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PENGENDALIAN PROSES
TK6543

MODEL MATEMATIKA

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BAGAIMANA MEMBANGUN SISTEM PENGENDALIAN ?

Langkah pertama, yaitu mengetahui kelakuan dinamik dari proses yang akan dikendalikan.

PENDEKATAN

- **Eksperimental**

  – Melalui pengukuran berbagai variabel input dan output untuk mengetahui kelakuan dinamika proses.

  – Model dinamik sesuai kenyataan.

  – *Time and effort consuming*.

- **Teoretikal**

  – Melalui penggambaran teoretik yang diterjemahkan dalam model matematik.

  – Model dinamik merupakan pendekatan, bergantung pada asumsi yang diambil.

  – Cepat, mudah, dan murah.

Model Matematika

- Menghindari percobaan yang rumit; melibatkan banyak variasi pada sejumlah variabel
- Menghindari percobaan yang membutuhkan biaya dan bahan
- Sebagai deskripsi proses sehingga rancangan sistem pengendalian dapat dilakukan dengan lebih mudah

Elemen Model Proses

- Persamaan Neraca
  - Neraca massa total dan komponen
  - Neraca energi
  - Neraca momentum
- Persamaan laju perpindahan
- Persamaan laju kinetik reaksi
- Persamaan kesetimbangan kimia dan fasa
- Persamaan Keaadaan

Principle of Conservation

$$\frac{\text{[accumulation of S within a system]}}{\text{time period}} = \frac{\text{[flow of S in the system]}}{\text{time period}} - \frac{\text{[flow of S out of the system]}}{\text{time period}}$$

$$+ \frac{\text{[amount of S generated within the system]}}{\text{time period}} - \frac{\text{[amount of S consumed within the system]}}{\text{time period}}$$

The quantity of S can be any of the following fundamental quantities :

- Total mass
- Mass of individual components
- Total energy
- Momentum

Model Proses : Perpindahan

- Perpindahan massa
  – Hukum Fick
- Perpindahan panas
  – Hukum Fourier
- Perpindahan momentum
  – Hukum Stokes

Model Proses : Kinetika Reaksi

- Persamaan laju reaksi

$$r = kC_A^{\alpha}C_B^{\beta}C_C^{\gamma}$$

- Pengaruh temperatur terhadap laju reaksi

$$k = k_o e^{-Ea/RT}$$

Model Proses : Kesetimbangan

- Kesetimbangan Reaksi

$$K = \frac{C_C^c C_D^d}{C_A^a C_B^b}$$

- Kesetimbangan Fasa

$$K = \frac{y}{x}$$

DEAD TIME

Pengaruh gangguan pada variabel-variabel keadaan yang teramati tidak terjadi secara spontan melainkan memerlukan tenggat waktu tertentu. Tenggat waktu tersebut biasa disebut sebagai *dead time,* atau *transportation lag*, atau *pure delay*, atau *distance-velocity lag*, atau singkatnya *lag.*

DEAD TIME

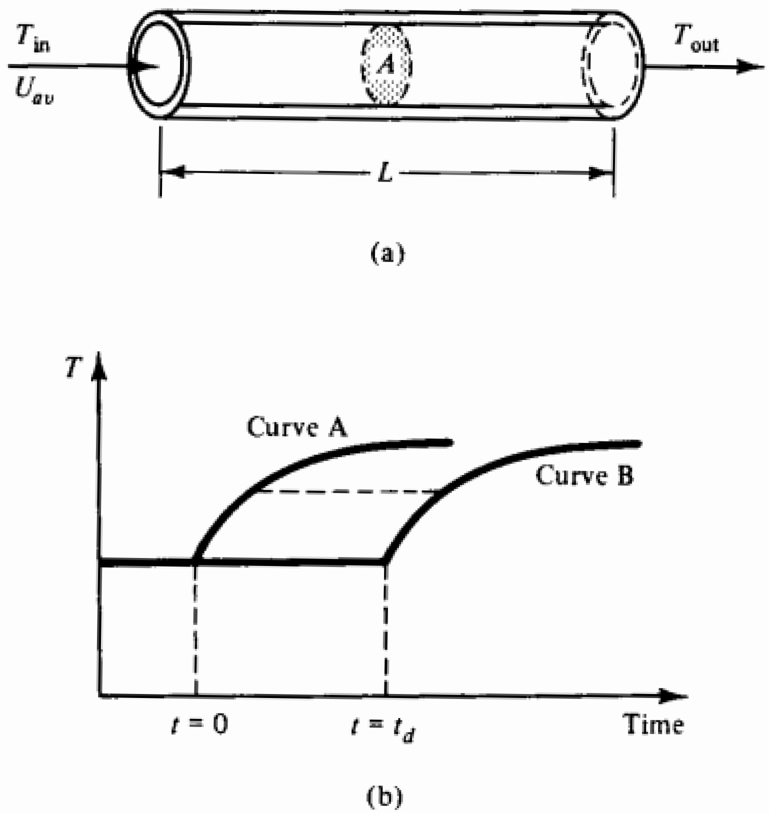


Figure 4.7 (a) Pipe flow of Example 4.9; (b) delayed response of exit temperature to inlet temperature change.

