Module 7: Permanent Magnet Machines for Hybrid and Electric Vehicles

Lecture 22: Permanent Magnet Motors

Permanent Magnet Motors

Introduction

The topics covered in this chapter are as follows:

- Permanent Magnet (PM) Machines
- Principle of Operation of PM Machine
- Operation of PM Machine Supplied by DC-AC Converter with 120^{o} Mode of Operation
- Operation of PM Machine Supplied by DC-AC Converter with 180^{o} Mode of Operation

Permanent Magnet (PM) Machines

By using high energy magnets such as rare earth based magnets, a PM machine drive can be designed with high power density, high speed and high operation efficiency. These advantages are attractive for their application in EVs and HEVs. The major advantages of PM machines are:

- ***High efficiency***: The PM machines have a very high efficiency due to the use of PMs for excitation which consume no power. Moreover, the absence of mechanical commutators and brushes results in low mechanical friction losses.
- ***High Power density***: The use of high energy density magnets has allowed achieving very high flux densities in the PM machines. As a result of high flux densities, high torque can be produced from a given volume of motor compared to other motors of same volume.
- ***Ease of Control***: THE PM motors can be controlled as easily as DC motors because the control variables are easily accessible and constant throughout the operation of the motor.

However, the PM machines also suffer from some disadvantages such as:

- ***Cost***: Rare-earth magnets commonly used in PM machines are very expensive.
- ***Magnet Demagnetization***: The magnets can be demagnetized by large opposing magnetomotive force and high temperatures.
- ***Inverter Failure***: Due to magnets on the rotor, PM motors present major risks in the case of short circuit failures of the inverters. The rotor is always energized and constantly induces EMF in the short circuited windings. A very large current circulates in those windings and an accordingly large torque tends to block the rotor. The dangers of blocking one or several wheels of a vehicle are non-negligible.

Based on the shape of the back e.m.f induced in the stator windings, the PM motors can be classified into two types:

- Permanent Magnet Synchronous Machine with sinusoidal back e.m.f (**Figure 1a**)
- Brushless Permanent Magnet DC Machines (BLDC) with trapezoidal back e.m.f (**Figure 1b**)

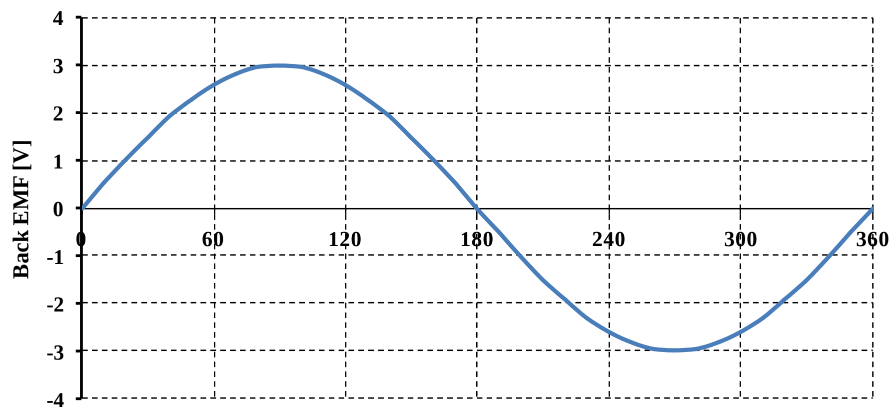

Figure 1a: Sinusoidal back emf

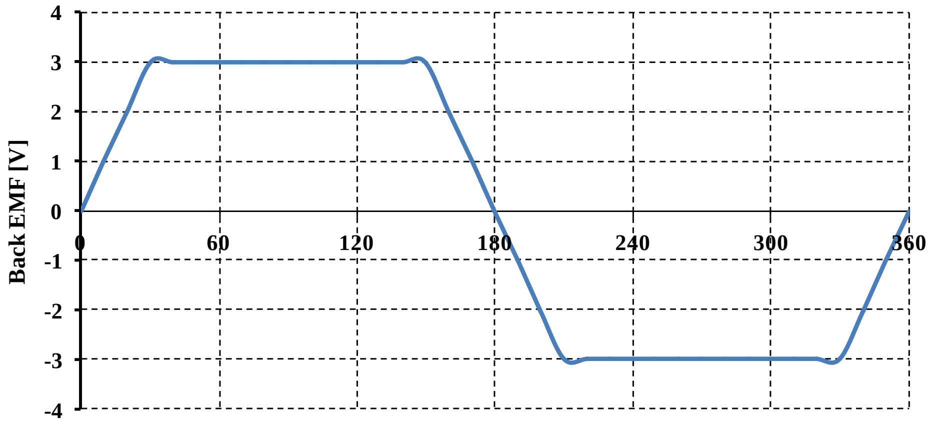

Figure 1b: Trapezoidal back emf

Based on the construction of the rotor, the PM machines can be broadly classified into three categories:

- Inner rotor machine (**Figure 2a**)
- Outer rotor machine (**Figure 2b**)
- Interior magnet rotor (**Figure 2c**)

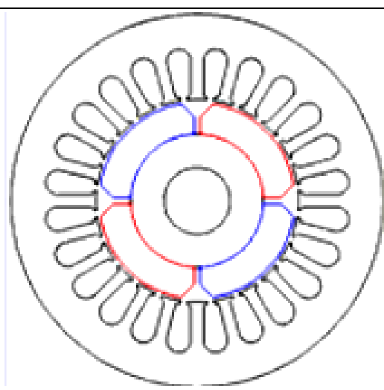

Figure 2a: Inner rotor machine

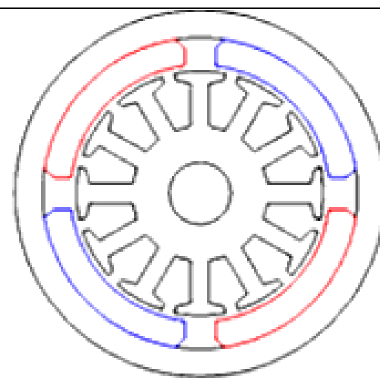

Figure 2b: Outer rotor machine

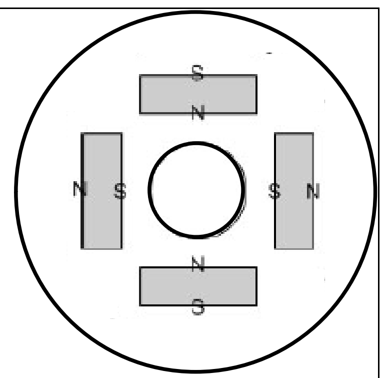

Figure 2c: Interior magnet rotor machine

Principle of Operation of PM Machine

To produce torque, in general, a rotor flux and a stator mmf has to be present that are stationary with respect to each other but having a nonzero phase shift between them. In PM machines, the necessary rotor flux is present due to rotor PMs. Currents in the stator windings generate the stator mmf. The zero relative speed between the stator mmf and the rotor flux is achieved if the stator mmf is revolving at the same speed as the rotor flux , that is, rotor speed and also in the same direction. The revolving stator mmf is the result of injecting a set of polyphase currents phase shifted from each other by the same amount of phase shift between the polyphase windings. For example, a three phase machine with three windings shifted in space by electrical 120^o between them produces a rotating magnetic field constant in magnitude and travelling at an angular frequency of the currents (just as in case of Induction machines). The rotor has permanent magnets on it, hence the flux produced by the rotor magnets start to chase the stator mmf and as a result torque is produced. Since the relative speed between the stator mmf and rotor flux has to be zero, the rotor moves at the same speed as the speed of the stator mmf. Hence, the PM machines are inherently synchronous machines.

As the coils in the stator experience a change of flux linkages caused by the moving magnets, there is an induced e.m.f in the windings. The shape of the induced e.m.f is very dependent on the shape of the flux linkage. If the rotational electrical speed of the machine ω_r and the air gap flux is sinusoidal then it can be expressed as (**Figure 3**)

$$\phi = \phi_m \sin(\omega_r t)$$

$$\omega_r = \frac{N_p}{2}\omega_{mech}$$

where

ϕ_m is the peak flux produced (1)

ω_r electrical speed of rotation of the rotor

ω_{mech} is the mechanical speed of the rotor

N_p is the number of poles of the motor

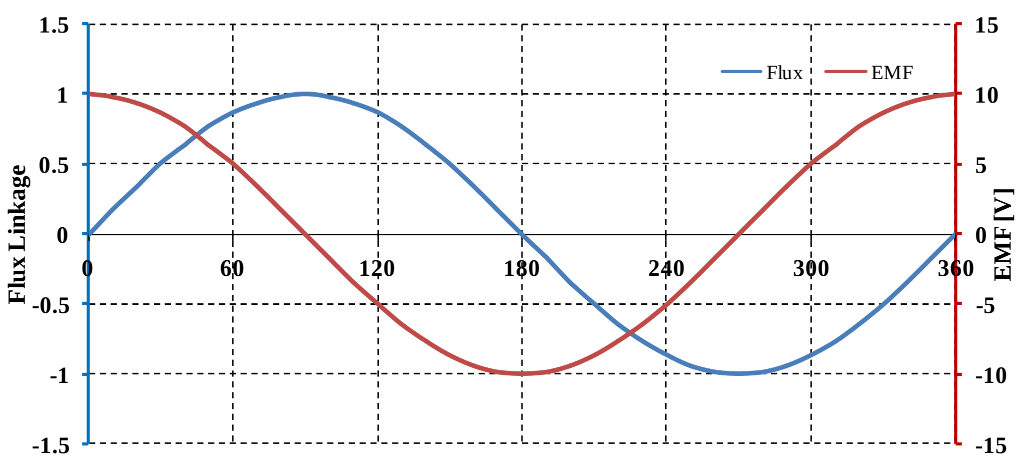

Figure 3: The flux linkage and back emf.

