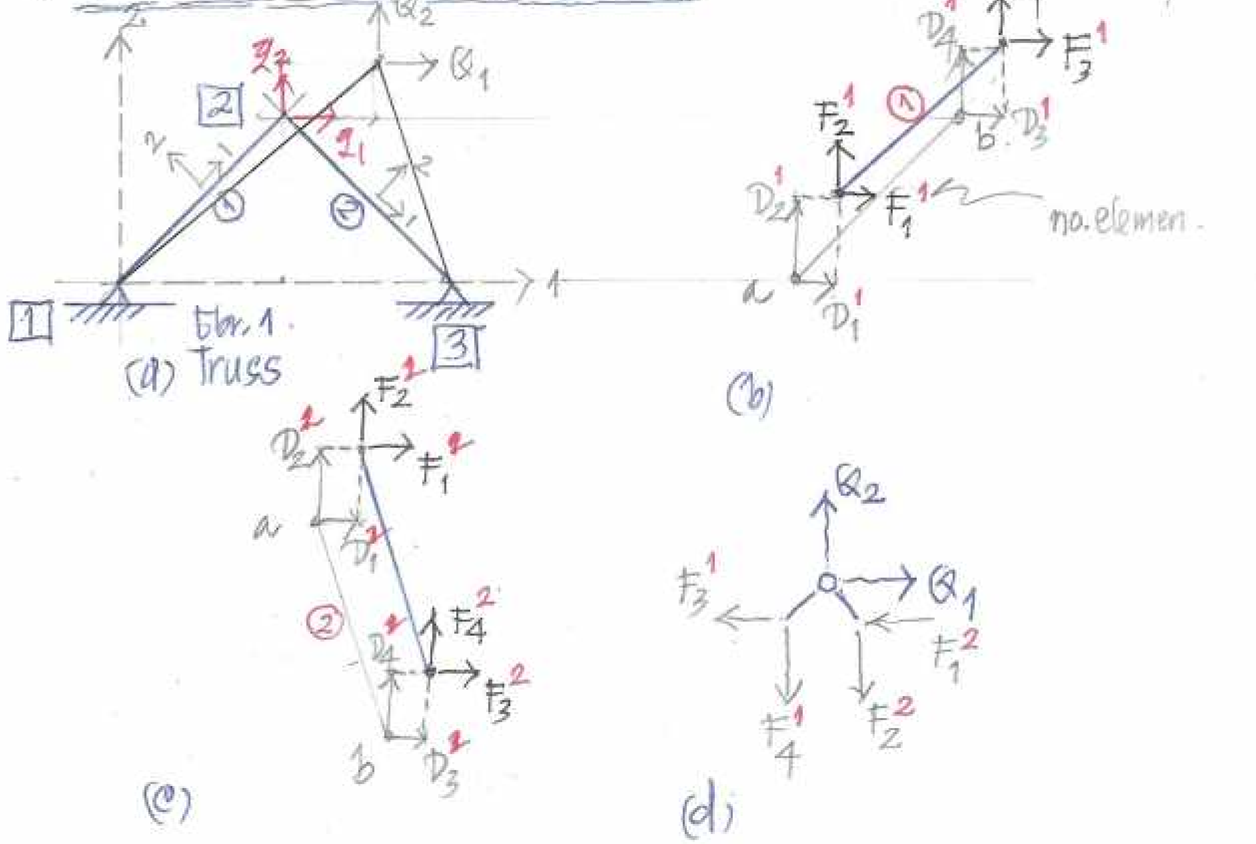
# TKS.665.METODE ELEMEN HINGGA

## S2-Teknik Sipil FT.UNS.
## TM-04. Truss Elemen & Tugas
## Senin, 16-Mart-2020

## TRUSS ELEMEN - COMPATIBILITY




- dof (degree of freedom) $n = l.N_j - N_C$ - - - - - - - - - - - - (2-1)
- Perhatikan gambar Truss, joint-1 & joint-3 constraint. joint-2 free. Simpangan $Q_1$ & $Q_2$ relatif thd. sb. global.
- Compatibility truss.

$$\left.\begin{array}{ll} D_1^1 = 0 \;;\; D_2^1 = 0 \;;\; D_3^1 = Q_1^1 \;;\; D_4^1 = Q_2^1 & \to \text{elemen } 1 \\ D_1^2 = Q_1 \;;\; D_2^2 = Q_2 \;;\; D_3^2 = 0 \;;\; D_4^2 = 0 & \to \text{elemen } 2 \end{array}\right\} \quad (2\text{-}2)$$