Asumsikan pipa terisolasi secara sempurna dan panas yang muncul karena gesekan diabaikan. Pada keadaan *steady state*, suhu keluar pipa Tout akan sama dengan suhu masuk pipa Tin. Suatu saat pada t=0, suhu umpan diubah sehingga mengikuti kurva A. Ketika itu, suhu keluar pipa tidak langsung berubah tetapi akan tetap sampai perubahan suhu tadi diujung pipa (mengikuti kurva B). Sehingga kita dapat menghubungkan Tin dan Tout dengan persamaan berikut :

Tout (t) = Tin (t-td)

MODEL DINAMIK

Perubahan variabel yang ditinjau terhadap waktu

⇩

Akumulasi ≠ 0

⇩

Persamaan Diferensial

PEMODELAN MATEMATIKA

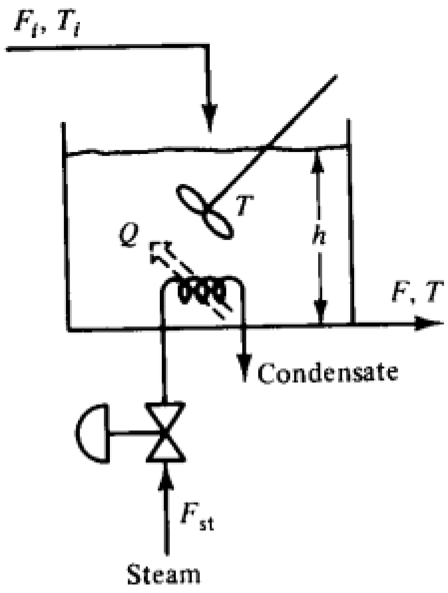
Pemodelan matematika yang akan dipelajari :

Example 4.4. : Stirred Tank Heater (Halaman 51)

Example 4.10. : Continuous Stirred Tank Reactor (Halaman 59)

Example 4.11. : Mixing Process (Halaman 64)

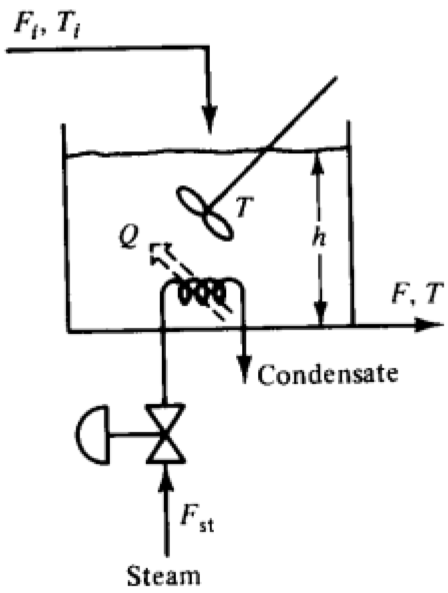
Example 4.4. : Stirred Tank Heater



The fundamental quantities whose values provide the information about the heater are :

a. The total mass of the liquid in the tank

b. The total energy of the material in the tank



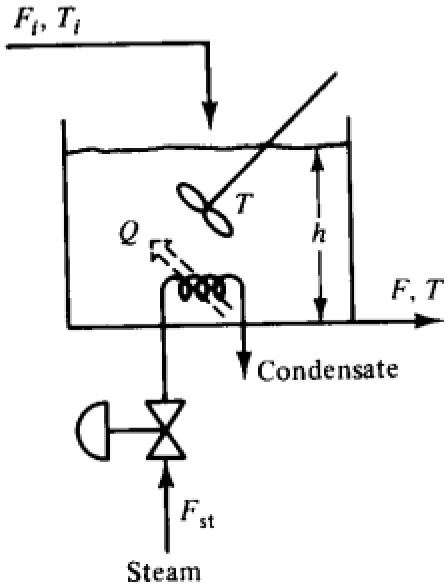
※ Neraca Massa

$$\frac{\text{massa yang terakumulasi}}{\text{waktu}} = \frac{\text{massa input}}{\text{waktu}} - \frac{\text{massa output}}{\text{waktu}}$$

$$\frac{d(\rho A h)}{dt} = \rho F_i - \rho F$$

ρ konstan, sehingga:

$$A \frac{dh}{dt} = F_i - F$$



* Neraca Energi

$$\frac{\text{Energi yang terakumulasi}}{\text{waktu}} = \frac{\text{Energi input}}{\text{waktu}} - \frac{\text{Energi output}}{\text{waktu}} + \frac{\text{Energi yang ditambahkan}}{\text{waktu}}$$

$$\frac{d[\rho A h\, C_p (T - T_{ref})]}{dt} = \rho F_i C_p (T_i - T_{ref}) - \rho F C_p (T - T_{ref}) + Q$$

