Tables

Laplace Transform Operations

$F(s)$	$f(t)$
$c_1F_1(s) + c_2F_2(s)$	$c_1f_1(t) + c_2f_2(t)$
$F(as) \quad (a > 0)$	$\frac{1}{a} f\left(\frac{t}{a}\right)$
$F(s-a)$	$e^{at}f(t)$
$e^{-as}F(s) \quad (a \geq 0)$	$u_a(t)f(t-a)$
$s\,F(s) - f(0^+)$	$f'(t)$
$s^2F(s) - s\,f(0^+) - f'(0^+)$	$f''(t)$
$s^nF(s) - s^{n-1}f(0^+) - s^{n-2}f'(0^+) - \cdots - f^{(n-1)}(0^+)$	$f^{(n)}(t)$
$\frac{F(s)}{s}$	$\int_0^t f(\tau)\,d\tau$
$F'(s)$	$-t\,f(t)$
$F^{(n)}(s)$	$(-1)^n t^n f(t)$
$\int_s^\infty F(x)\,dx$	$\frac{1}{t}\,f(t)$
$F(s)\,G(s)$	$\int_0^t f(\tau)\,g(t-\tau)\,d\tau$
$\lim_{s\to\infty} s\,F(s)$	$\lim_{t\to 0^+} f(t) = f(0^+)$
$\lim_{s\to 0} s\,F(s)$	$\lim_{t\to\infty} f(t)$

Table of Laplace Transforms

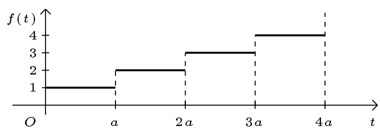
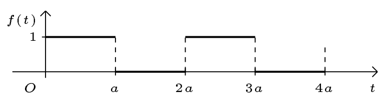
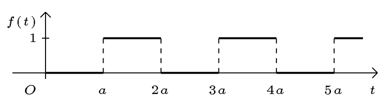
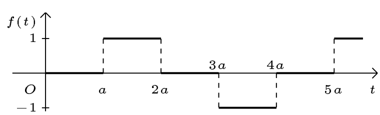
$F(s)$	$f(t)$
1	$\delta(t)$
$\frac{1}{s}$	1
$\frac{1}{s^2}$	t
$\frac{1}{s^n}$ $(n = 1, 2, 3, \ldots)$	$\frac{t^{n-1}}{(n-1)!}$
$\frac{1}{s^\nu}$ $(\nu > 0)$	$\frac{t^{\nu-1}}{\Gamma(\nu)}$
$\frac{(s-1)^n}{s^{n+1}}$ $(n = 0, 1, 2, \ldots)$	$L_n(t) = \frac{e^t}{n!}\frac{d^n}{dt^n}(t^n e^{-t})$ Laguerre polynomials
$\frac{1}{s-a}$	e^{at}
$\frac{1}{s(s-a)}$	$\frac{1}{a}(e^{at}-1)$
$\frac{1}{(s-a)(s-b)}$ $(a \neq b)$	$\frac{e^{at}-e^{bt}}{a-b}$
$\frac{s}{(s-a)(s-b)}$ $(a \neq b)$	$\frac{a\,e^{at}-b\,e^{bt}}{a-b}$
$\frac{s}{(s-a)^2}$	$(1+at)\,e^{at}$
$\frac{a}{s^2+a^2}$	$\sin at$
$\frac{s}{s^2+a^2}$	$\cos at$
$\frac{a}{(s-b)^2+a^2}$	$e^{bt}\sin at$
$\frac{s-b}{(s-b)^2+a^2}$	$e^{bt}\cos at$

