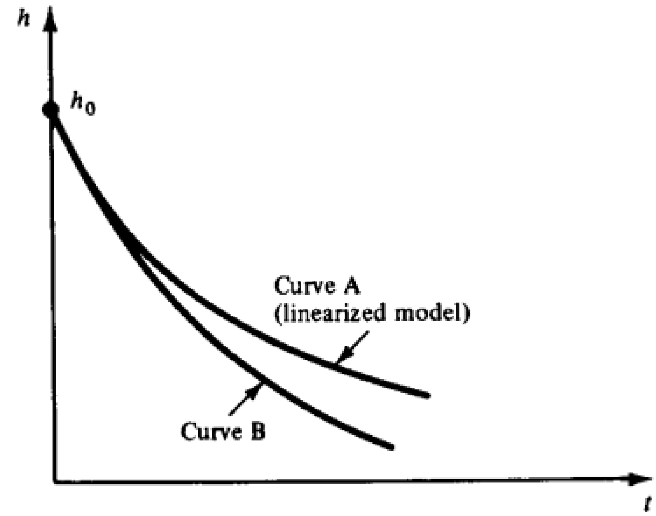
$$A\frac{dh}{dt} + \frac{\beta}{2\sqrt{h_o}}h = F_i - \frac{\beta}{2}\sqrt{h_o}$$

Persamaan kurva A:

$$A\frac{dh}{dt} + \frac{\beta}{2\sqrt{h_0}}h = F_i - \frac{\beta}{2}\sqrt{h_0}$$

Persamaan kurva B:

$$A\frac{dh}{dt} + \beta\sqrt{h} = F_i$$

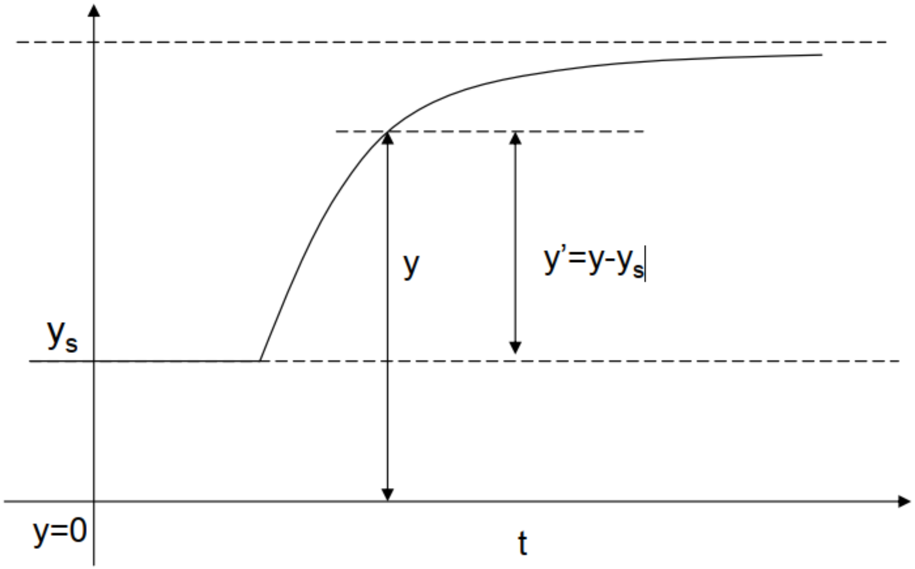


Tangki berada dalam kondisi *steady state* dengan ketinggian cairan ho. Kemudian pada t = 0, aliran ke dalam tangki ditutup dan aliran keluar tangki tetap dibuka. Sehingga akan terjadi penurunan ketinggian cairan di dalam tangki.

Kedua kurva memiliki nilai yang sangat dekat satu sama lain pada periode waktu tertentu. Dengan meningkatnya waktu dan ketinggian cairan terus menurun, nilainya semakin menyimpang dari nilai awal ho di sekitar model linierisasi. Hal ini menunjukkan bahwa ketika perbedaan ho-h meningkat, pendekatan linierisasi semakin tidak akurat, seperti yang diharapkan.

Penyimpangan Variabel

- Kepentingannya:
  – Penyimpangan variabel (yang dikendalikan) terhadap keadaan setimbangnya (bukan dari keadaan awal proses)
- Keuntungannya:
  – Model menjadi lebih sederhana
  – Nilai awal menjadi tentu (nol)



Penyimpangan Variabel

xs pada kondisi *steady state* :

$$\frac{dx_s}{dt} = 0 = f(x_s)$$

Linierisasi persamaan di atas di sekitar xo = xs akan diperoleh:

$$\frac{dx}{dt} = f(x_s) + \left(\frac{df}{dx}\right)_{x_s}(x - x_s)$$

Dikurangkan kedua persamaan di atas:

$$\frac{d(x - x_s)}{dt} = \left(\frac{df}{dx}\right)_{x_s}(x - x_s)$$

$$x' = x - x_s$$

$$\frac{dx'}{dt} = \left(\frac{df}{dx}\right)_{x_s} x'$$

Example 6.2.

Model linierisasi pada tangki sebagai berikut:

$$A\frac{dh}{dt} + \frac{\beta}{2\sqrt{h_0}}h = F_i - \frac{\beta}{2}\sqrt{h_0}$$

hs merupakan ketinggian cairan dan Fi,s merupakan laju alir umpan pada *steady state*.

Linierisasi di sekitar hs:

$$A\frac{dh}{dt} + \frac{\beta}{2\sqrt{h_s}}h = F_i - \frac{\beta}{2}\sqrt{h_s}$$

Pada kondisi *steady state* sebagai berikut:

$$A\frac{dh}{dt} + \beta\sqrt{h} = F_i \quad \Rightarrow \quad A\frac{dh_s}{dt} + \beta\sqrt{h_s} = F_{i,s}$$

$$0 = \frac{dh_s}{dt}$$

Example 6.2.

Dikurangkan kedua persamaan di atas:

$$h' = h - h_s \qquad \text{and} \qquad F_i' = F_i - F_{i,s}$$

$$A\frac{dh}{dt} + \frac{\beta}{2\sqrt{h_s}}h = F_i - \frac{\beta}{2}\sqrt{h_s}$$

$$A\frac{dh_s}{dt} = F_{i,s} - \beta\sqrt{h_s}$$

$$A\frac{d(h-h_s)}{dt} + \frac{\beta}{2\sqrt{h_s}}h = (F_i - F_{is}) + \frac{\beta}{2}\sqrt{h_s} \rightarrow \frac{\beta}{2\sqrt{h_s}}h_s$$

$$A\frac{d(h-h_s)}{dt} + \frac{\beta}{2\sqrt{h_s}}(h-h_s) = (F_i - F_{is}) \longrightarrow A\frac{dh'}{dt} + \frac{\beta}{2\sqrt{h_s}}h' = F_i'$$

TERIMA KASIH