$: \rho C_p$

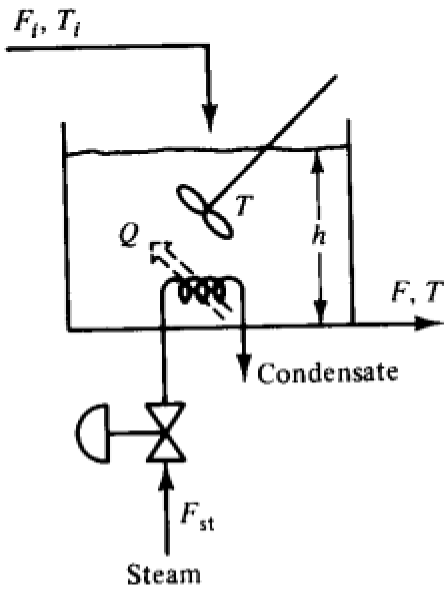
$T_{ref} = 0$

$$A \frac{d(hT)}{dt} = F_i T_i - F.T + \frac{Q}{\rho C_p}$$

$$A h \frac{dT}{dt} + A T \frac{dh}{dt} = F_i T_i - F.T + \frac{Q}{\rho C_p}$$

$$A h \frac{dT}{dt} + (F_i - F) T = F_i T_i - F.T + \frac{Q}{\rho C_p}$$

$$A\, h. \frac{dT}{dt} = F_i (T_i - T) + \frac{Q}{\rho C_p}$$



State equations:

$$A \frac{dh}{dt} = F_i - F$$

$$Ah \frac{dT}{dt} = F_i(T_i - T) + \frac{Q}{\rho c_p}$$

State variables: h, T

Output variables: h, T (both measured)

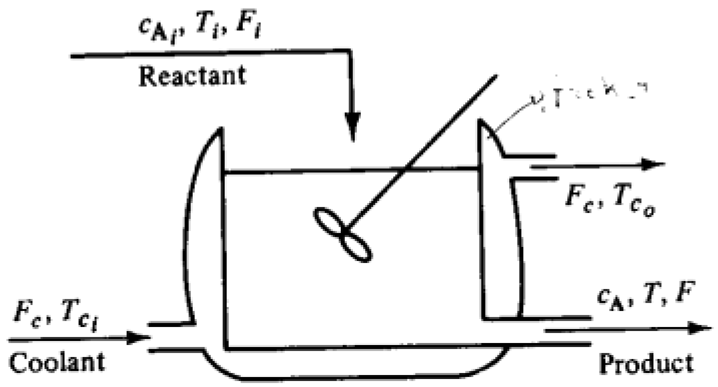
Input variables

Disturbances: T_i, F_i

Manipulated variables: Q, F (for feedback control)
F_i (for feedforward control)

Parameters: A, ρ, c_p

Example 4.10. : Continuous Stirred Tank Reactor



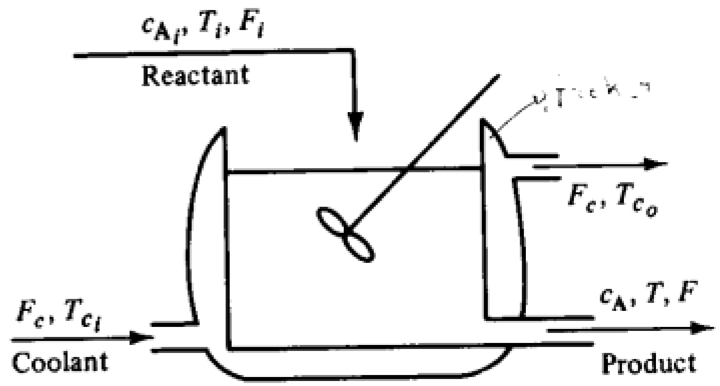
A simple exothermic reaction A → B takes place in the reactor, which is in turn cooled by coolant that flows through a jacket around the reactor.