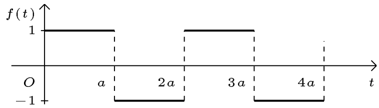
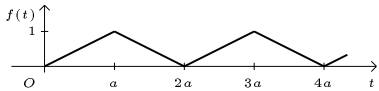
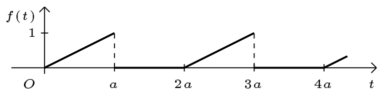
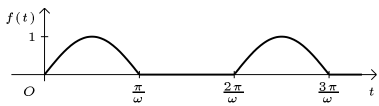
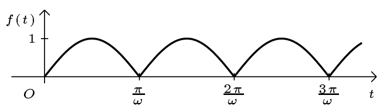
$F(s)$	$f(t)$
$\dfrac{a}{s^2 - a^2}$	$\sinh at$
$\dfrac{s}{s^2 - a^2}$	$\cosh at$
$\dfrac{a}{(s-b)^2 - a^2}$	$e^{bt} \sinh at$
$\dfrac{s-b}{(s-b)^2 - a^2}$	$e^{bt} \cosh at$
$\dfrac{1}{(s^2 + a^2)^2}$	$\dfrac{1}{2a^3}(\sin at - at\cos at)$
$\dfrac{s}{(s^2 + a^2)^2}$	$\dfrac{1}{2a}(t\sin at)$
$\dfrac{s^2}{(s^2 + a^2)^2}$	$\dfrac{1}{2a}(\sin at + at\cos at)$
$\dfrac{s^3}{(s^2 + a^2)^2}$	$\cos at - \frac{1}{2}\, at\sin at$
$\dfrac{s^2 - a^2}{(s^2 + a^2)^2}$	$t\cos at$
$\dfrac{1}{(s^2 - a^2)^2}$	$\dfrac{1}{2a^3}(at\cosh at - \sinh at)$
$\dfrac{s}{(s^2 - a^2)^2}$	$\dfrac{1}{2a}(t\sinh at)$

$F(s)$		$f(t)$
$\dfrac{s^2}{(s^2-a^2)^2}$		$\dfrac{1}{2a}(\sinh at + at\cosh at)$
$\dfrac{s^3}{(s^2-a^2)^2}$		$\cosh at + \frac{1}{2}\,at\sinh at$
$\dfrac{s^2+a^2}{(s^2-a^2)^2}$		$t\,\cosh at$
$\dfrac{ab}{(s^2+a^2)(s^2+b^2)}$	$(a^2 \neq b^2)$	$\dfrac{a\sin bt - b\sin at}{a^2-b^2}$
$\dfrac{s}{(s^2+a^2)(s^2+b^2)}$	$(a^2 \neq b^2)$	$\dfrac{\cos bt - \cos at}{a^2-b^2}$
$\dfrac{s^2}{(s^2+a^2)(s^2+b^2)}$	$(a^2 \neq b^2)$	$\dfrac{a\sin at - b\sin bt}{a^2-b^2}$
$\dfrac{s^3}{(s^2+a^2)(s^2+b^2)}$	$(a^2 \neq b^2)$	$\dfrac{a^2\cos at - b^2\cos bt}{a^2-b^2}$
$\dfrac{ab}{(s^2-a^2)(s^2-b^2)}$	$(a^2 \neq b^2)$	$\dfrac{b\sinh at - a\sinh bt}{a^2-b^2}$
$\dfrac{s}{(s^2-a^2)(s^2-b^2)}$	$(a^2 \neq b^2)$	$\dfrac{\cosh at - \cosh bt}{a^2-b^2}$
$\dfrac{s^2}{(s^2-a^2)(s^2-b^2)}$	$(a^2 \neq b^2)$	$\dfrac{a\sinh at - b\sinh bt}{a^2-b^2}$
$\dfrac{s^3}{(s^2-a^2)(s^2-b^2)}$	$(a^2 \neq b^2)$	$\dfrac{a^2\cosh at - b^2\cosh bt}{a^2-b^2}$