Given the number of turns (N_{turns}), then the flux linkages (λ) are equal to the product $N_{turns}\phi$. The induced emf is equal to the rate of flux linkages and is given by (**Figure 3**):

$$e = -\frac{d\lambda}{dt} = -N_{turns}\phi_m\omega_r\cos(\omega_r t) = -\lambda_m\omega_r\cos(\omega_r t) = -\lambda_m\omega_r\cos(\theta_r)$$

where (2)

$$\lambda_m = N_{turns}\phi_m$$
$$\theta_r = \omega_r t$$

The –ve sign in **equation 2** indicates that the induced e.m.f opposes the applied voltage. Some observations based on **equation 2** are:

- The emf is proportional to the product of the rotational frequency and air gap for a constant number of turns.
- Assuming that air gap flux is constant, it can be seen that the e.m.f is influenced only by the rotational speed of rotor ω_r which is same as the stator current frequency (because the PM machines are synchronous speed)
- By changing the frequency of stator current, the speed of the motor can be changed and a speed control of the motor can be achieved. However, beyond a certain speed known as base speed, an increase in stator frequency will result in voltage demand exceeding the supply capability. During that operation, keeping the voltage constant and increasing the excitation frequency reduces the airgap flux and thus allowing the excitation frequency reduces the air gap flux, thus allowing going to higher speed over and above the base speed. This operation is known as ***flux weakening***.

The PM machines are fed by DC-AC converter. By changing the frequency at which the gates are turned on, the frequency of the output wave can be varied. In the next sections the operation of a three phase PM machines with 120^o and 180^o conduction modes are explained. The following assumptions are made in the following analysis:

- The phases of the machines are **Y** connected.
- The current entering the neutral point (n) is considered to be positive and leaving it is considered to be negative.
- The back e.m.f induced in the phases is sinusoidal.
- All the phases of the machine are balanced, that is, the inductances and resistances of the phases are equal.

Operation of PM Machine Supplied by DC-AC Converter with 120^o Mode of Operation

The equivalent circuit of the PM machine motor is shown in **Figure 4**. When the motor is operated in 120^o then at given point of time only two switches conduct and six switching combinations are possible. The gating signals are shown in **Figure 5**. At angle $\theta = \frac{\pi}{3}$ the gate S_6 turns off and the gate S_2 is turned ***on***. Hence, phase **B** is the ***outgoing phase*** and **C** is the ***incoming phase***. Since the machine windings have inductance, the current through the phase **B** (i_b) cannot become zero instantaneously. Thus, the current through the phase **B** continues to flow through the freewheeling diode D_3 if $i_b < 0$ or D_6 if $i_b > 0$.

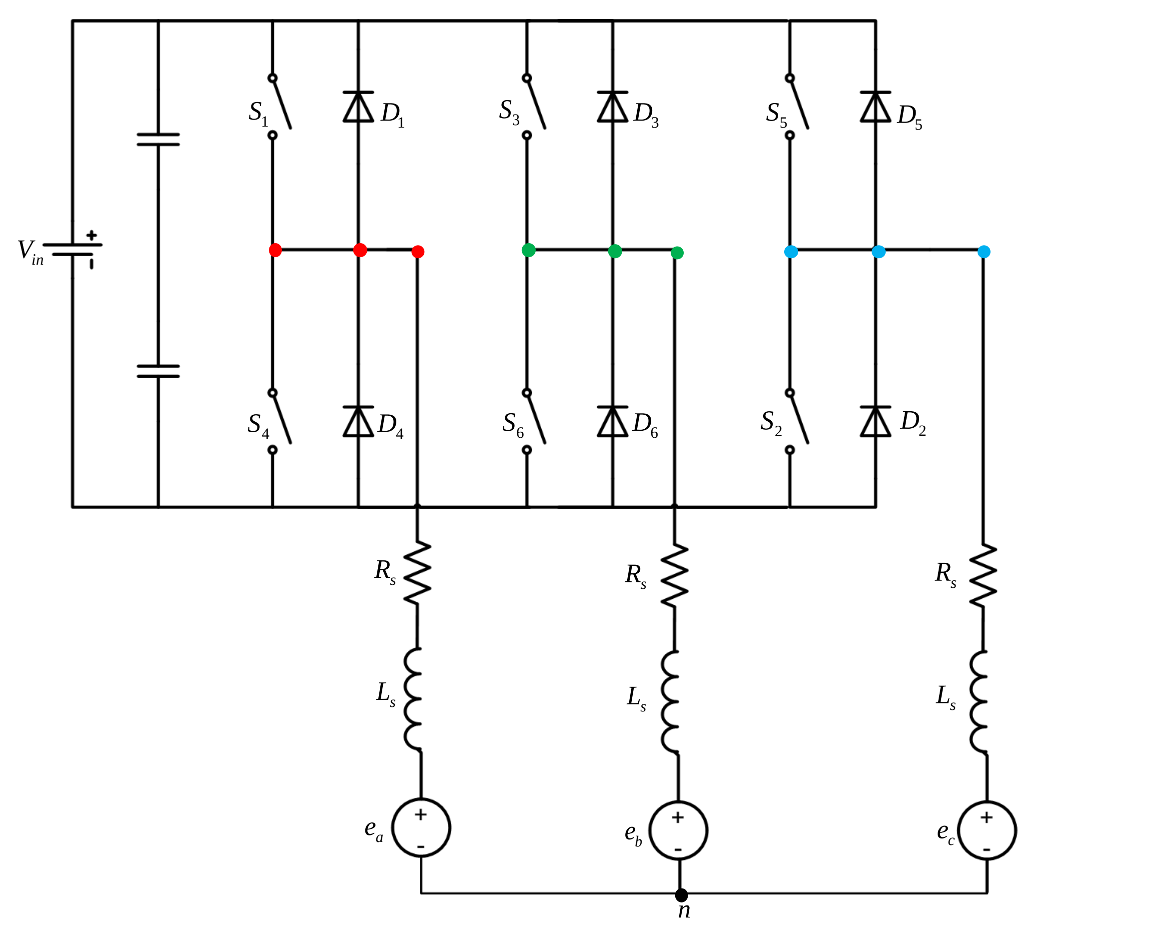

Figure 4: Equivalent circuit of PM machine

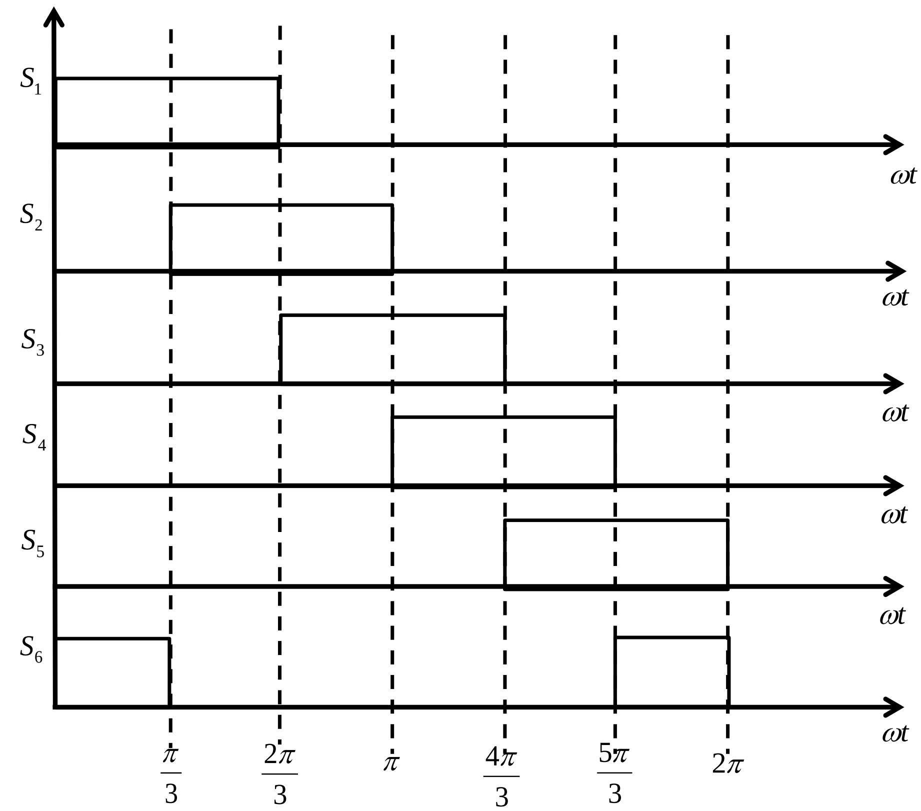

Figure 5: Gating sequence for 120° mode of conduction

As a result of non zero value of i_b, all the three phases will conduct and once $i_b = 0$, then only phases **A** and **C** will conduct. The equivalent circuits for all the three conditions are given in **Figures 6**. In case of $i_b < 0$ (**Figure 6a**) the system equations are

$$V_{an} = R_s i_a + L_s \frac{di_a}{dt} + e_a \tag{3a}$$

$$V_{bn} = R_s i_b + L_s \frac{di_b}{dt} + e_b \tag{3b}$$

$$V_{cn} = R_s i_c + L_s \frac{di_c}{dt} + e_c \tag{3c}$$

Since the phases are connected in star the following condition holds true

$$i_a + i_b + i_c = 0 \tag{3d}$$

The three induced e.m.fs are

$$e_a = \lambda_m \omega_r \cos(\omega_r t); e_b = \lambda_m \omega_r \cos\left(\omega_r t - \frac{2\pi}{3}\right); e_c = \lambda_m \omega_r \cos\left(\omega_r t + \frac{2\pi}{3}\right) \tag{3e}$$