- maka: dof $\Rightarrow n = 2 * (3) - (4) = 2$
- Matrik u/ Truss tsb.
(Boundary Condition)

$$\begin{array}{cc} & \begin{array}{cc} 1 & 2 \end{array} \to \text{elemen} \\ M = & \left[\begin{array}{cc} 0 & 1 \\ 0 & 2 \\ 1 & 0 \\ 2 & 0 \end{array}\right] \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \end{array}$$

$\leftarrow$ simpangan

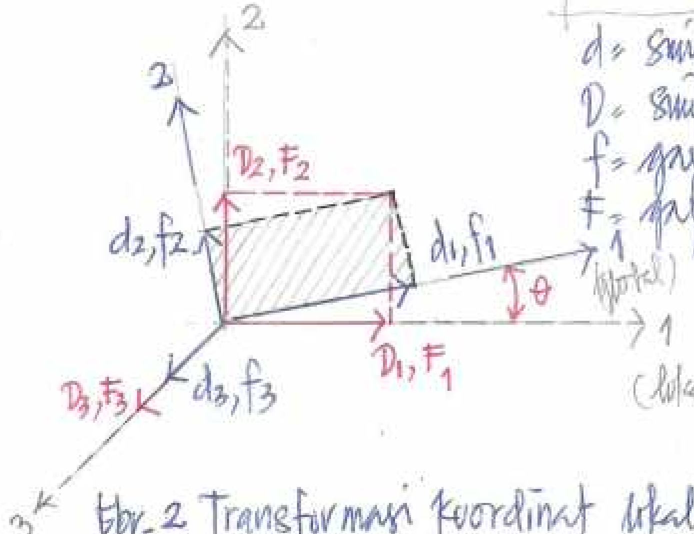
(2) q

## TRANSFORMASI KOORDINAT.

$$d = \Lambda D \; ; \quad F = \Lambda^T . f \quad \text{---} (2)$$

d = simpangan lokal
D = simpangan pd. koord global
f = gaya pd koord lokal
F = gaya pd koord global.

$\Lambda$ = matrik transformasi.

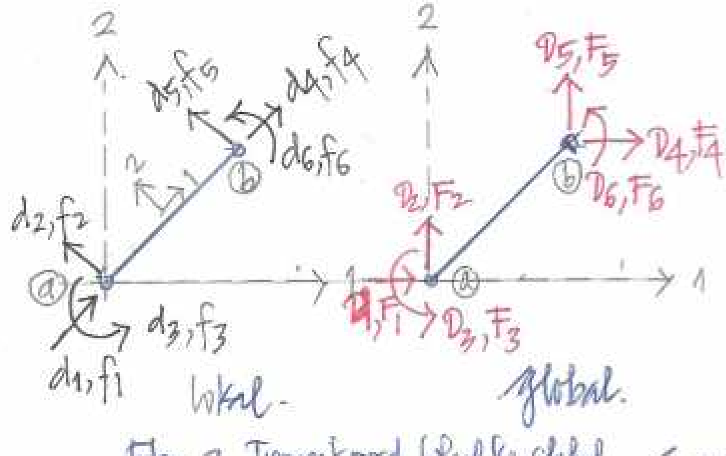



Gbr. 2 Transformasi koordinat lokal & global.

$$\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} c_1 & c_2 & 0 \\ -c_2 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} \quad \text{atau : } d_a = \lambda . D_a \quad \text{---} (3)$$

$$\lambda = \begin{bmatrix} c_1 & c_2 & 0 \\ -c_2 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} , \quad c_1 = \cos\theta \; , \; c_2 = \sin\theta \quad \text{---} (4)$$

$\theta$ = sudut putar berlawanan arah jarum jam, dari koord global-1 ke lokal-1.