The fundamental dependent quantities for the reactor are :

a. Total mass of the reacting mixture in tank

b. Mass of chemical A in the reacting mixture

c. Total energy of the reacting mixture in the tank

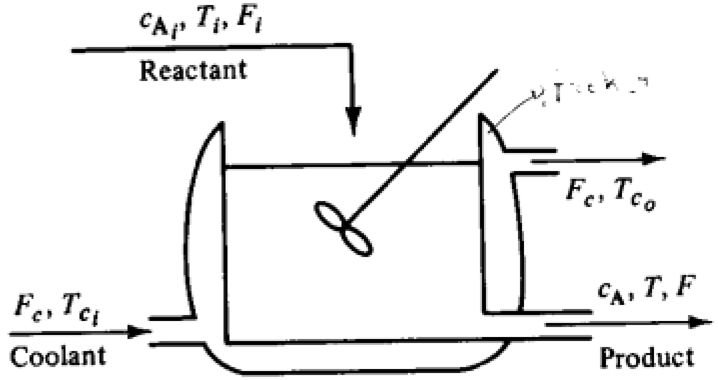


* Neraca Massa

$$\frac{\text{Massa yang terakumulasi}}{\text{waktu}} = \frac{\text{Massa input}}{\text{waktu}} - \frac{\text{Massa output}}{\text{waktu}} \pm \frac{\text{Massa yang tergenerasi / terkonsumsi}}{\text{waktu}}$$

$$\frac{d(\rho V)}{dt} = \rho_i F_i - \rho F \pm 0$$

ρ konstan, sehingga $\frac{dV}{dt} = F_i - F$



* Neraca massa komponen A

$$\frac{\text{akumulasi A}}{\text{waktu}} = \frac{\text{input A}}{\text{waktu}} - \frac{\text{output A}}{\text{waktu}} - \frac{\text{pengurangan A dikarenakan reaksi}}{\text{waktu}}$$

$$\frac{d(n_A)}{dt} = \frac{d(C_A . V)}{dt} = C_{Ai} . F_i - C_A . F - r . V$$

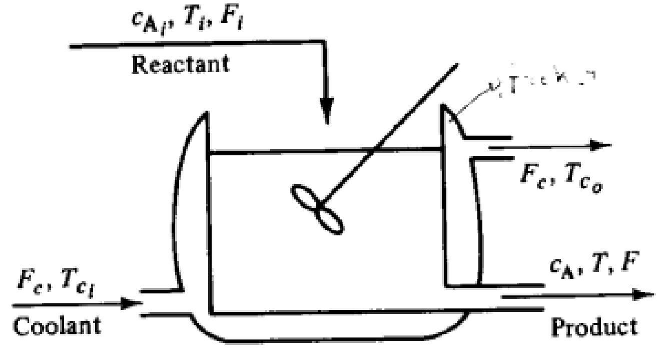
$$\frac{d(C_A V)}{dt} = C_A \frac{dV}{dt} + V \frac{dC_A}{dt} = C_{Ai} . \bar{F_i} - C_A . F - r . V$$

$$C_A (F_i - F) + V \frac{dC_A}{dt} = C_{Ai} . F_i - C_A F - r . V$$

$$V \frac{dC_A}{dt} = -C_A (F_i - \cancel{F}) + C_{Ai} . F_i - \cancel{C_A F} - r . V$$

$$V \frac{dC_A}{dt} = F_i (C_{Ai} - C_A) - k_0 e^{-E/RT} . C_A . V$$

$$\frac{dC_A}{dt} = \frac{F_i}{V} (C_{Ai} - C_A) - k_0 e^{-E/RT} . C_A$$



※ Neraca Energi

$$\frac{\text{Energi yang terakumulasi}}{\text{waktu}} = \frac{\text{Energi masuk}}{\text{waktu}} - \frac{\text{Energi keluar}}{\text{waktu}} - \frac{\text{Energi yang terambil oleh pendingin}}{\text{waktu}}$$

Energi total :

$$E = U + K + P$$

U = internal energy

K = kinetic energy

P = potential energy

Diasumsikan reaksi tidak berpindah ⇒ $\frac{dK}{dt} = \frac{dP}{dt} = 0$

sehingga :

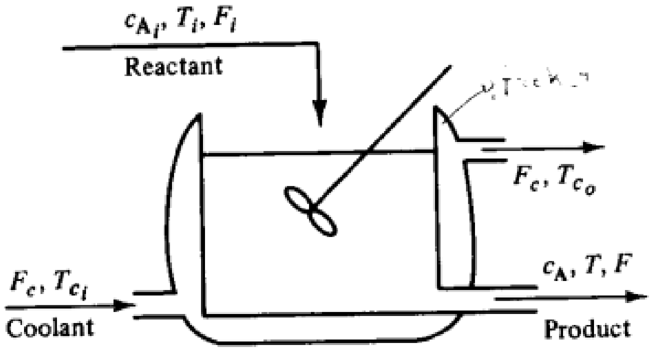
$$\frac{dE}{dt} = \frac{d(U+K+P)}{dt} = \frac{dU}{dt}$$

$$\frac{dU}{dt} \simeq \frac{dH}{dt}$$

$$\frac{dH}{dt} = \rho_i F_i h_i(T_i) - \rho F h(T) - Q$$

Q = amout of heat removed by coolant

h = specific ~~entalphy~~ enthalpy (enthalpy per unit mass)



* Neraca Energi

Entalpi merupakan fungsi temperatur dan komposisi

$$H = H(T, n_A, n_B)$$

$$\frac{dH}{dt} = \frac{\partial H}{\partial T}\frac{dT}{dt} + \frac{\partial H}{\partial n_A}\frac{dn_A}{dt} + \frac{\partial H}{\partial n_B}\frac{dn_B}{dt}$$

$$\frac{\partial H}{\partial T} = \rho V C_p \qquad \frac{\partial H}{\partial n_A} = \bar{H}_A(T) \qquad \frac{\partial H}{\partial n_B} = \bar{H}_B(T)$$

C_p : specific heat capacity

$\bar{H}_A$ dan $\bar{H}_B$: partial molar enthalpies of A and B.