Since the e.m.fs are sinusoidal, the following condition holds true

$$e_a + e_b + e_c = 0 \tag{3f}$$

Using **equations 3a** to **3f**, the following set of equations are obtained

$$\begin{aligned} \frac{di_a}{dt} &= -\frac{R_s}{L_s} i_a - \frac{1}{L_s} e_a + \frac{1}{L_s}\frac{V_{in}}{3} \\ \frac{di_b}{dt} &= -\frac{R_s}{L_s} i_b - \frac{1}{L_s} e_b + \frac{1}{L_s}\frac{V_{in}}{3} \\ \frac{di_c}{dt} &= -\frac{R_s}{L_s} i_c - \frac{1}{L_s} e_c - \frac{1}{L_s}\frac{2V_{in}}{3} \end{aligned} \tag{4a}$$

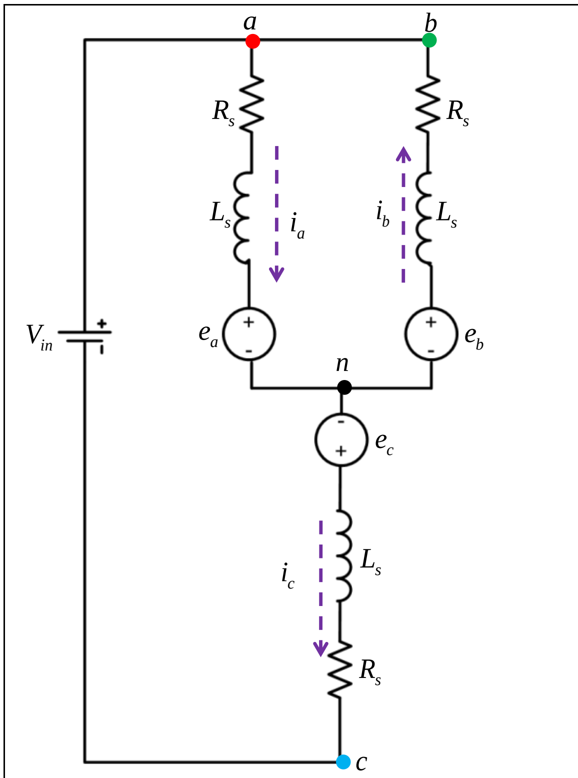

Figure 6a: Equivalent circuit when $i_b < 0$

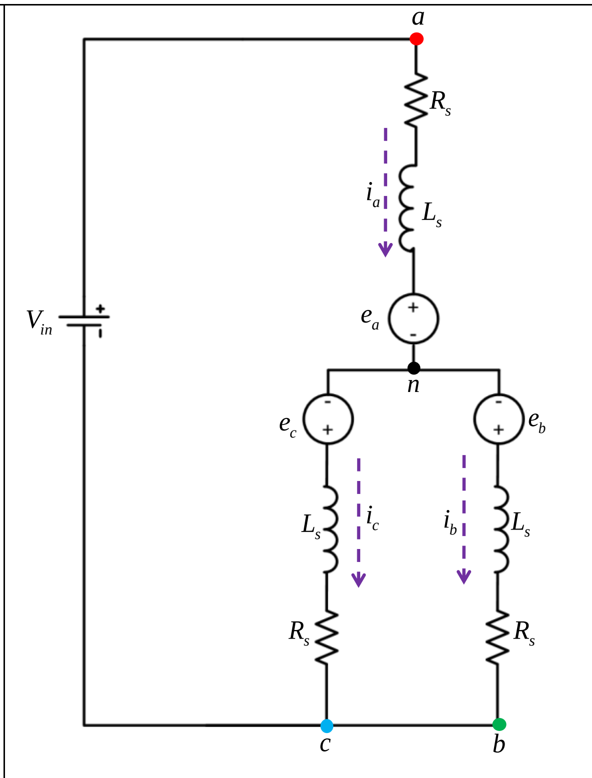

Figure 6b: Equivalent circuit when $i_b > 0$

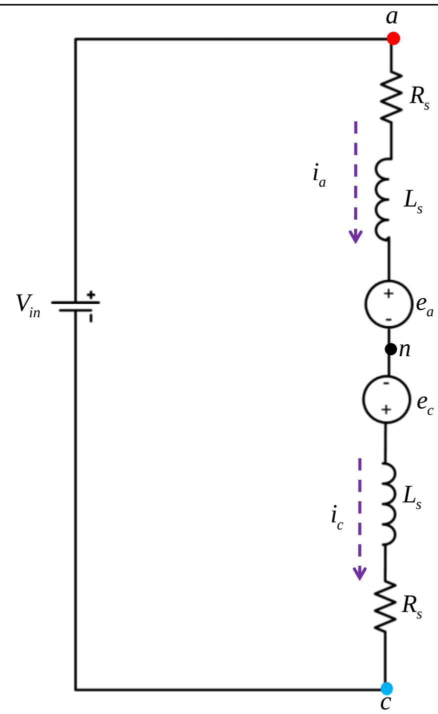

Figure 6c: Equivalent circuit when $i_b = 0$

Equation 4a can be written in matrix form as

$$p\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} -\frac{R_s}{L_s} & 0 & 0 \\ 0 & -\frac{R_s}{L_s} & 0 \\ 0 & 0 & -\frac{R_s}{L_s} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} \frac{1}{L_s} & 0 & 0 \\ 0 & \frac{1}{L_s} & 0 \\ 0 & 0 & \frac{1}{L_s} \end{bmatrix} \begin{bmatrix} V_{an} - e_a \\ V_{bn} - e_b \\ V_{cn} - e_c \end{bmatrix} \tag{4b}$$

where

$$\begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = \begin{bmatrix} \frac{V_{in}}{3} \\ \frac{V_{in}}{3} \\ -\frac{2V_{in}}{3} \end{bmatrix}; \; p = \frac{d}{dt}$$

In case of $i_b > 0$ (**Figure 6b**) using the **equations 3a** to **3f**, the following set of equations can be obtained

$$\frac{di_a}{dt} = -\frac{R_s}{L_s} i_a - \frac{1}{L_s} e_a + \frac{1}{L_s} \frac{2V_{in}}{3}$$
$$\frac{di_b}{dt} = -\frac{R_s}{L_s} i_b - \frac{1}{L_s} e_b - \frac{1}{L_s} \frac{V_{in}}{3} \qquad (4c)$$
$$\frac{di_c}{dt} = -\frac{R_s}{L_s} i_c - \frac{1}{L_s} e_c - \frac{1}{L_s} \frac{V_{in}}{3}$$

The matrix form of **equation 4c** is same as in **equation 4b** with change only is applied voltage. The applied voltage in this case is

$$p\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = \begin{bmatrix} \frac{2V_{in}}{3} \\ -\frac{V_{in}}{3} \\ -\frac{V_{in}}{3} \end{bmatrix} \qquad (4d)$$

When $i_b = 0$, using the **equations 3a** to **3f** (**Figure 6c**), the following set of equations can be obtained

$$\frac{di_a}{dt} = -\frac{R_s}{L_s} i_a - \frac{1}{2L_s} e_a + \frac{1}{L_s} \frac{V_{in}}{2}$$
$$\frac{di_c}{dt} = -\frac{R_s}{L_s} i_c - \frac{1}{2L_s} e_c - \frac{1}{L_s} \frac{V_{in}}{2} \qquad (4e)$$

The duration when all the three phases conduct is known as the ***commutation period*** (**equations 4b** and **4d**) and the duration when only two phases conduction is known as ***conduction period***. At the end of commutation period $0 \le t \le t_c$, the outgoing phase current becomes zero. Hence, the following conditions are obtained:

$$i_b(t_c) = 0;\ i_a(0) = I_o;\ i_b(0) = -I_o;\ i_c(0) = 0 \qquad (5)$$

By utilizing the systems of **equations 4a** to **4e** and the conditions given in **equation 5** the solution for the current can be obtained as discussed in Lecture 22.

Operation of PM Machine Supplied by DC-AC Converter with 180^{o} Mode of Operation

In this mode of operation at any point of time at least three switches are on. The gating sequence for this mode of operation is shown in **Figure 7**. The equivalent circuit of the PM machine, when switches S_1, S_5 and S_6 are ***on***, is shown in **Figure 8**. The set of equations defining the circuit configuration in **Figure 8** is

$$p\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} -\frac{R_s}{L_s} & 0 & 0 \\ 0 & -\frac{R_s}{L_s} & 0 \\ 0 & 0 & -\frac{R_s}{L_s} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} \frac{1}{L_s} & 0 & 0 \\ 0 & \frac{1}{L_s} & 0 \\ 0 & 0 & \frac{1}{L_s} \end{bmatrix} \begin{bmatrix} V_{an} - e_a \\ V_{bn} - e_b \\ V_{cn} - e_c \end{bmatrix} \tag{6}$$

where $\begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = \begin{bmatrix} \frac{V_{in}}{3} \\ -\frac{2V_{in}}{3} \\ \frac{V_{in}}{3} \end{bmatrix}$

The solution of **equation 6** is given in Lecture 22.

