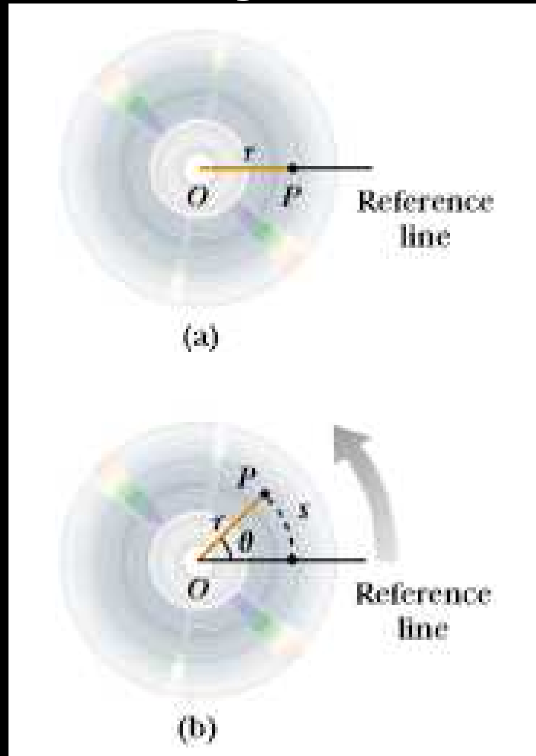
07 Rotasi

Angular Position, Velocity, and Acceleration

$$s = r\theta$$

$$\theta = \frac{s}{r}$$



one radian is the angle subtended by an arc length equal to the radius of the arc.

360° corresponds to an angle of $(2\pi r/r)$ rad $= 2\pi$ rad.

angular displacement

$$\Delta\theta = \theta_f - \theta_i$$

average angular speed

$$\bar{\omega} \equiv \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$

instantaneous angular speed

$$\omega \equiv \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

instantaneous angular acceleration

$$\bar{\alpha} \equiv \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

$$\alpha \equiv \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

Rotational Kinematics

Kinematic Equations for Rotational and Linear Motion Under Constant Acceleration

Rotational Motion About Fixed Axis	Linear Motion
$\omega_f = \omega_i + \alpha t$	$v_f = v_i + at$
$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$	$x_f = x_i + v_i t + \frac{1}{2}at^2$
$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$	$v_f^2 = v_i^2 + 2a(x_f - x_i)$
$\theta_f = \theta_i + \frac{1}{2}(\omega_i + \omega_f)t$	$x_f = x_i + \frac{1}{2}(v_i + v_f)t$

Angular and Linear Quantities

$$v = \frac{ds}{dt} = r\,\frac{d\theta}{dt}$$

$$v = r\omega$$

$$a_t = \frac{dv}{dt} = r\,\frac{d\omega}{dt}$$

$$a_t = r\alpha$$

Rotational Kinetic Energy

$$K_R = \sum_i K_i = \sum_i \frac{1}{2} m_i v_i^2 = \frac{1}{2} \sum_i m_i r_i^2 \omega^2$$