$F(s)$	$f(t)$
$\dfrac{a^2}{s^2(s^2+a^2)}$	$t - \dfrac{1}{a}\sin at$
$\dfrac{a^2}{s^2(s^2-a^2)}$	$\dfrac{1}{a}\sinh at - t$
$\dfrac{1}{\sqrt{s}}$	$\dfrac{1}{\sqrt{\pi t}}$
$\dfrac{1}{\sqrt{s+a}}$	$\dfrac{e^{-at}}{\sqrt{\pi t}}$
$\dfrac{1}{s\sqrt{s+a}}$	$\dfrac{1}{\sqrt{a}}\operatorname{erf}(\sqrt{at})$
$\dfrac{1}{\sqrt{s+a}+\sqrt{s+b}}$	$\dfrac{e^{-bt}-e^{-at}}{2(b-a)\sqrt{\pi t^3}}$
$\dfrac{1}{s\sqrt{s}}$	$2\sqrt{\dfrac{t}{\pi}}$
$\dfrac{1}{(s-a)\sqrt{s}}$	$\dfrac{1}{\sqrt{a}}e^{at}\operatorname{erf}\sqrt{at}$
$\dfrac{1}{\sqrt{s-a}+b}$	$e^{at}\left(\dfrac{1}{\sqrt{\pi t}} - b\,e^{b^2 t}\operatorname{erfc}(b\sqrt{t})\right)$
$\dfrac{1}{\sqrt{s^2+a^2}}$	$J_0(at)$
$\dfrac{1}{\sqrt{s^2-a^2}}$	$I_0(at)$
$\dfrac{(\sqrt{s^2+a^2}-s)^\nu}{\sqrt{s^2+a^2}} \quad (\nu > -1)$	$a^\nu J_\nu(at)$
$\dfrac{(s-\sqrt{s^2-a^2})^\nu}{\sqrt{s^2-a^2}} \quad (\nu > -1)$	$a^\nu I_\nu(at)$
$\dfrac{1}{(s^2+a^2)^\nu} \quad (\nu > 0)$	$\dfrac{\sqrt{\pi}}{\Gamma(\nu)}\left(\dfrac{t}{2a}\right)^{\nu-\frac{1}{2}} J_{\nu-\frac{1}{2}}(at)$
$\dfrac{1}{(s^2-a^2)^\nu} \quad (\nu > 0)$	$\dfrac{\sqrt{\pi}}{\Gamma(\nu)}\left(\dfrac{t}{2a}\right)^{\nu-\frac{1}{2}} I_{\nu-\frac{1}{2}}(at)$

$F(s)$	$f(t)$
$(\sqrt{s^2+a^2}-s)^\nu \quad (\nu > 0)$	$\frac{\nu a^\nu}{t} J_\nu(at)$
$(s-\sqrt{s^2-a^2})^\nu \quad (\nu > 0)$	$\frac{\nu a^\nu}{t} I_\nu(at)$
$\sqrt{s-a}-\sqrt{s-b}$	$\frac{1}{2t\sqrt{\pi t}}(e^{bt}-e^{at})$
$\frac{e^{-a/s}}{\sqrt{s}}$	$\frac{\cos 2\sqrt{at}}{\sqrt{\pi t}}$
$\frac{e^{-a/s}}{s\sqrt{s}}$	$\frac{\sin 2\sqrt{at}}{\sqrt{\pi a}}$
$\frac{e^{-a/s}}{s^{\nu+1}} \quad (\nu > -1)$	$\left(\frac{t}{a}\right)^{\nu/2} J_\nu(2\sqrt{at})$
$\frac{e^{-a\sqrt{s}}}{\sqrt{s}} \quad (a > 0)$	$\frac{e^{-a^2/4t}}{\sqrt{\pi t}}$
$e^{-a\sqrt{s}} \quad (a > 0)$	$\frac{a}{2\sqrt{\pi t^3}} e^{-a^2/4t}$
$\frac{e^{-a\sqrt{s}}}{s} \quad (a > 0)$	$\operatorname{erfc}\left(\frac{a}{2\sqrt{t}}\right)$
$\frac{e^{-k\sqrt{s^2+a^2}}}{\sqrt{s^2+a^2}}$	$\begin{cases} 0 & 0 < t < k \\ J_0(a\sqrt{t^2-k^2}) & t > k \end{cases}$
$\frac{e^{-k\sqrt{s^2-a^2}}}{\sqrt{s^2-a^2}}$	$\begin{cases} 0 & 0 < t < k \\ I_0(a\sqrt{t^2-k^2}) & t > k \end{cases}$