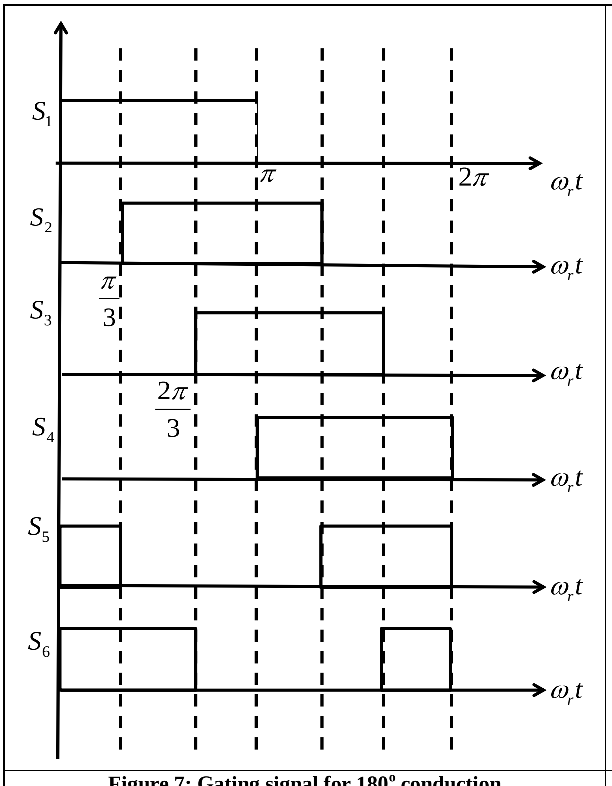

Figure 7: Gating signal for 180^{o} conduction

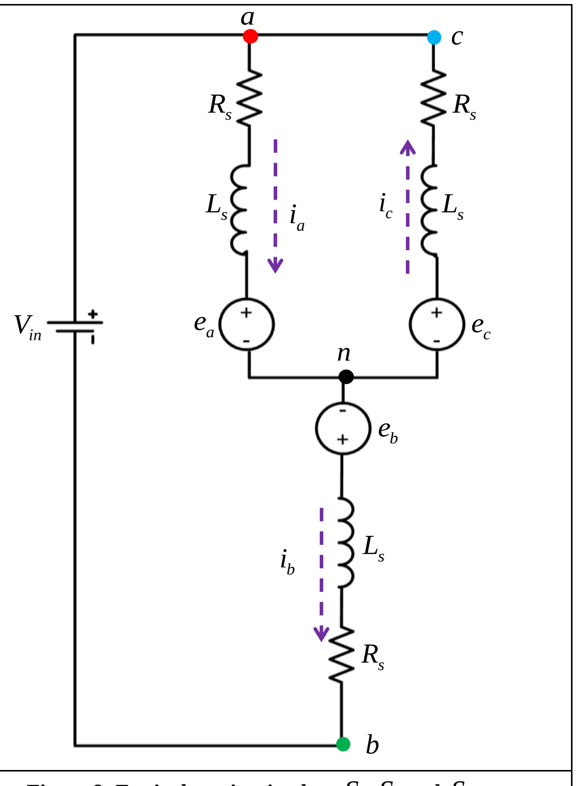

Figure 8: Equivalent circuit when S_1, S_2 and S_3 are on

Suggested Reading:

[1] D. C. Hanselman, *Brushless Permanent Magnet Motor Design*, Magna Physics Pub, 2006

Lecture 23: Steady State Characteristics of Permanent Magnet Motors

Steady State Characteristics of Permanent Magnet Motors

Introduction

The topics covered in this chapter are as follows:

- Steady State Modelling of Permanent Magnet Machines
- Steady State Solution for 120^o Conduction of the DC-AC Converter
- Steady State Solution for 180^o Conduction of the DC-AC Converter

Steady State Modelling of Permanent Magnet Machines

In Lecture 21 the general operation of the PM machine is described. The PM machines are driven by the inverter and the triggering of the DC-AC converter switches is symmetric, the waveforms of the applied stator voltage exhibits the following relationships

$$V_{an}\left(\omega_r t+\frac{\pi}{3}\right)=-V_{bn}\left(\omega_r t\right);\ V_{bn}\left(\omega_r t+\frac{\pi}{3}\right)=-V_{cn}\left(\omega_r t\right);\ V_{cn}\left(\omega_r t+\frac{\pi}{3}\right)=-V_{an}\left(\omega_r t\right) \quad (1)$$

The voltage and current relations given in Lecture 21 are repeated below

$$p\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}=\begin{bmatrix} -\frac{R_s}{L_s} & 0 & 0 \\ 0 & -\frac{R_s}{L_s} & 0 \\ 0 & 0 & -\frac{R_s}{L_s} \end{bmatrix}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}+\begin{bmatrix} \frac{1}{L_s} & 0 & 0 \\ 0 & \frac{1}{L_s} & 0 \\ 0 & 0 & \frac{1}{L_s} \end{bmatrix}+\begin{bmatrix} V_{an}-e_a \\ V_{bn}-e_b \\ V_{cn}-e_c \end{bmatrix} \quad (2a)$$

$$e_a=\lambda_m\omega_r\cos\left(\omega_r t\right); e_b=\lambda_m\omega_r\cos\left(\omega_r t-\frac{2\pi}{3}\right); e_c=\lambda_m\omega_r\cos\left(\omega_r t+\frac{2\pi}{3}\right) \quad (2b)$$

The differential equations given in **equation 2** are time-invariant. Hence, the stator currents which form the response of the system, obey the same symmetry relations as the input voltage and can be written as

$$i_a\left(\omega_r t+\frac{\pi}{3}\right)=-i_b\left(\omega_r t\right);\ i_b\left(\omega_r t+\frac{\pi}{3}\right)=-i_c\left(\omega_r t\right);\ i_c\left(\omega_r t+\frac{\pi}{3}\right)=-i_a\left(\omega_r t\right) \quad (3)$$

Steady State Solution for 120^o Conduction of the DC-AC Converter

Due to the symmetries for the voltages and currents given by **equations 1** and **3**, if the solution is known for one basic switching interval, it can be used to generate the solution for the remaining intervals. In 120^o conduction mode of DC-AC inverter, each switch conducts for 60^o, the switching scheme is shown already shown in Lecture 21. The analysis starts when switch S_2 is turned ***on*** and S_6 is turned ***off*** Due to the inductance of the stator windings, the current in phase **B** does not become zero instantaneously and continues to flow through the freewheeling diodes D_3 or D_6 depending on the direction of the current (**Figure 6** of **Lecture 21**). Once the current through the phase **B** becomes zero, the diode stops conducting and only phase **A** and **C** conduct and the equivalent circuit is shown in **Figure 6c** of **Lecture 21**. The duration for which the freewheeling diodes conduct is known as ***commutation period*** and the duration when only two phases conduct is known as ***conduction period***. At the start of the commutation period (when switch S_6 is turned off), the rotor angle is defined to be

$$\theta_r = -\phi + \frac{\pi}{6} \tag{4}$$

where

ϕ is trhe advance firing angle

The duration of the ***commutation period*** is given by the commutation angle θ_c and is a function of ϕ , the winding inductances and resistances and rotor speed ω_r making it difficult to estimate. The determination of current is achieved in two steps:

- **Step 1:** In this step the general solution of the currents is obtained
- **Step 2:** In this step the angle θ_c is determined using the symmetries given in **equations 1** and **3**.

Step 1: General Solution

At time $t=0$ the switch S_2 is turned ***on*** and S_6 is turned ***off***. As discussed in the previous section, the current i_b does not become zero immediately and remains nonzero till the time $t=t_c$. Hence, the commutation period lasts for $0 \le t \le t_c$. In this period all the three phases are connected to the DC-AC converter and the stator voltages for $i_b < 0$ are

$$\begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = \begin{bmatrix} \frac{V_{in}}{3} \\ \frac{V_{in}}{3} \\ -\frac{2V_{in}}{3} \end{bmatrix} \tag{5}$$

In case $i_b > 0$, the stator phase voltages are

$$\begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = \begin{bmatrix} \frac{2V_{in}}{3} \\ -\frac{V_{in}}{3} \\ -\frac{V_{in}}{3} \end{bmatrix} \tag{6}$$

Substituting e_a, e_b, e_c from **equation 2b** and replacing θ_r with $\theta_r - \phi + \frac{\pi}{6}$ in **equation 2a** gives

$$p\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} -\frac{R_s}{L_s} & 0 & 0 \\ 0 & -\frac{R_s}{L_s} & 0 \\ 0 & 0 & -\frac{R_s}{L_s} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} \frac{1}{L_s} & 0 & 0 \\ 0 & \frac{1}{L_s} & 0 \\ 0 & 0 & \frac{1}{L_s} \end{bmatrix} \begin{bmatrix} V_{an} - \omega_r \lambda_m \cos\left(\theta_r - \phi + \frac{\pi}{6}\right) \\ V_{bn} - \omega_r \lambda_m \sin\left(\theta_r - \phi\right) \\ V_{cn} - \omega_r \lambda_m \cos\left(\theta_r - \phi + \frac{5\pi}{6}\right) \end{bmatrix} \tag{7}$$

The system of first order differential **equation 7** can be expressed in standard state variable form as

$$pi_{abc}(t) = Ai_{abc}(t) + Bu(t)$$

where

$$i_{abc} = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}, A = \begin{bmatrix} -\frac{R_s}{L_s} & 0 & 0 \\ 0 & -\frac{R_s}{L_s} & 0 \\ 0 & 0 & -\frac{R_s}{L_s} \end{bmatrix} \tag{8}$$

$$B = \begin{bmatrix} \frac{1}{L_s} & 0 & 0 \\ 0 & \frac{1}{L_s} & 0 \\ 0 & 0 & \frac{1}{L_s} \end{bmatrix}, u(t) = \begin{bmatrix} V_{an} - \omega_r \lambda_m \cos\left(\theta_r - \phi + \frac{\pi}{6}\right) \\ V_{bn} - \omega_r \lambda_m \sin\left(\theta_r - \phi\right) \\ V_{cn} - \omega_r \lambda_m \cos\left(\theta_r - \phi + \frac{5\pi}{6}\right) \end{bmatrix}$$

The solution of **equation 8** is

$$i_{abc}(t) = e^{At} i_o + \int_0^{t_c} e^{A(-\tau)} Bu(\tau) d\tau \tag{9}$$

where the initial conditions vector is

$$i_o = \begin{bmatrix} i_a(0) \\ i_b(0) \\ i_c(0) \end{bmatrix} = \begin{bmatrix} i_a(0) \\ i_b(0) \\ -\left(i_a(0) + i_b(0)\right) \end{bmatrix} \tag{10}$$