$$\begin{bmatrix} d_4 \\ d_5 \\ d_6 \end{bmatrix} = \begin{bmatrix} c_1 & c_2 & 0 \\ -c_2 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} D_4 \\ D_5 \\ D_6 \end{bmatrix}$$

$$d_b = \lambda . D_b \quad \text{---} (5)$$

Gbr. 3. Trans koord lokal ke global

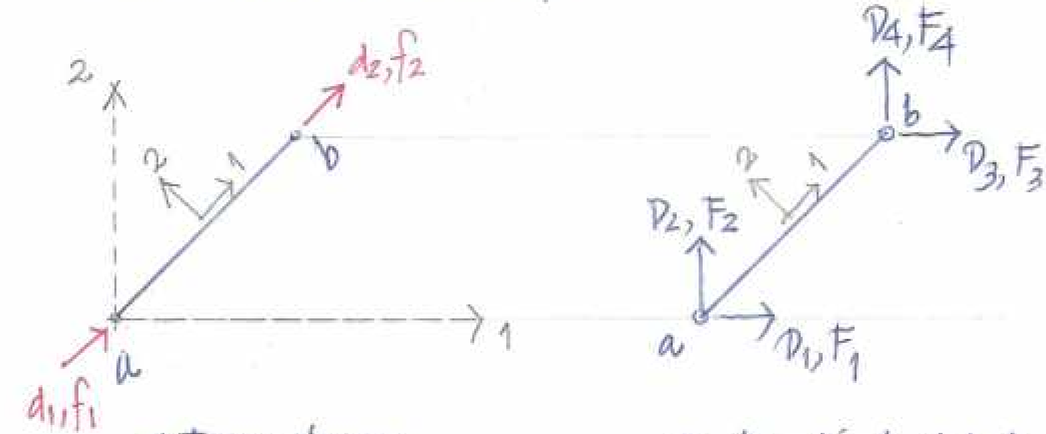
$$\begin{bmatrix} d_a \\ d_b \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} D_a \\ D_b \end{bmatrix} \rightarrow \boxed{d = \Lambda D} \quad \text{---} (6)$$

Analog pers (6) untuk transformasi gaya: $F_a = \lambda^T f_a$ ; $F_b = \lambda^T . f_b$ --- (7)

$$\begin{bmatrix} F_a \\ F_b \end{bmatrix} = \begin{bmatrix} \lambda^T & 0 \\ 0 & \lambda^T \end{bmatrix} \begin{bmatrix} f_a \\ f_b \end{bmatrix} \quad \text{atau : } \underline{\underline{F = \Lambda^T . f}} \; . \; ---- (8)$$

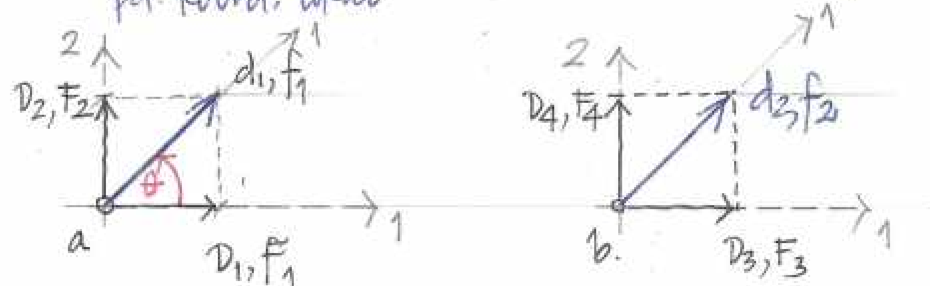
$$\Lambda^T = \begin{bmatrix} \lambda^T & 0 \\ 0 & \lambda^T \end{bmatrix}$$

- TRUSS (ELEMEN BATANG).




(a) Truss elemen pd. koord. lokal

(b) Koordinat global.




(c) ujung ttk. a.

(d) ujung. ttk. b.

Gambar. 4, Truss elemen.

- Pada titik-a. ujung dari elemen truss. gbr. 4 (c)

$$d_1 = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} \text{ atau : } d_a = \lambda . D_a \quad \text{----} \quad (9).$$

- Pada ttk. b. ujung dari elemen truss gbr. 4. (d)

$$d_2 = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \begin{bmatrix} D_3 \\ D_4 \end{bmatrix} \text{ atau : } d_b = \lambda D_b \quad \text{----} \quad (10)$$

- Gabung pers (9) & (10) diperoleh :

$$\begin{bmatrix} d_a \\ d_b \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} D_a \\ D_b \end{bmatrix} \text{ atau : } d = \Lambda D \quad \text{----} \quad (11)$$

- Dimana : $\lambda = \begin{bmatrix} c_1 & c_2 \end{bmatrix}$ ; $c_1 = \cos\theta$ ; $c_2 = \sin\theta$ --- (12)

# ○ GAYA PADA JOINT

Gaya pd. joint × reaksinya pada joint, dapat dihitung dengan kondisi keseimbangan pd. joint.
Kondisi keseimbangan pd. joint. sbb.

$$P_j = \sum_{i=1}^{NE} F_j^i \; ; \; j = 1, 2, \ldots, NJ. \quad \text{------} (9)$$

dimana:

$P_j$ = vektor gaya pd joint

$$F_j^i = \begin{cases} F_a^i & : \text{ttk. a, elemen. } i; \\ F_b^i & : \text{ttk-b, elemen. } i. \\ 0 & : \text{atau yg lain} \end{cases}$$

NE = jumlah elemen

NJ = jumlah joint.