$F(s)$	$f(t)$
$e^{-as} \quad (a > 0)$	$\delta_a(t)$
$\dfrac{e^{-as}}{s} \quad (a > 0)$	$u_a(t)$
$e^{s^2/4}\,\mathrm{erfc}\dfrac{s}{2}$	$\dfrac{2}{\sqrt{\pi}}\, e^{-t^2}$
$\dfrac{1}{s(e^{as}-1)} = \dfrac{e^{-as}}{s(1-e^{-as})}$	$\left[\dfrac{t}{a}\right]$ ($[t]$: greatest integer $\leq t$)
$\dfrac{1}{s(e^{s}-a)} = \dfrac{e^{-s}}{s(1-ae^{-s})} \quad (a \neq 1)$	$\dfrac{a^{[t]}-1}{a-1}$
$\dfrac{e^{s}-1}{s(e^{s}-a)} = \dfrac{1-e^{-s}}{s(1-ae^{-s})}$	$a^{[t]}$
$\dfrac{1}{s(1-e^{-as})}$	
$\dfrac{1}{s(1+e^{-as})}$	
$\dfrac{1}{s(1+e^{as})}$	
$\dfrac{1-e^{-as}}{s(e^{as}+e^{-as})}$	

$F(s)$	$f(t)$
$\dfrac{(1-e^{-as})}{s(1+e^{-as})} = \dfrac{1}{s}\tanh\dfrac{as}{2}$	
$\dfrac{1-e^{-as}}{as^2(1+e^{-as})} = \dfrac{1}{as^2}\tanh\dfrac{as}{2}$	
$\dfrac{1-(1+as)e^{-as}}{as^2(1-e^{-2as})}$	
$\dfrac{\omega}{(s^2+\omega^2)(1-e^{-\pi s/\omega})}$	
$\dfrac{\omega}{s^2+\omega^2}\left(\dfrac{1+e^{-\pi s/\omega}}{1-e^{-\pi s/\omega}}\right) = \dfrac{\omega}{s^2+\omega^2}\coth\dfrac{\pi s}{2\omega}$	
$\log\left(\dfrac{s+a}{s+b}\right)$	$\dfrac{e^{-bt}-e^{-at}}{t}$
$\dfrac{-(\log s+\gamma)}{s}$ (γ: Euler constant)	$\log t$

$F(s)$	$f(t)$
$\dfrac{\log s}{s}$	$-(\log t + \gamma)$
$\log\left(\dfrac{s^2+a^2}{s^2+b^2}\right)$	$\dfrac{2}{t}(\cos bt - \cos at)$
$\tan^{-1}\left(\dfrac{a}{s}\right)$	$\dfrac{1}{t}\sin at$
$\dfrac{\sinh xs}{s\sinh as}$	$\dfrac{x}{a} + \dfrac{2}{\pi}\sum_{n=1}^{\infty}\dfrac{(-1)^n}{n}\sin\dfrac{n\pi x}{a}\cos\dfrac{n\pi t}{a}$
$\dfrac{\sinh xs}{s\cosh as}$	$\dfrac{4}{\pi}\sum_{n=1}^{\infty}\dfrac{(-1)^{n-1}}{2n-1}\sin\left(\dfrac{2n-1}{2a}\right)\pi x\sin\left(\dfrac{2n-1}{2a}\right)\pi t$
$\dfrac{\cosh xs}{s\sinh as}$	$\dfrac{t}{a} + \dfrac{2}{\pi}\sum_{n=1}^{\infty}\dfrac{(-1)^n}{n}\cos\dfrac{n\pi x}{a}\sin\dfrac{n\pi t}{a}$
$\dfrac{\cosh xs}{s\cosh as}$	$1 + \dfrac{4}{\pi}\sum_{n=1}^{\infty}\dfrac{(-1)^n}{2n-1}\cos\left(\dfrac{2n-1}{2a}\right)\pi x\cos\left(\dfrac{2n-1}{2a}\right)\pi t$
$\dfrac{\sinh xs}{s^2\sinh as}$	$\dfrac{xt}{a} + \dfrac{2a}{\pi^2}\sum_{n=1}^{\infty}\dfrac{(-1)^n}{n^2}\sin\dfrac{n\pi x}{a}\sin\dfrac{n\pi t}{a}$
$\dfrac{\sinh xs}{s^2\cosh as}$	$x + \dfrac{8a}{\pi^2}\sum_{n=1}^{\infty}\dfrac{(-1)^n}{(2n-1)^2}\sin\left(\dfrac{2n-1}{2a}\right)\pi x\cos\left(\dfrac{2n-1}{2a}\right)\pi t$
$\dfrac{\cosh xs}{s^2\sinh as}$	$\dfrac{1}{2a}\left(x^2 + t^2 - \dfrac{a^2}{3}\right) - \dfrac{2a}{\pi^2}\sum_{n=1}^{\infty}\dfrac{(-1)^n}{n^2}\cos\dfrac{n\pi x}{a}\cos\dfrac{n\pi t}{a}$
$\dfrac{\cosh xs}{s^2\cosh as}$	$t + \dfrac{8a}{\pi^2}\sum_{n=1}^{\infty}\dfrac{(-1)^n}{(2n-1)^2}\cos\left(\dfrac{2n-1}{2a}\right)\pi x\sin\left(\dfrac{2n-1}{2a}\right)\pi t$
$\dfrac{\sinh x\sqrt{s}}{\sinh a\sqrt{s}}$	$\dfrac{2\pi}{a^2}\sum_{n=1}^{\infty}(-1)^n n e^{-n^2\pi^2 t/a^2}\sin\dfrac{n\pi x}{a}$