The solution of **equation 9** for the time interval $0 \le t \le t_c$, using the initial conditions given in **equation 10**, is

$$\begin{aligned} i_a(t) = {} & i_a(0) e^{-\frac{R_s}{L_s}t} + \frac{V_{an}}{R_s}\left(1 - e^{-\frac{R_s}{L_s}t}\right) + \frac{\omega_r \lambda_m}{2\left(R_s^2 + \omega_r^2 L_s^2\right)}\left[\left(\omega_r L_s + \sqrt{3} R_s\right)\cos\phi + \left(R_s - \sqrt{3}\omega_r L_s\right)\sin\phi\right] e^{-\frac{R_s}{L_s}t} \\ & - \frac{\omega_r \lambda_m}{2\left(R_s^2 + \omega_r^2 L_s^2\right)}\left[\left(\omega_r L_s + \sqrt{3} R_s\right)\cos\left(\theta_r - \phi\right) - \left(R_s - \sqrt{3}\omega_r L_s\right)\sin\left(\theta_r - \phi\right)\right] \end{aligned} \tag{11}$$

$$\begin{aligned} i_b(t) = {} & i_b(0) e^{-\frac{R_s}{L_s}t} + \frac{V_{bn}}{R_s}\left(1 - e^{-\frac{R_s}{L_s}t}\right) - \frac{\omega_r \lambda_m}{\left(R_s^2 + \omega_r^2 L_s^2\right)}\left[\omega_r L_s \cos\phi + R_s \sin\phi\right] e^{-\frac{R_s}{L_s}t} \\ & + \frac{\omega_r \lambda_m}{\left(R_s^2 + \omega_r^2 L_s^2\right)}\left[\omega_r L_s \cos\left(\theta_r - \phi\right) - R_s \sin\left(\theta_r - \phi\right)\right] \end{aligned} \tag{12}$$

Since the three phases are connected in Y, the phase **C** current is given by

$$i_c(t) = -\left(i_a(t) + i_b(t)\right) \tag{13}$$

When the i_b becomes zero at $t = t_c$, the commutation period ends and the conduction period starts with just phases **A** and **C** conducting. The duration of conduction period is $t_c \le t \le \frac{\pi}{3\omega_r}$. The differential equation given in **equation 11** holds for the conduction period and the only change is in $u(t)$ and initial values of current given by

$$u(t) = \begin{bmatrix} \frac{V_{in}}{2} - \frac{\omega_r \lambda_m}{2}\cos\left(\theta_r - \phi + \frac{\pi}{6}\right) \\ \omega_r \lambda_m \sin\left(\theta_r - \phi\right) \\ -\frac{V_{in}}{2} - \frac{\omega_r \lambda_m}{2}\cos\left(\theta_r - \phi + \frac{5\pi}{6}\right) \end{bmatrix}; \; i_o = \begin{bmatrix} i_a(t_c) \\ i_b(t_c) \\ i_c(t_c) \end{bmatrix} = \begin{bmatrix} i_a(t_c) \\ 0 \\ -i_a(t_c) \end{bmatrix} \tag{14}$$

The solution of **equation 9** for conduction period is

$$i_{abc}(t) = e^{A(t - t_c)} i_o + \int_0^{\frac{\pi}{3\omega_r}} e^{A(t-\tau)} Bu(\tau) d\tau \tag{15}$$

The evaluation of the integration given in **equation 16** gives

$$\begin{aligned} i_a(t) = i_a(t_c)e^{-\frac{R_s}{L_s}(t-t_c)} + \frac{V_{in}}{2R_s}\left(1-e^{-\frac{R_s}{L_s}(t-t_c)}\right) + \frac{\sqrt{3}\omega_r\lambda_m}{2\left(R_s^2+\omega_r^2L_s^2\right)}\left[R_s\cos\left(\phi-\theta_c\right)-\omega_rL_s\sin\left(\phi-\theta_c\right)\right]e^{-\frac{R_s}{L_s}(t-t_c)} \\ -\frac{\sqrt{3}\omega_r\lambda_m}{2\left(R_s^2+\omega_r^2L_s^2\right)}\left[R_s\cos\left(\theta_r-\phi\right)+\omega_rL_s\sin\left(\theta_r-\phi\right)\right] \end{aligned} \quad (18)$$

The phase **C** current is same as phase **A** current ($i_a(t)=-i_c(t)$) and phase **B** current is zero ($i_b(t)=0$)

Step 2: Determination of Commutation Angle

At time $t=0$, the switch S_6 is turned off and S_2 is turned on. Hence, at $t=0$ the current in phase **C** is zero and the phase **A** and **B** currents are equal in magnitude. Therefore, the initial conditions are given by

$$i_o = \begin{bmatrix} i_a(0) \\ i_b(0) \\ i_c(0) \end{bmatrix} = \begin{bmatrix} I_o \\ -I_o \\ 0 \end{bmatrix} \quad (19)$$

At the end of the conduction period, the currents are

$$\begin{bmatrix} i_a\left(\frac{\pi}{3\omega_r}\right) \\ i_b\left(\frac{\pi}{3\omega_r}\right) \\ i_c\left(\frac{\pi}{3\omega_r}\right) \end{bmatrix} = \begin{bmatrix} I_o \\ 0 \\ -I_o \end{bmatrix} \quad (20)$$

The commutation period ends when phase **B** current becomes zero, that is, $i_b\left(t_c\right)=0$. Using this condition and initial conditions given by **equation 19** in **equation 18** gives

$$\begin{aligned} 0 = -I_oe^{-\frac{R_s}{L_s}t_c} + \frac{V_{in}}{R_s}\left(1-e^{-\frac{R_s}{L_s}t_c}\right) - \frac{\omega_r\lambda_m}{\left(R_s^2+\omega_r^2L_s^2\right)}\left[\omega_rL_s\cos\phi+R_s\sin\phi\right]e^{-\frac{R_s}{L_s}t_c} \\ +\frac{\omega_r\lambda_m}{\left(R_s^2+\omega_r^2L_s^2\right)}\left[\omega_rL_s\cos\left(\theta_c-\phi\right)-R_s\sin\left(\theta_c-\phi\right)\right] \end{aligned} \quad (21)$$

Using the boundary condition given by **equation 20** in **equation 18** gives

$$\begin{aligned} I_o = i_a(t_c)e^{-\frac{R_s}{L_s}\left(\frac{\pi}{3\omega_r}-t_c\right)} + \frac{V_{in}}{2R_s}\left(1-e^{-\frac{R_s}{L_s}\left(\frac{\pi}{3\omega_r}-t_c\right)}\right) + \frac{\sqrt{3}\omega_r\lambda_m}{2\left(R_s^2+\omega_r^2L_s^2\right)}\left[R_s\cos\left(\phi-\theta_c\right)-\omega_rL_s\sin\left(\phi-\theta_c\right)\right]e^{-\frac{R_s}{L_s}\left(\frac{\pi}{3\omega_r}-t_c\right)} \\ -\frac{\sqrt{3}\omega_r\lambda_m}{2\left(R_s^2+\omega_r^2L_s^2\right)}\left[R_s\cos\left(\frac{\pi}{3}-\phi\right)+\omega_rL_s\sin\left(\frac{\pi}{3}-\phi\right)\right] \end{aligned} \quad (22)$$

The expression for $i_a(t_c)$ can be obtained by substituting $t = t_c$ in **equation 18**.

$$
\begin{aligned}
I_o = i_a(t_c) e^{-\frac{R_s}{L_s}t_c} + \frac{V_{an}}{R_s}\left(1 - e^{-\frac{R_s}{L_s}t_c}\right) + \frac{\omega_r\lambda_m}{2\left(R_s^2+\omega_r^2L_s^2\right)}\left[\left(\omega_r L_s + \sqrt{3}R_s\right)\cos\phi + \left(R_s - \sqrt{3}\omega_r L_s\right)\sin\phi\right]e^{-\frac{R_s}{L_s}t_c} \\
-\frac{\omega_r\lambda_m}{2\left(R_s^2+\omega_r^2L_s^2\right)}\left[\left(\omega_r L_s + \sqrt{3}R_s\right)\cos\left(\frac{\pi}{3}-\phi\right) - \left(R_s - \sqrt{3}\omega_r L_s\right)\sin\left(\frac{\pi}{3}-\phi\right)\right]
\end{aligned}
\qquad (23)
$$

From **equation 23** the value of $i_a(t_c)$ into **equation 22** and in turn substituting the resulting value of I_o into **equation 21** gives:

$$
\begin{aligned}
\frac{1}{R_s}\left[V_a + V_b - V_b e^{\frac{R_s}{\omega_r L_s}\frac{\pi}{3}} - \frac{V_{in}}{2}\right] + \left[\frac{1}{2} - e^{\frac{R_s}{\omega_r L_s}\frac{\pi}{3}}\right]\frac{\omega_r\lambda_m}{\sqrt{\omega_r^2L_s^2+R_s^2}}\cos\left(\omega_r t_c - \phi + \delta\right) \\
+e^{-\frac{R_s}{L_s}t_c}\left[\frac{V_{in}}{R_s}\left(e^{\frac{R_s}{\omega_r L_s}\frac{\pi}{3}} - 1\right) - \frac{V_a}{R_s} + \frac{V_{in}}{2R_s}e^{\frac{R_s}{\omega_r L_s}\frac{\pi}{3}} + \left(\frac{1}{2}e^{\frac{R_s}{\omega_r L_s}\frac{\pi}{3}} - 1\right)\frac{\omega_r\lambda_m}{\sqrt{\omega_r^2L_s^2+R_s^2}}\right]\cos\left(\frac{\pi}{3} - \phi + \delta\right) = 0
\end{aligned}
\qquad (24)
$$

where

$$\delta = \tan^{-1}\left(\frac{R_s}{\omega_r L_s}\right)$$

In **equation 24** the unknown variable is t_c. However, this is transcendental function of t_c and hence it is impossible to get an analytical solution of this equation. An iterative process, such as ***Newton-Raphson*** method, can be used to determine t_c. Once the value of t_c is determined, from **equation 23** the value of I_o can be determined.