NM komponen A : $\frac{dn_A}{dt} = \frac{d(C_A.V)}{dt} = C_{Ai}.F_i - C_A.F - r.V$

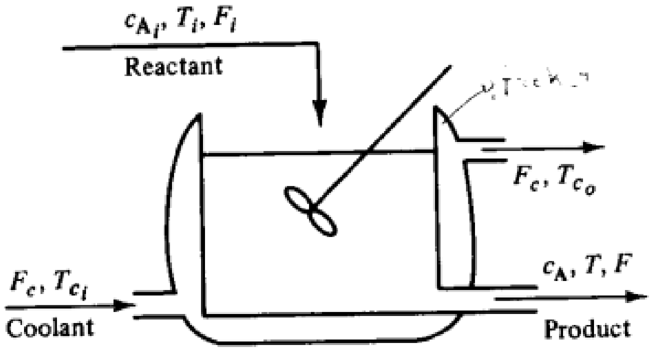
NM komponen B : $\frac{dn_B}{dt} = \frac{d(C_B.V)}{dt} = 0 - C_B.F + r.V$

sehingga :

$$\frac{dH}{dt} = \rho V C_p \frac{dT}{dt} + \bar{H}_A(C_{Ai}.F_i - C_A.F - r.V) + \bar{H}_B(-C_B.F + rV)$$

$$\rho_i.F_i.h_i(T_i) - \rho F h(T) - Q = \rho V C_p \frac{dT}{dt} + \bar{H}_A(C_{Ai}.F_i - C_A.F - r.V) + \bar{H}_B(-C_B F + r.V)$$

$$\rho V C_p \frac{dT}{dt} = \rho_i F_i h_i(T_i) - \rho F h(T) - Q - \bar{H}_A(C_{Ai}.F_i - C_A.F - r.V) - \bar{H}_B(-C_B.F + r.V)$$



- $\rho_i F_i h_i(T_i) = F_i \left[\rho_i h_i(T) + \rho_i c_{pi} (T_i - T) \right]$
  $= F_i \left[C_{Ai} \bar{H}_A(T) + \rho_i c_{pi} (T_i - T) \right]$
- $\rho F h(T) = F \left[C_A \bar{H}_A(T) + C_B \bar{H}_B(T) \right]$

$$\rho V C_p \frac{dT}{dt} = F_i \left[C_{Ai} \bar{H}_A + \rho_i c_{pi} (T_i - T) \right] - F \left[C_A \bar{H}_A + C_B \bar{H}_B \right] - Q - \bar{H}_A (C_{Ai} F_i - C_A F - r.V) - \bar{H}_B (-C_B F + r.V)$$

$$\rho V C_p \frac{dT}{dt} = F_i C_{Ai} \bar{H}_A + F_i \rho_i c_{pi} (T_i - T) - F C_A \bar{H}_A - F C_B \bar{H}_B - Q - \bar{H}_A C_{Ai} . F_i + \bar{H}_A C_A . F + \bar{H}_A . r . V + \bar{H}_B C_B . F - \bar{H}_B . r . V$$

$$\rho V c_p \frac{dT}{dt} = F_i \rho_i c_{pi} (T_i - T) - Q + (\bar{H}_A - \bar{H}_B) r.V$$

$\rho_i = \rho$ & $c_{pi} = C_p$
$\bar{H}_A - \bar{H}_B = (-\Delta H_r)$
persamaan dibagi ρC_p :

$$V \frac{dT}{dt} = F_i (T_i - T) - \frac{Q}{\rho C_p} + \frac{(-\Delta H_r) . r . V}{\rho C_p}$$

State variables: V, c_A, T

State equations:

$$\frac{dV}{dt} = F_i - F \tag{4.8a}$$

$$\frac{dc_A}{dt} = \frac{F_i}{V}(c_{A_i} - c_A) - k_0 e^{-E/RT} c_A \tag{4.9a}$$

$$\frac{dT}{dt} = \frac{F_i}{V}(T_i - T) + Jk_0 e^{-E/RT} c_A - \frac{Q}{\rho c_p V} \tag{4.10b}$$

where $J = (-\Delta H_r)/\rho c_p$.