$F(s)$	$f(t)$
$\dfrac{\cosh x\sqrt{s}}{\cosh a\sqrt{s}}$	$\dfrac{\pi}{a^2}\sum_{n=1}^{\infty}(-1)^{n-1}(2n-1)e^{-(2n-1)^2\pi^2 t/4a^2}\cos\left(\dfrac{2n-1}{2a}\right)\pi x$
$\dfrac{\sinh x\sqrt{s}}{\sqrt{s}\cosh a\sqrt{s}}$	$\dfrac{2}{a}\sum_{n=1}^{\infty}(-1)^{n-1}e^{-(2n-1)^2\pi^2 t/4a^2}\sin\left(\dfrac{2n-1}{2a}\right)\pi x$
$\dfrac{\cosh x\sqrt{s}}{\sqrt{s}\sinh a\sqrt{s}}$	$\dfrac{1}{a}+\dfrac{2}{a}\sum_{n=1}^{\infty}(-1)^{n}e^{-n^2\pi^2 t/a^2}\cos\dfrac{n\pi x}{a}$
$\dfrac{\sinh x\sqrt{s}}{s\sinh a\sqrt{s}}$	$\dfrac{x}{a}+\dfrac{2}{\pi}\sum_{n=1}^{\infty}\dfrac{(-1)^n}{n}e^{-n^2\pi^2 t/a^2}\sin\dfrac{n\pi x}{a}$
$\dfrac{\cosh x\sqrt{s}}{s\cosh a\sqrt{s}}$	$1+\dfrac{4}{\pi}\sum_{n=1}^{\infty}\dfrac{(-1)^n}{2n-1}e^{-(2n-1)^2\pi^2 t/4a^2}\cos\left(\dfrac{2n-1}{2a}\right)\pi x$
$\dfrac{\sinh x\sqrt{s}}{s^2\sinh a\sqrt{s}}$	$\dfrac{xt}{a}+\dfrac{2a^2}{\pi^3}\sum_{n=1}^{\infty}\dfrac{(-1)^n}{n^3}(1-e^{-n^2\pi^2 t/a^2})\sin\dfrac{n\pi x}{a}$
$\dfrac{\cosh x\sqrt{s}}{s^2\cosh a\sqrt{s}}$	$\dfrac{x^2-a^2}{2}+t-\dfrac{16a^2}{\pi^3}\sum_{n=1}^{\infty}\dfrac{(-1)^n}{(2n-1)^3}e^{-(2n-1)^2\pi^2 t/4a^2}\cos\left(\dfrac{2n-1}{2a}\right)\pi x$