Steady State Solution for 180^o Conduction of the DC-AC Converter

In 180^o operation, ***three switches*** are always ***on***. Since the basic configuration of the circuit does not change, a closed form solution can be developed. The solution can be obtained in two steps:

- ***Step1***: General solution
- ***Step 2***: Determination of initial values

Since the commutation process is not involved, the solution process is simpler.

Step 1: General Solution

For time duration $0 \le t \le \frac{\pi}{3\omega_r}$ the currents are given by

$$\begin{aligned} i_a(t) &= i_a(0)e^{-\frac{R_s}{L_s}t} + \frac{V_{an}}{R_s}\left(1-e^{-\frac{R_s}{L_s}t}\right) + \frac{\omega_r \lambda_m}{2\left(R_s^2+\omega_r^2 L_s^2\right)}\left[\left(\omega_r L_s + \sqrt{3}R_s\right)\cos\phi + \left(R_s - \sqrt{3}\omega_r L_s\right)\sin\phi\right]e^{-\frac{R_s}{L_s}t} \\ &\quad - \frac{\omega_r \lambda_m}{2\left(R_s^2+\omega_r^2 L_s^2\right)}\left[\left(\omega_r L_s + \sqrt{3}R_s\right)\cos\left(\theta_r - \phi\right) - \left(R_s - \sqrt{3}\omega_r L_s\right)\sin\left(\theta_r - \phi\right)\right] \end{aligned} \tag{25}$$

$$\begin{aligned} i_b(t) &= i_b(0)e^{-\frac{R_s}{L_s}t} + \frac{V_{bn}}{R_s}\left(1-e^{-\frac{R_s}{L_s}t}\right) - \frac{\omega_r \lambda_m}{\left(R_s^2+\omega_r^2 L_s^2\right)}\left[\omega_r L_s \cos\phi + R_s \sin\phi\right]e^{-\frac{R_s}{L_s}t} \\ &\quad + \frac{\omega_r \lambda_m}{\left(R_s^2+\omega_r^2 L_s^2\right)}\left[\omega_r L_s \cos\left(\theta_r - \phi\right) - R_s \sin\left(\theta_r - \phi\right)\right] \end{aligned} \tag{26}$$

$$i_c(t) = -\left(i_a(t) + i_b(t)\right) \tag{27}$$

Step 2: Determination of Initial values

In 180^{o} conduction the stator phases are not subject to open circuit condition and hence it is no longer true that the current in any phase will be zero at the start of commutation period. Hence, the following conditions hold:

$$\begin{aligned} i_a\left(\frac{\pi}{3\omega_r}\right) &= -i_b\left(0\right) \\ i_b\left(\frac{\pi}{3\omega_r}\right) &= i_a(0) + i_b\left(0\right) \end{aligned} \tag{28}$$

The above relations are enough to determine the initial conditions. The steps involved in determining the initial conditions are:

- Set $t = \frac{\pi}{3\omega_r}$ in **equations 25** and **26**. This results in two equations with two unknowns $i_a\left(0\right)$ and $i_b\left(0\right)$.
- The solution of these two equations give the values of $i_a\left(0\right)$ and $i_b\left(0\right)$. Once the initial conditions are known, the complete solution is obtained for the time duration $0 \le t \le \frac{\pi}{3\omega_r}$

Suggested Reading:

[1] D. C. Hanselman, *Brushless Permanent Magnet Motor Design*, Magna Physics Pub, 2006

Lecture 24: Dynamic Model of PM Machines

Dynamic Model of PM Machines

Introduction

The topics covered in this chapter are as follows:

- The $d-q$ axis model of Two Phase Permanent Magnet (PM) Machine
- Transformation to Rotor Reference Frames
- Three Phase to Two Phase Transformation
- The Power Equivalence
- The Electromagnetic Torque
- The Steady State Torque Characteristics
- Models in Flux Linkages

The $d-q$ axis model of Two Phase Permanent Magnet (PM) Machine

The variable speed PM machines are fed by an inverter and hence it becomes necessary to evaluate the dynamics of the machine to determine the adequacy of the converter switch ratings. The dynamic model becomes a convenient method of studying the machine's dynamic behavior. In order to derive the dynamic model, the following assumptions are made:

- The stator windings are balanced and mmf produced by the windings is sinusoidal.
- The variation of the inductance with respect to the rotor position is sinusoidal
- The effects of magnetic saturation are neglected.

A two phase PM machine with windings and magnets on the rotor is shown in **Figure 1a**. From **Figure 1** the following can be observed:

- The windings are displaced by 90 electrical degrees.
- The rotor pole is at an angle of θ_r from the stator's d axis.
- The q axis leads the d axis for anti-clockwise direction of rotation of the rotor.

In **Figure 1a** only two poles are shown, however the model derived for two pole machine is valid for any number of poles.

The d and q axes stator voltages are given by:

$$v_{qs} = R_q i_{qs} + p\lambda_{qs} \tag{1}$$

$$v_{ds} = R_d i_{ds} + p\lambda_{ds} \tag{2}$$

where

$p = \frac{d}{dt}$

v_{qs}, v_{ds} are the voltages in the q and d axes windings

i_{qs}, i_{ds} are the q and d axes currents

R_q, R_d are the q and d axes winding resistances

$\lambda_{qs}, \lambda_{ds}$ are the q and d axes stator flux linkages

The stator winding flux linkages is the sum of flux linkages due to the self excitation and mutual flu linkage resulting from the current in other winding and magnet sources. Hence, the q and d axes flux linkages can be written as:

$$\lambda_{qs} = L_{qq} i_{qs} + L_{qd} i_{ds} + \lambda_{af} \sin\theta_r \quad (3)$$

$$\lambda_{ds} = L_{dd} i_{ds} + L_{qd} i_{qs} + \lambda_{af} \cos\theta_r \quad (4)$$

where

θ_r is the instantaneous rotor position

L_{dd}, L_{qq} are the self inductances of the d and q axis windings respectively

L_{qd} is the mutual inductance between the d and q axis windings

Taking into account that the stator windings are balanced, that is $R_d = R_q = R_s$, the substitution of **equations 3** and **4** into **equation 1** and **2** gives:

$$v_{qs} = R_s i_{qs} + i_{qs} pL_{qq} + L_{qq} pi_{qs} + i_{ds} pL_{qd} + L_{qd} pi_{ds} + \lambda_{af} p(\sin\theta_r) \quad (5)$$

$$v_{ds} = R_s i_{ds} + i_{ds} pL_{dd} + L_{dd} pi_{ds} + i_{qs} pL_{qd} + L_{qd} pi_{qs} + \lambda_{af} p(\cos\theta_r) \quad (6)$$

The inductances in PM machine are functions of the rotor position. Consider the case when the d axis of the stator is aligned with the magnet's axis (**Figure 1a**). Since the relative permeability of the magnets is almost equal to that of air, the length of the flux path in air is increased by the magnet's thickness. Hence, the reluctance of the flux in this path increases and the winding inductance decreases. This position of the rotor corresponds to the minimum inductance position and this inductance is denoted by L_d. As the rotor moves, the inductance increases and reaches the maximum value when the rotor has rotated by 90^o and the interpolar axis aligns with the q axis of the stator winding (**Figure 1b**).