Output variables: V, c_A, T

Input variables: c_{A_i}, F_i, T_i, Q, F (when feedback control is used)

Among the input variables the most common disturbances are:

Disturbances: c_{A_i}, F_i, T_i

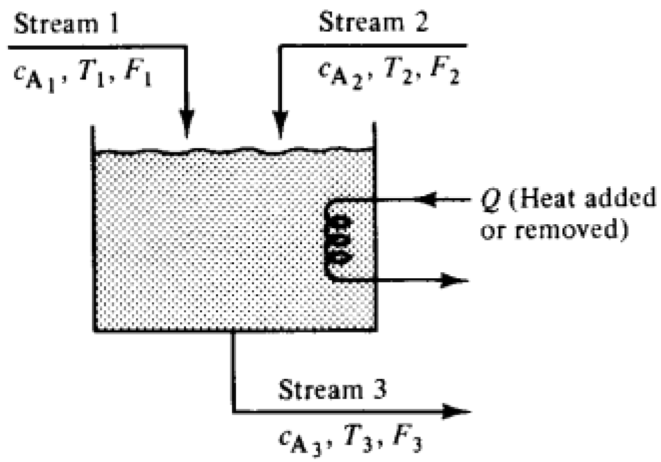
while the usual manipulated variables are:

Manipulated variables: Q, F (occasionally F_i or T_i)

The remaining variables are parameters characteristic of the reactor system:

Constant parameters: ρ, c_p, $(-\Delta H_r)$, k_0, E (activation energy), R

Example 4.11. : Mixing Process

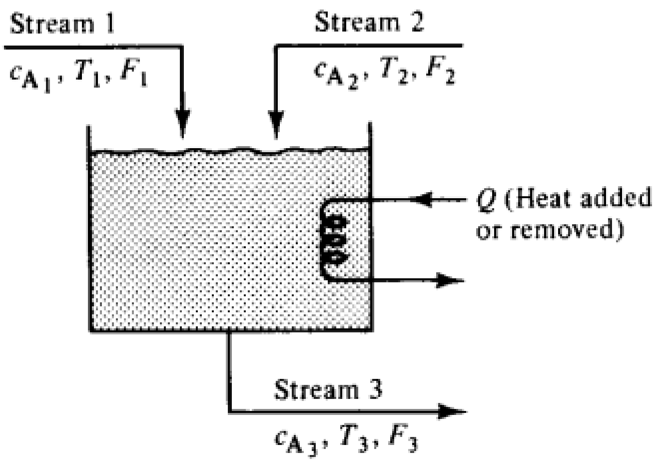


The fundamental quantities needed to describe the mixing process are :

a. Total mass in the tank

b. Amounts of components A and B in the tank

c. Total energy



NM:

akumulasi = input - output

$$\frac{d(\rho V)}{dt} = (\rho_1 F_1 + \rho_2 F_2) - \rho_3 F_3$$

$$\rho = \rho_1 = \rho_2 = \rho_3$$

$$\frac{dV}{dt} = A\frac{dh}{dt} = (F_1 + F_2) - F_3$$

NM komponen A

akumulasi = input - output

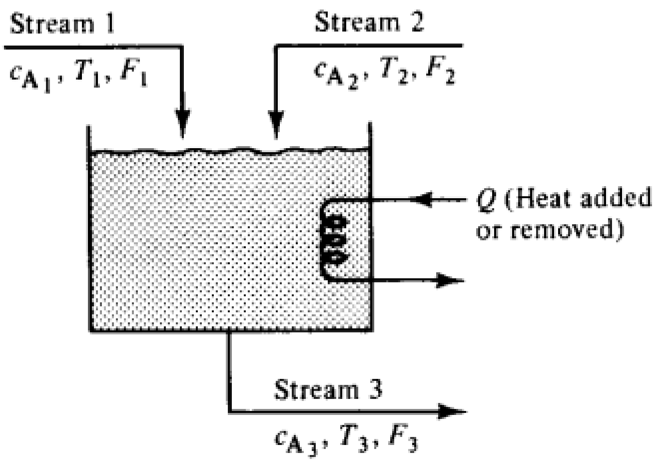
$$\frac{d(C_A . V)}{dt} = (C_{A1} F_1 + C_{A2} F_2) - C_{A3} F_3$$

$$V\frac{dC_A}{dt} + C_A\frac{dV}{dt} = (C_{A1} F_1 + C_{A2} F_2) - C_{A3}.F_3$$

$$V\frac{dC_A}{dt} + C_A(F_1 + F_2 - F_3) = C_{A1} F_1 + C_{A2} F_2 - C_{A3} F_3$$

$C_A = C_{A3}$ karena pencampuran sempurna

$$V\frac{dC_{A3}}{dt} = (C_{A1} - C_{A3}) F_1 + (C_{A2} - C_{A3}) F_2 + (C_{A3} - C_{A3}) F_3 \quad (= 0)$$