$$K_R = \frac{1}{2}\left(\sum_i m_i r_i^2\right)\omega^2$$

$$I = \sum_i m_i r_i^2$$

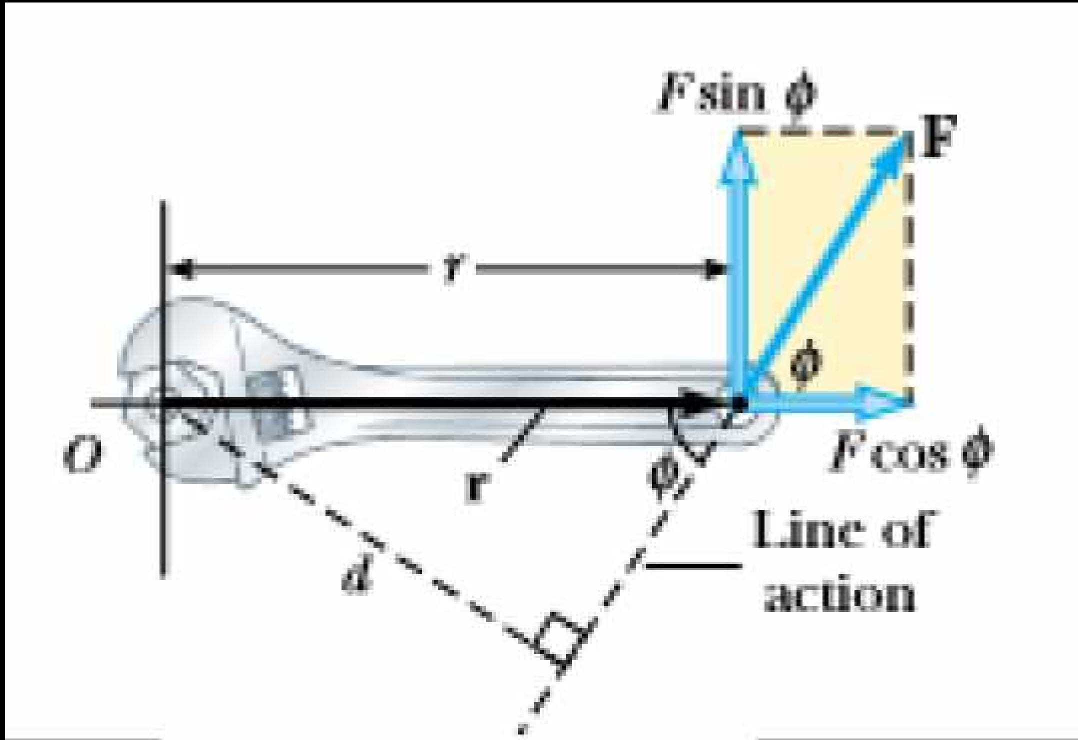
$$K_R = \frac{1}{2} I \omega^2$$

Moments of Inertia

$$I = \lim_{\Delta m_i \to 0} \sum_i r_i^2 \Delta m_i = \int r^2 \, dm$$

$$I = \int \rho r^2 \, dV$$

Torque



$$\tau = rF\sin\phi = Fd$$

Work, Power, and Energy in Rotational Motion

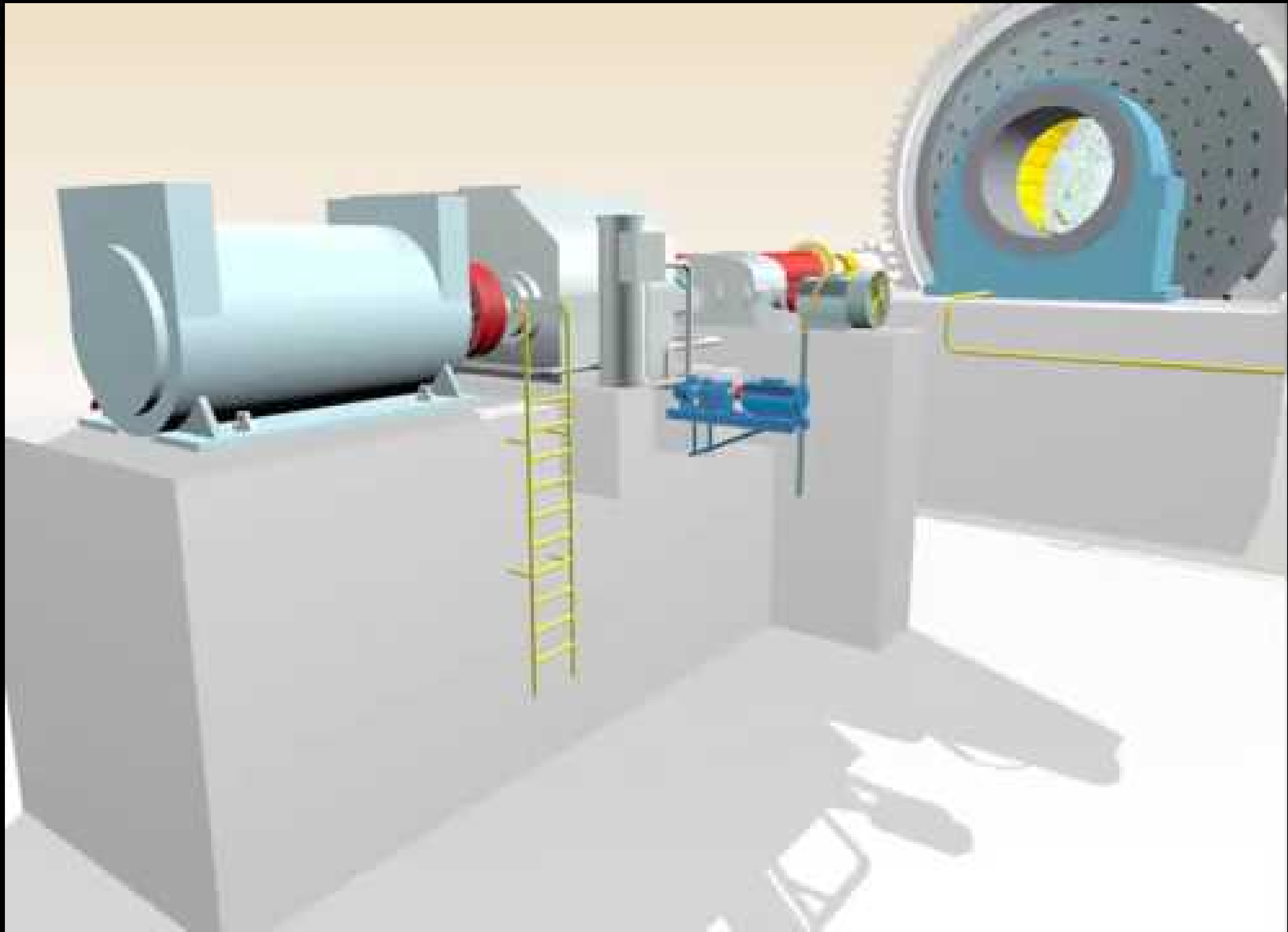
$$dW = \mathbf{F} \cdot d\mathbf{s} = (F \sin \phi)\, r\, d\theta$$