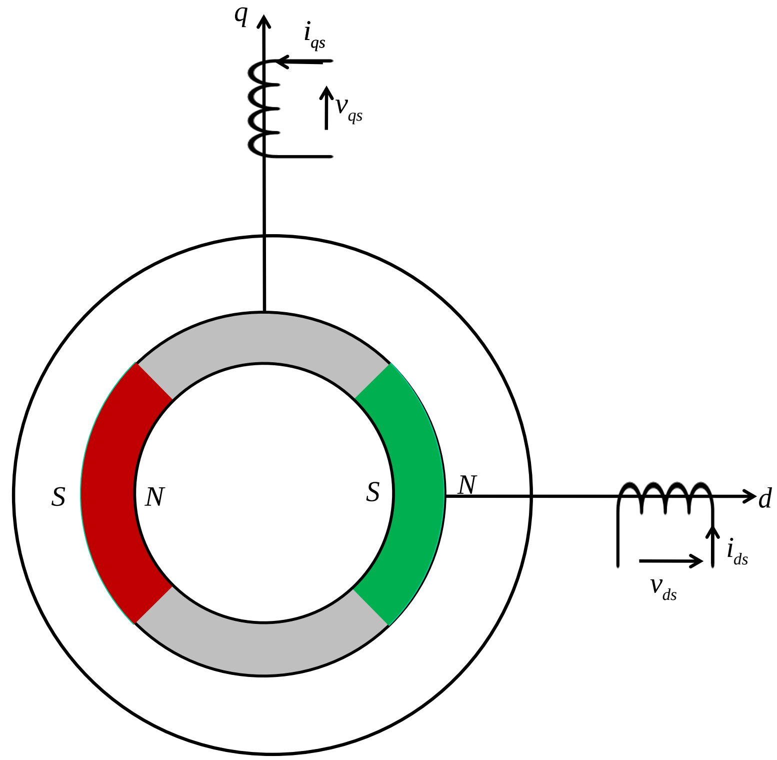

Figure 1a: The rotor axis aligned with the *d* axis

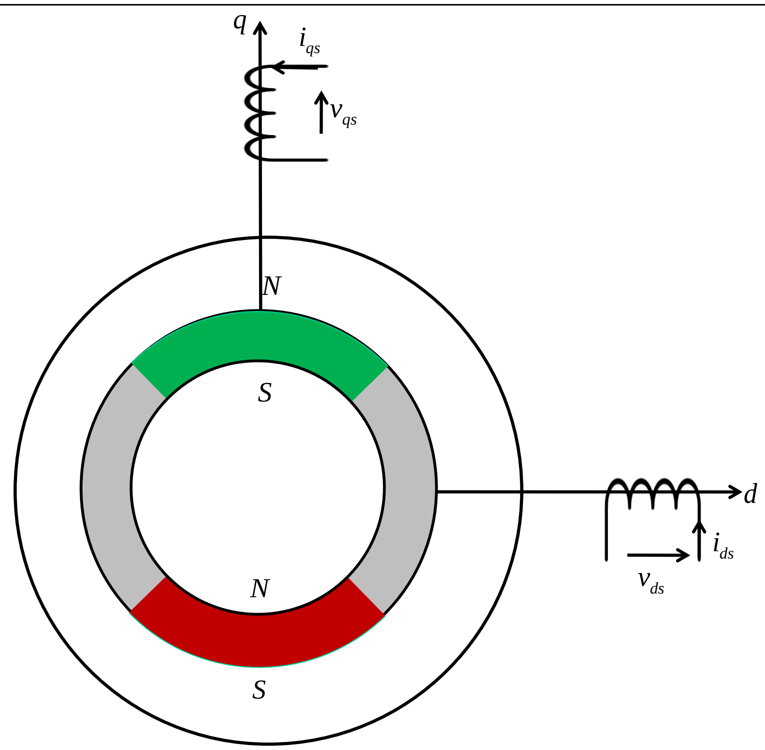

Figure 1b: The rotor axis aligned with the *q* axis

The inductance is denoted by L_q and the variation of the reluctance and the inductance with respect to rotor position is shown in **Figure 2**. The self inductances of the q and d windings can be expressed in terms of L_q and L_d as

$$L_{qq} = \frac{1}{2}\left[\left(L_q + L_d\right) + \left(L_q - L_d\right)\cos\left(2\theta_r\right)\right]$$

(7)

$$L_{dd} = \frac{1}{2}\left[\left(L_q + L_d\right) - \left(L_q - L_d\right)\cos\left(2\theta_r\right)\right] \tag{8}$$

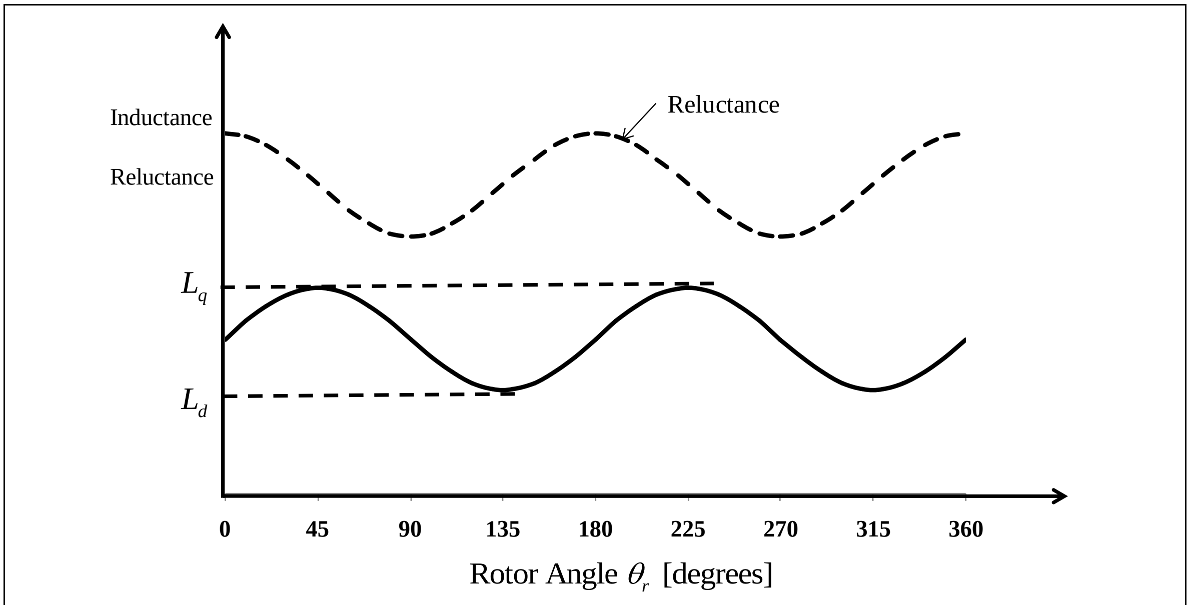

Figure 2: Variation of reluctance and inductance with respect to rotor position

The **equations 7** and **8** can be expressed in more compact form as

$$L_{qq} = L_1 + L_2\cos\left(2\theta_r\right) \tag{9}$$

$$L_{dd} = L_1 - L_2\cos\left(2\theta_r\right) \tag{10}$$

where

$$L_1 = \frac{1}{2}\left(L_q + L_d\right);\ L_2 = \frac{1}{2}\left(L_q - L_d\right) \tag{11}$$

The mutual inductance between the q and d axis windings is zero if the rotor is cylindrical and smooth, as the flux set up by a current in one winding will not link with the other winding displaced by 90°. For the configuration of the machine shown in, due to the saliency of the rotor a part of the d axis winding flux will be linked by the q axis winding and the mutual inductance will not be zero. The mutual inductance in case of salient pole motor will depend on the position of the rotor. When the rotor position is zero or 90° (**Figure 1a**), the mutual coupling is zero and is maximum when the rotor angle is 45° (**Figure 1b**). The mutual inductance also varies sinusoidally with the rotor position and is given as

$$L_{qd} = \frac{1}{2}\left(L_d - L_q\right)\sin\left(2\theta_r\right) = -L_2 \sin\left(2\theta_r\right) \tag{12}$$

Substituting the value of mutual inductance from **equation 12** into **equations 5** and **6** and can be written in matrix form as

$$\begin{aligned} \begin{bmatrix} v_{qs} \\ v_{ds} \end{bmatrix} &= R_s \begin{bmatrix} i_{qs} \\ i_{ds} \end{bmatrix} + \begin{bmatrix} L_1 + L_2\cos\left(2\theta_r\right) & -L_2\sin\left(2\theta_r\right) \\ -L_2\sin\left(2\theta_r\right) & L_1 - L_2\cos\left(2\theta_r\right) \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_{qs} \\ i_{ds} \end{bmatrix} \\ &\quad + 2\omega_r L_2 \begin{bmatrix} -\sin\left(2\theta_r\right) & -\cos\left(2\theta_r\right) \\ -\cos\left(2\theta_r\right) & \sin\left(2\theta_r\right) \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \end{bmatrix} + \lambda_{af} \begin{bmatrix} \cos\theta_r \\ -\sin\theta_r \end{bmatrix} \end{aligned} \tag{13}$$

Transformation to Rotor Reference Frames

From **equation 13** it can be seen that there are many quantities dependent on the rotor position. Hence, if the entire system is observed from the rotor, the system inductance matrix becomes independent of the rotor position. This leads to the simplification of **equation 13**. The d and q axis on the stator are stationary and are denoted by a subscript s whereas the rotor d and q axis rotate with the rotor and are denoted a subscript r. In **Figure 3**, the alignment of the stator and rotor axes is shown. From **Figure 3** it can be seen that the angle between the rotor and stator axis (θ_r) change as the rotor rotates.

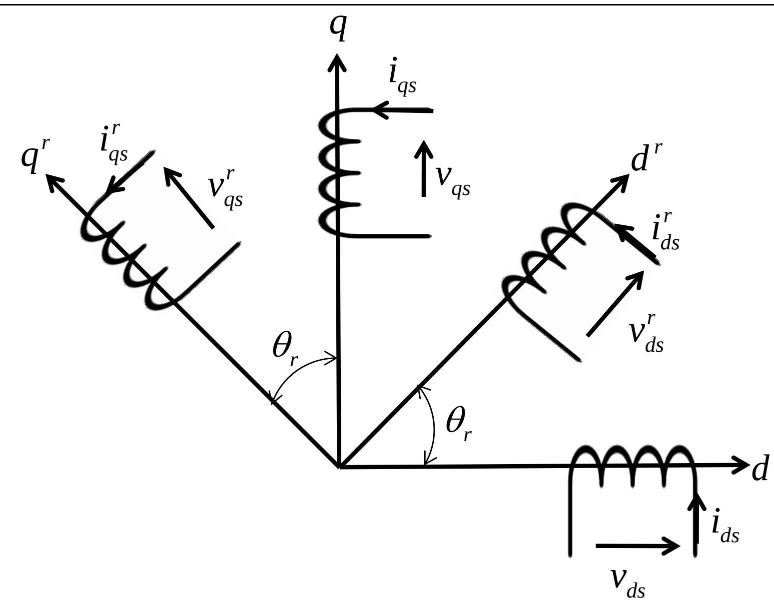

Figure 3: Rotor and start *d* and *q* axis

The transformation to obtain constant inductances is achieved by replacing the actual stator windings with a fictitious winding on the d_r and q_r axis (**Figure 3**). The fictitious stator winding has following properties:

- It has same number of turns as the original winding on the stator axis.
- The mmf produced by the fictitious winding is same as that produced by the original winding.