NE

akumulasi = input − output ± heat yg diambil / ditambahkan

$$\frac{dU}{dt} \simeq \frac{dH}{dt}$$

- H : total entalpi di dalam tangki

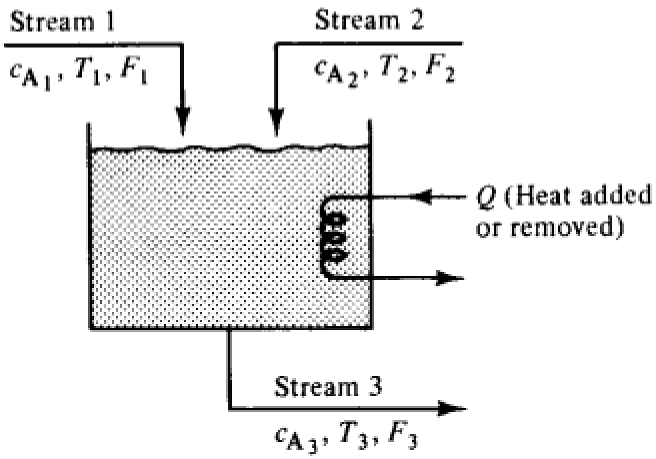
energi input = $\rho\,(F_1 h_1 + F_2 h_2)$

energi output = $\rho F_3 h_3$

} → $\frac{dH}{dt}$

- h_1, h_2, h_3 ⇒ entalpi spesifik (entalpi / massa)

entalpi spesifik pada aliran 3 dianggap sama dengan yang ditangki



total energi balance :

$$\frac{d(\rho V h_3)}{dt} = \rho(F_1 h_1 + F_2 h_2) - \rho F_3 h_3 \pm Q$$

$\Delta H_s \rightarrow$ panas larutan

$h_3(T_3) = h_3(T_0) + Cp_3(T_3 - T_0)$

$h_2(T_2) = h_2(T_0) + Cp_2(T_2 - T_0)$

$h_1(T_1) = h_1(T_0) + Cp_1(T_1 - T_0)$

$\tilde{H}_A$, $\tilde{H}_B$ > partial molaR entulphy (entalpi per mol)

To adalah T referensi

$\rho h_3(T_0) = C_{A3}\hat{H}_A + C_{B3}\hat{H}_B + C_{A3}\Delta\tilde{H}_{s3}(T_0)$

$\rho h_2(T_0) = C_{A2}\tilde{H}_A + C_{B2}\tilde{H}_B + C_{A2}\Delta\tilde{H}_{s2}(T_0)$

$\rho h_1(T_0) = C_{A1}\tilde{H}_A + C_{B1}\tilde{H}_B + C_{A1}\Delta\tilde{H}_{s1}(T_0)$

$$\frac{d[V(C_{A3}\tilde{H}_A + C_{B3}\hat{H}_B + C_{A3}\Delta\hat{H}_{s3}) + \rho V Cp_3(T_3 - T_0)]}{dt}$$

$= F_1(C_{A1}\tilde{H}_A + C_{B1}\tilde{H}_B + C_{A1}\Delta\tilde{H}_{s1}) + \rho F Cp_1(T_1 - T_0)$

$+ F_2(C_{A2}\tilde{H}_A + C_{B2}\tilde{H}_B + C_{A2}\Delta\tilde{H}_{s2}) + \rho F Cp_2(T_2 - T_0)$

$- F_3(C_{A3}\tilde{H}_A + C_{B3}\tilde{H}_B + C_{B3}\Delta\hat{H}_{s3}) + \rho F Cp_3(T_3 - T_0) \pm Q$

$\rho Cp_3 \frac{d[V(T_3 - T_0)]}{dt} + \tilde{H}_A\left(\frac{d(VC_{A3})}{dt} - F_1 C_{A1} - F_2 C_{A2} + F_3 C_{A3}\right)$ =0 NM komp A

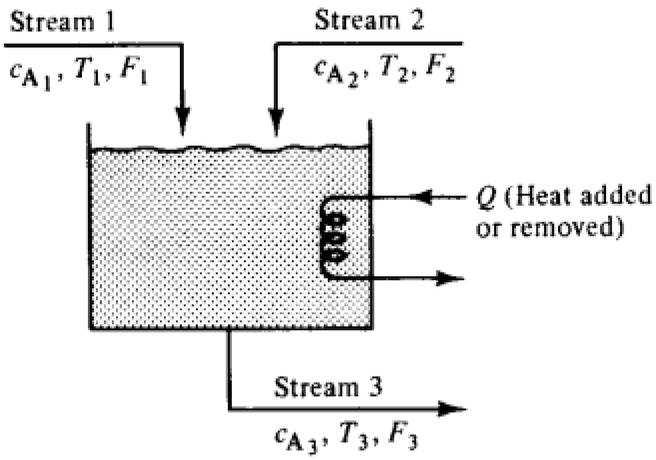
$+ \hat{H}_B\left(\frac{d(VC_{B3})}{dt} - C_{B1}F_1 - C_{B2}F_2 \; C_{B3}F_3\right)$ =0 NM komp B