$$\mathcal{P} = \frac{dW}{dt} = \tau\omega$$

$$\sum W = \int_{\omega_i}^{\omega_f} I\omega\, d\omega = \tfrac{1}{2} I\omega_f^{\,2} - \tfrac{1}{2} I\omega_i^{\,2}$$

Useful Equations in Rotational and Linear Motion

Rotational Motion About a Fixed Axis	Linear Motion
Angular speed $\omega = d\theta/dt$	Linear speed $v = dx/dt$
Angular acceleration $\alpha = d\omega/dt$	Linear acceleration $a = dv/dt$
Net torque $\Sigma\tau = I\alpha$	Net force $\Sigma F = ma$
If $\alpha =$ constant $\begin{cases} \omega_f = \omega_i + \alpha t \\ \theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 \\ \omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i) \end{cases}$	If $a =$ constant $\begin{cases} v_f = v_i + at \\ x_f = x_i + v_i t + \frac{1}{2}at^2 \\ v_f^2 = v_i^2 + 2a(x_f - x_i) \end{cases}$
Work $W = \int_{\theta_i}^{\theta_f} \tau\, d\theta$	Work $W = \int_{x_i}^{x_f} F_x\, dx$
Rotational kinetic energy $K_R = \frac{1}{2}I\omega^2$	Kinetic energy $K = \frac{1}{2}mv^2$
Power $\mathcal{P} = \tau\omega$	Power $\mathcal{P} = Fv$
Angular momentum $L = I\omega$	Linear momentum $p = mv$
Net torque $\Sigma\tau = dL/dt$	Net force $\Sigma F = dp/dt$



A wheel rotates with a constant angular acceleration of 3.50 rad/s^2.

(A) If the angular speed of the wheel is 2.00 rad/s at ti = 0, through what angular displacement does the wheel rotate in 2.00 s?

(B) Through how many revolutions has the wheel turned during this time interval?

(C) What is the angular speed of the wheel at t = 2.00 s?

$$\Delta\theta = \theta_f - \theta_i = \omega_i t + \tfrac{1}{2}\alpha t^2$$
$$= (2.00\ \text{rad/s})(2.00\ \text{s}) + \tfrac{1}{2}(3.50\ \text{rad/s}^2)(2.00\ \text{s})^2$$
$$= \boxed{11.0\ \text{rad}} = (11.0\ \text{rad})(57.3^\circ/\text{rad}) = \boxed{630^\circ}$$

$$\Delta\theta = 630^\circ\left(\frac{1\ \text{rev}}{360^\circ}\right) = \boxed{1.75\ \text{rev}}$$

$$\omega_f = \omega_i + \alpha t = 2.00\ \text{rad/s} + (3.50\ \text{rad/s}^2)(2.00\ \text{s})$$
$$= \boxed{9.00\ \text{rad/s}}$$

Consider an oxygen molecule (O_2) rotating in the xy plane about the z axis. The rotation axis passes through the center of the molecule, perpendicular to its length. The mass of each oxygen atom is 2.66 x 10^{-26} kg, and at room temperature the average separation between the two atoms is d = 1.21 x 10^{-10} m. (The atoms are modeled as particles.)

(A) Calculate the moment of inertia of the molecule about the z axis.