Perhatikan gbr-5 berikut:

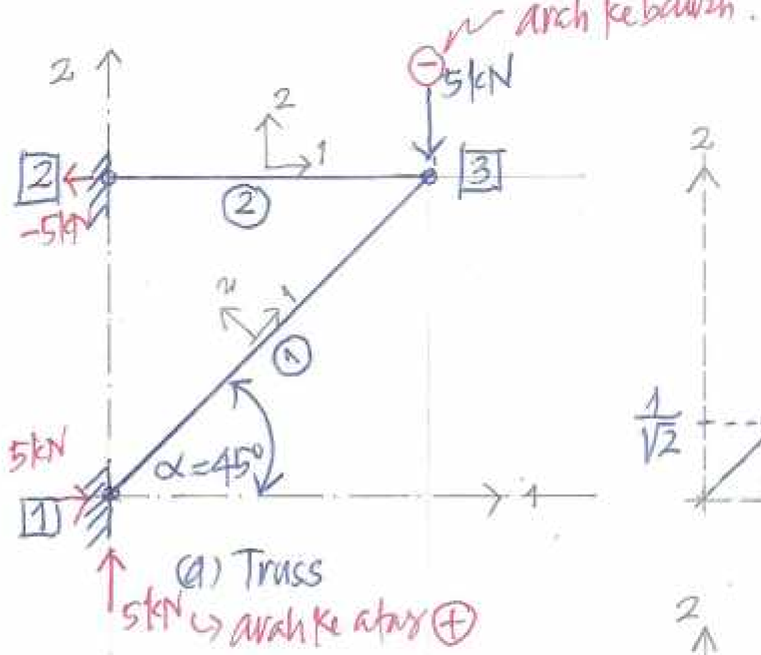
$$\left.\begin{aligned} P_1 &= F_a^1 = \lambda^{1^T} f_a^1 \\ P_2 &= F_a^2 = \lambda^{2^T} f_a^2 \\ P_3 &= F_b^1 + F_b^2 = \lambda^{1^T} \cdot f_b^1 + \lambda^{2^T} \cdot f_b^2 \end{aligned}\right\} \quad \text{-----} (10)$$

lihat gbr. 5. (b):

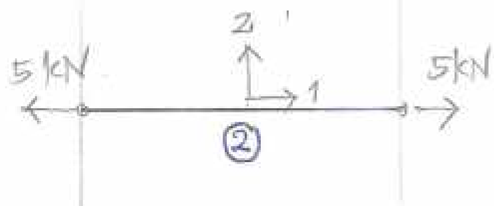
$$f^1 = \begin{bmatrix} f_{a_1}^1 \\ f_b^1 \end{bmatrix} = \begin{bmatrix} 5\sqrt{2} \\ \text{---} \\ 5\sqrt{2} \end{bmatrix} \; ; \; f^2 = \begin{bmatrix} f_a^2 \\ f_b^2 \end{bmatrix} = \begin{bmatrix} -5 \\ \text{---} \\ 5 \end{bmatrix} \quad \text{-----} (11)$$

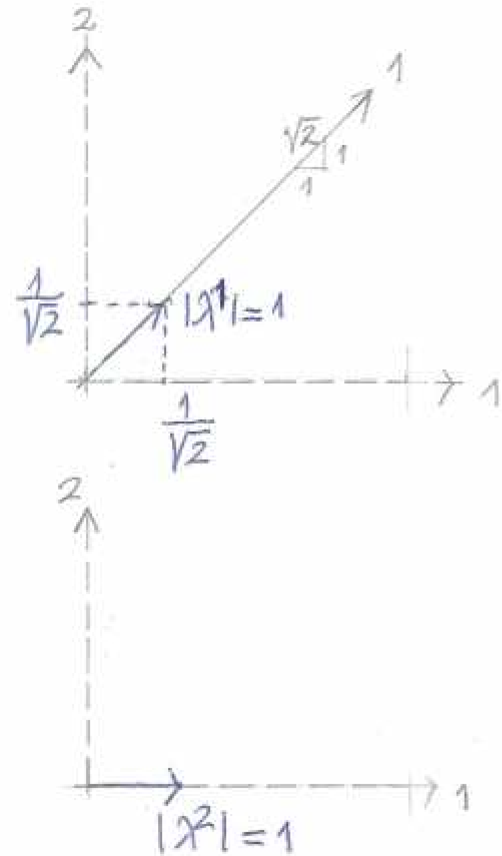
Menurut gbr-5 (a); $\theta^1 = \pi/4$; $\theta^2 = 0°$; maka