$+ \Delta\hat{H}_{s3}\frac{d(VC_{A3})}{dt}$

$= F_1 C_{A1}\Delta\hat{H}_{s1} + \rho F Cp_1(T_1 - T_0)$

$+ F_2 C_{A2}\Delta\hat{H}_{s2} + \rho F Cp_2(T_2 - T_0)$

$- F_3 C_{A3}\Delta\hat{H}_{s3} + \rho F Cp_3(T_3 - T_0) \pm Q$



$(F_1 + F_2) - F_3$ ☆

$$\rho C_{p3} V \frac{dT}{dt} + \rho C_{p3} (T_3 - T_0) \frac{dV}{dt} + \Delta H_{s3} \left(C_{A1} F_1 + C_{A2} F_2 - C_{A3} F_3 \right)$$

$$= F_1 C_{A1} \Delta \hat{H}_{s1} + \rho F_1 C_{p1} (T_1 - T_0) + F_2 C_{A2} \Delta \hat{H}_{s2} + \rho F_2 C_{p2} (T_2 - T_0)$$

$$- F_3 C_{A3} \Delta \hat{H}_{s3} - \rho F_3 C_{p3} (T_3 - T_0) \pm Q$$

finally :

$$\rho C_{p3} V \frac{dT_3}{dt} = C_{A1} F_1 \left(\Delta \hat{H}_{s1} - \Delta \hat{H}_{s3} \right) + C_{A2} F_2 \left(\Delta \hat{H}_{s2} - \Delta \hat{H}_{s3} \right)$$

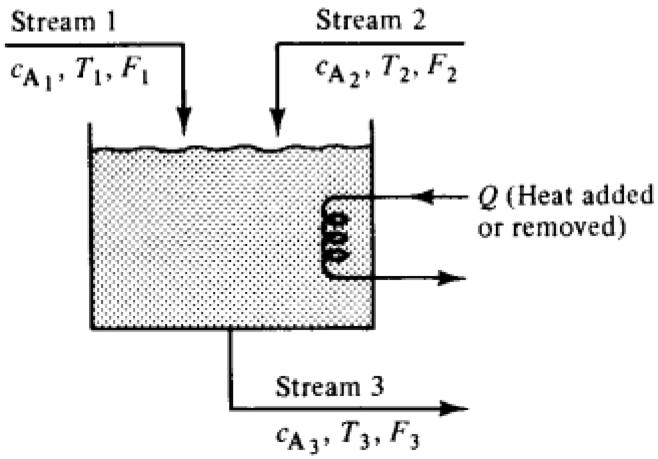
$$+ \rho F_1 \left(C_{p1} (T_1 - T_0) - C_{p3} (T_3 - T_0) \right)$$

$$+ \rho F_2 \left(C_{p2} (T_2 - T_0) - C_{p3} (T_3 - T_0) \right) \pm Q$$

$$C_{p1} = C_{p2} = C_{p3} = C_p$$

$$\rho C_p V \frac{dT_3}{dt} = C_{A1} F_1 \left(\Delta \hat{H}_{s1} - \Delta \hat{H}_{s3} \right) + C_{A2} F_2 \left(\Delta \hat{H}_{s2} - \Delta \hat{H}_{s3} \right)$$

$$+ \rho F_1 C_p (T_1 - T_3) + \rho F_2 C_p (T_2 - T_3) \pm Q$$



State variables: V, c_{A_3}, T_3

State equations:

$$\frac{dV}{dt} = (F_1 + F_2) - F_3 \tag{4.12a}$$

$$V\frac{dc_{A_3}}{dt} = (c_{A_1} - c_{A_3})F_1 + (c_{A_2} - c_{A_3})F_2 \tag{4.13a}$$

$$\rho c_p V \frac{dT_3}{dt} = c_{A_1}F_1[\Delta\tilde{H}_{S_1} - \Delta\tilde{H}_{S_3}] + c_{A_2}F_2[\Delta\tilde{H}_{S_2} - \Delta\tilde{H}_{S_3}] + \rho F_1 c_p(T_1 - T_3) + \rho F_2 c_p(T_2 - T_3) \pm Q \tag{4.14a}$$

Input variables: $F_1, c_{A_1}, T_1, F_2, c_{A_2}, T_2, F_3$ (for feedback control)
Output variables: V (or equivalently the height of liquid level, h), c_{A_3}, T_3
Parameters (*constant*): $\rho, c_p, \Delta\tilde{H}_{S_1}, \Delta\tilde{H}_{S_2}, \Delta\tilde{H}_{S_3}$

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