(B) If the angular speed of the molecule about the z axis is 4.60 x 10^{12} rad/s, what is its rotational kinetic energy?

$$I = \sum_i m_i r_i^2 = m\left(\frac{d}{2}\right)^2 + m\left(\frac{d}{2}\right)^2 = \frac{md^2}{2}$$

$$= \frac{(2.66 \times 10^{-26}\ \text{kg})(1.21 \times 10^{-10}\ \text{m})^2}{2}$$

$$= \boxed{1.95 \times 10^{-46}\ \text{kg}\cdot\text{m}^2}$$

$$K_R = \tfrac{1}{2} I\omega^2$$

$$= \tfrac{1}{2}(1.95 \times 10^{-46}\ \text{kg}\cdot\text{m}^2)(4.60 \times 10^{12}\ \text{rad/s})^2$$

$$= \boxed{2.06 \times 10^{-21}\ \text{J}}$$

During a certain period of time, the angular position of a swinging door is described by $\theta = 5.00 + 10.0t + 2.00t^2$, where θ is in radians and t is in seconds. Determine the angular position, angular speed, and angular acceleration of the door (a) at t = 0 (b) at t = 3.00 s.

(a) $\theta\big|_{t=0} = \boxed{5.00\ \text{rad}}$

$$\omega\big|_{t=0} = \left.\frac{d\theta}{dt}\right|_{t=0} = 10.0 + 4.00t\big|_{t=0} = \boxed{10.0\ \text{rad/s}}$$

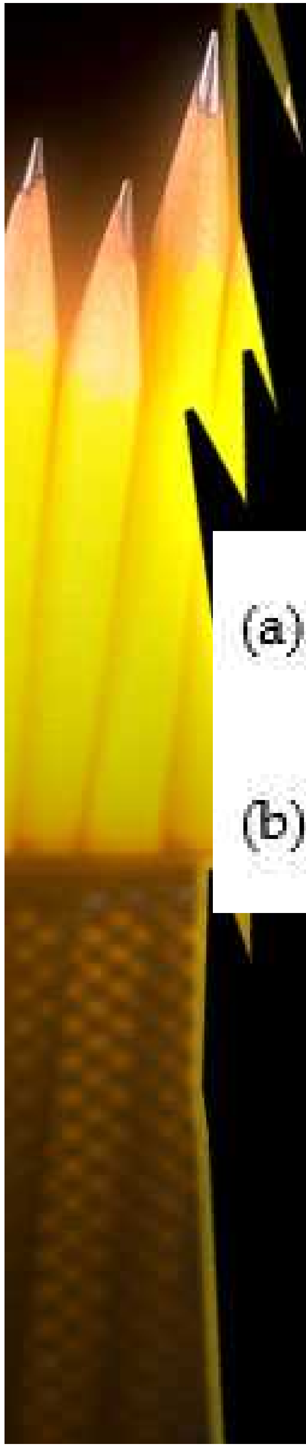
$$\alpha_{t=0} = \left.\frac{d\omega}{dt}\right|_{t=0} = \boxed{4.00\ \text{rad/s}^2}$$

(b) $\theta\big|_{t=3.00\ \text{s}} = 5.00 + 30.0 + 18.0 = \boxed{53.0\ \text{rad}}$

$$\omega\big|_{t=3.00\ \text{s}} = \left.\frac{d\theta}{dt}\right|_{t=3.00\ \text{s}} = 10.0 + 4.00t\big|_{t=3.00\ \text{s}} = \boxed{22.0\ \text{rad/s}}$$

$$\alpha\big|_{t=3.00\ \text{s}} = \left.\frac{d\omega}{dt}\right|_{t=3.00\ \text{s}} = \boxed{4.00\ \text{rad/s}^2}$$

A wheel starts from rest and rotates with constant angular acceleration to reach an angular speed of 12.0 rad/s in 3.00 s. Find (a) the magnitude of the angular acceleration of the wheel and (b) the angle in radians through which it rotates in this time.



(a) $$\alpha = \frac{\omega - \omega_i}{t} = \frac{12.0 \text{ rad/s}}{3.00 \text{ s}} = \boxed{4.00 \text{ rad/s}^2}$$

(b) $$\theta = \omega_i t + \frac{1}{2}\alpha t^2 = \frac{1}{2}\left(4.00 \text{ rad/s}^2\right)\left(3.00 \text{ s}\right)^2 = \boxed{18.0 \text{ rad}}$$

A centrifuge in a medical laboratory rotates at an angular speed of 3600 rev/min. When switched off, it rotates 50.0 times before coming to rest. Find the constant angular acceleration of the centrifuge.

$$\omega_i = 3\,600 \text{ rev/min} = 3.77 \times 10^2 \text{ rad/s}$$

$$\theta = 50.0 \text{ rev} = 3.14 \times 10^2 \text{ rad and } \omega_f = 0$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\theta$$

$$0 = \left(3.77 \times 10^2 \text{ rad/s}\right)^2 + 2\alpha\left(3.14 \times 10^2 \text{ rad}\right)$$

$$\alpha = \boxed{-2.26 \times 10^2 \text{ rad/s}^2}$$