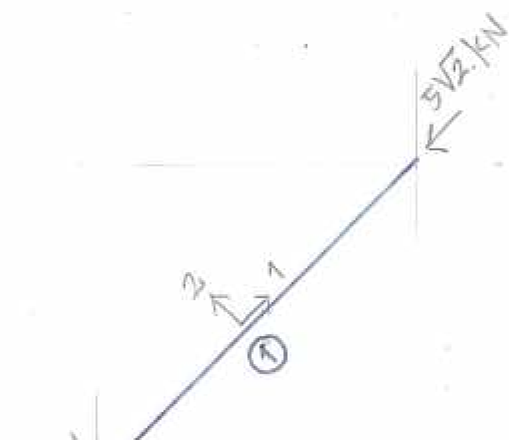
$$\lambda^1 = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \; ; \; \lambda^2 = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad \text{------} (12)$$




(a) Truss




(c) verifikasi. $\lambda^1$ & $\lambda^2$

jadi: $P_1 = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$ ; $P_2 = \begin{bmatrix} -5 \\ 0 \end{bmatrix}$

$P_3 = \begin{bmatrix} -5 \\ -5 \end{bmatrix} + \begin{bmatrix} 5 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \end{bmatrix}$

(b) Gaya² elemen.

Gambar-5 Perhitungan gaya² pada joint.

• SOAL. PR (1).

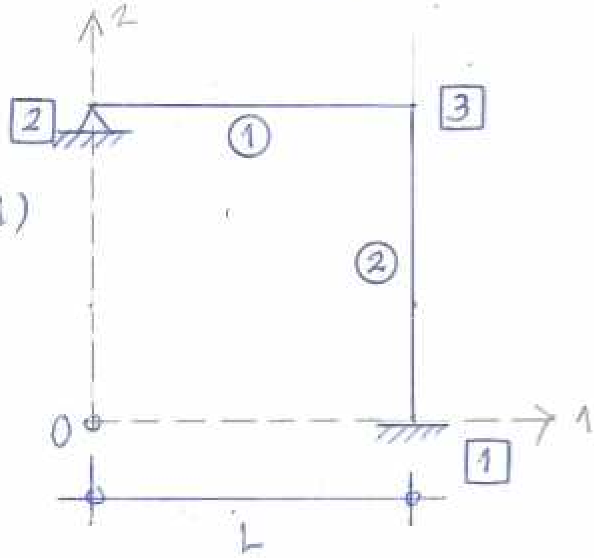
~~Perhat~~ gambar-1 berikut; pd. str.(a); (b); & (c)

Tentukan :

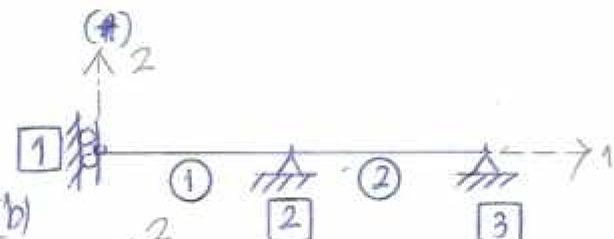
(1), DOF (Degree of Freedom) .
     (derajat kebebasan struktur).

(2). Susun matrik Compatibility. (BC =).
     (BC: Boundary Condition).

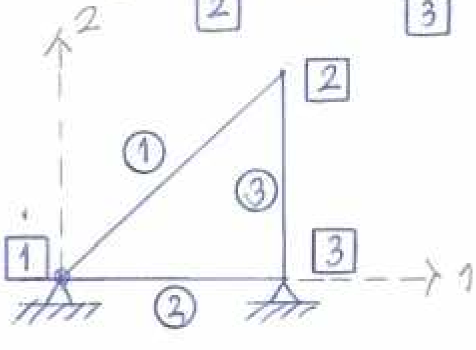
(a)




(b)




(c)



# Selesai.