From **Figure 3** the translation of the stator currents in stator winding on the d_s and q_s (i_{ds}, i_{qs}) to the currents in the fictitious winding on d_r and q_r (i_{ds}^r, i_{qs}^r) is given by

$$i_{qds} = [K] i_{qds}^r \quad (14)$$

where

$$i_{qds} = \begin{bmatrix} i_{qs} & i_{ds} \end{bmatrix}^T ;\ i_{qds}^r = \begin{bmatrix} i_{qs}^r & i_{ds}^r \end{bmatrix}^T ;\ K = \begin{bmatrix} \cos\theta_r & \sin\theta_r \\ -\sin\theta_r & \cos\theta_r \end{bmatrix}$$

Similarly, the relation between the stator voltage (v_{ds}, v_{qs}) in stationary d_s and q_s reference frame and the stator voltage (v_{ds}^r, v_{qs}^r) in rotor reference frame's voltage is

$$v_{qds} = [K] v_{qds}^r \quad (15)$$

where

$$v_{qds} = \begin{bmatrix} v_{qs} & v_{ds} \end{bmatrix}^T ;\ v_{qds}^r = \begin{bmatrix} v_{qs}^r & v_{ds}^r \end{bmatrix}^T$$

Substituting **equations 14** and **15** into **equation 13** gives

$$\begin{bmatrix} v_{qs}^r \\ v_{ds}^r \end{bmatrix} = \begin{bmatrix} R_s + L_q p & \omega_r L_d \\ -\omega_r L_q & R_s + L_d p \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \end{bmatrix} + \begin{bmatrix} \omega_r \lambda_{af} \\ 0 \end{bmatrix} \quad (16)$$

where

$$\omega_r = \frac{d\theta}{dt}$$

Thus, from **equation 16** it can be seen that the voltage equation is greatly simplified.

Three Phase to Two Phase Transformation

A dynamic model for the three phase PMSM can be derived from the two phase machine if the equivalence between the three and two phases is established. **Figure 4** shows the three phase and two phase winding axes. Assuming that each of the three phase windings has N number of turns per phase, and equal current magnitudes, the two phase windings will have $\frac{3}{2}N$ turns per phase for mmf equality.

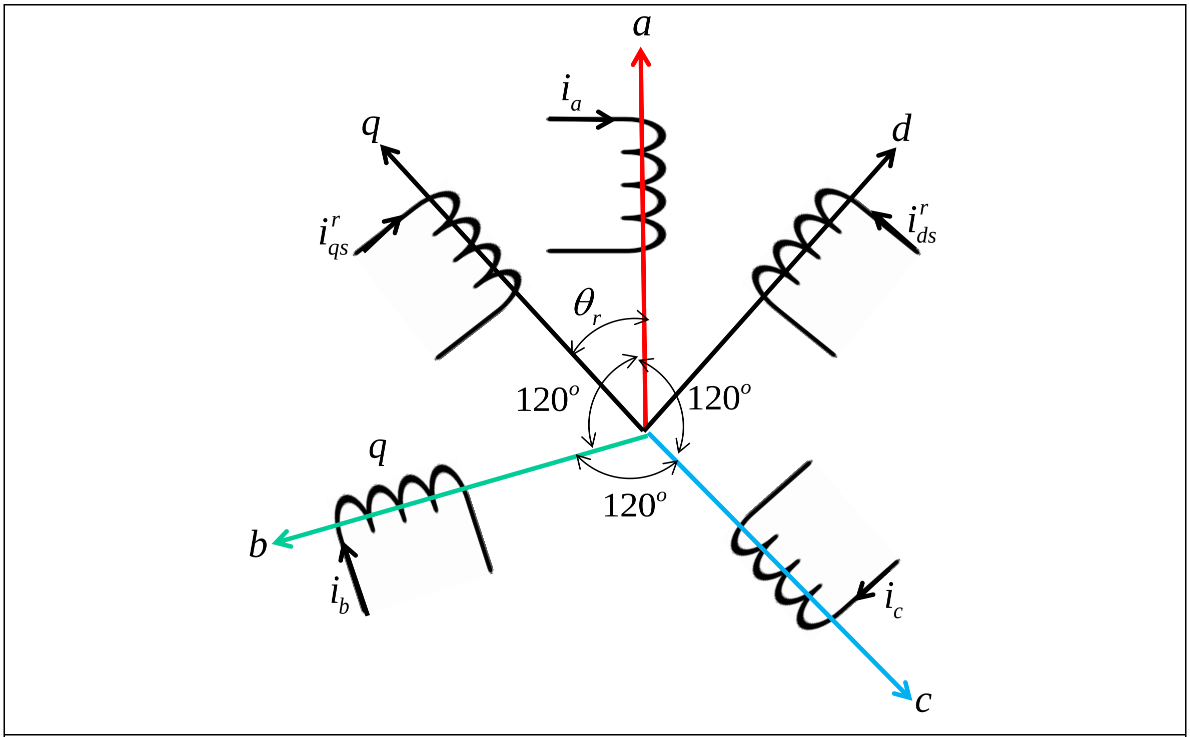

Figure 4: Three phase and two phase winding axis

Resolving the three phase currents along the d_r and q_r axis is gives

$$\begin{bmatrix} i_{qs}^r \\ i_{ds}^r \\ i_o^r \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \cos\theta_r & \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\left(\theta_r + \frac{2\pi}{3}\right) \\ \sin\theta_r & \sin\left(\theta_r - \frac{2\pi}{3}\right) & \sin\left(\theta_r + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} \tag{17}$$

The **equation 17** can be expressed in compact form as

$$i_{qdo}^{r} = \left[K_{abc}\right] i_{abcs}$$

where (18)

$$i_{qdo}^{r} = \left[i_{qs}^{r} \quad i_{ds}^{r} \quad i_{0}\right]^{T};\ i_{abcs} = \left[i_{as} \quad i_{bs} \quad i_{cs}\right]^{T}$$

Similarly, the three phase voltages can be transformed as

$$v_{qdo}^{r} = \left[K_{abc}\right] v_{abcs}$$

where (19)

$$v_{qdo}^{r} = \left[v_{qs}^{r} \quad v_{ds}^{r} \quad v_{0}\right]^{T};\ v_{abcs} = \left[v_{as} \quad v_{bs} \quad v_{cs}\right]^{T}$$

The Power Equivalence

The power input to the three phase machine has to be equal to the power input to the two phase machine in order to achieve linear transformation. Consider the power produced by a three phase machine

$$p_{in} = v_{abcs}^{T} i_{abcs}^{T} = v_{as} i_{as} + v_{bs} i_{bs} + v_{cs} i_{cs} \tag{20}$$

From **equation 18** and **19** the three phase currents can be expressed in terms of two phase quantities as

$$i_{abcs} = \left[K\right]^{-1} i_{qdo}^{r};\ v_{abcs} = \left[K\right]^{-1} v_{qdo}^{r} \tag{21}$$

Substituting the values of three phase quantities from **equation 21** into **equation 20** gives

$$p_{in} = \left(v_{qdo}^{r}\right)^{T} \left(\left[K_{abcs}\right]^{-1}\right)^{T} \left[K_{abcs}\right]^{-1} i_{qdo}^{r}$$

$$\Rightarrow p_{in} = \frac{3}{2}\left(v_{qs}^{r} i_{qs}^{r} + v_{ds}^{r} i_{ds}^{r} + 2 v_{0} i_{0}\right) \tag{23}$$

For a balanced three phase machine, the zero sequence currents do not exist, hence the **equation 23** can be expressed as

$$p_{in} = \frac{3}{2}\left[v_{qs}^{r} i_{qs}^{r} + v_{ds}^{r} i_{ds}^{r}\right] \tag{24}$$

The Electromagnetic Torque

The dynamic equation of a PM machine can be written as

$$V = [R]i + [L]pi + [G]\omega_r t$$

where

$[R]$ matrix consists of resistive element (25)

$[L]$ matrix consists of the coefficients of the derivative operator p

$[G]$ matrix has elements that are the coefficients of the electrical rotor speed ω_r

Hence, the input power is given by

$$p_{in} = i^T V = i^T [R]i + i^T [L]pi + i^T [G]\omega_r i \quad (26)$$

The term $i^T [R]i$ represents the stator resistive losses (Ohmic losses). The term $i^T [L]pi$ denotes the rate of change of stored magnetic energy. The power component $i^T [G]\omega_r t$ is the air gap power. Hence, the air gap torque is derived from the terms involving the rotor mechanical speed ω_m as

$$\begin{aligned} \omega_m T_e &= p_{in} = \frac{N_p}{2} i^T [G]\omega_r i \\ \Rightarrow T_e &= \frac{N_p}{2} i^T [G] i \end{aligned} \quad (27)$$

Substituting the values of $[G]$ from **equation 16** gives

$$T_e = \frac{3}{2}\frac{N_p}{2}\left[\lambda_{af} + \left(L_d - L_q\right) i_{ds}^r\right] i_{qs}^r \quad (28)$$

The Steady State Torque Characteristics

A salient pole PM machine is considered in order to generalize the steady state characteristics of the machine. A set of balanced polyphase currents is assumed to be input to the stator windings and is given by

$$\begin{aligned} I_{qs} &= I_m \sin\left(\omega_r t + \delta\right) \\ I_{ds} &= I_m \cos\left(\omega_r t + \delta\right) \end{aligned} \quad (29)$$

By using the transformation matrix K from **equation 14**, the stator currents in the rotor reference frames are obtained as

$$\begin{aligned} I_{qs}^r &= I_m \sin\left(\delta\right) \\ I_{ds}^r &= I_m \cos\left(\delta\right) \end{aligned} \quad (30)$$

Substituting the currents from **equation 30** into **equation 28** gives

$$T_e = \frac{3}{2}\frac{N_p}{2}\left[\lambda_{af} + \left(L_d - L_q\right) i_{ds}^r\right] i_{qs}^r = \frac{3}{2}\frac{N_p}{2}\left[\lambda_{af} + \left(L_d - L_q\right) I_m \cos\delta\right] I_m \sin\delta$$
$$\Rightarrow T_e = \frac{3}{2}\frac{N_p}{2}\left[\lambda_{af} I_m \sin\delta + \frac{1}{2}\left(L_d - L_q\right) I_m^2 \sin(2\delta)\right] \quad (31)$$

The angle δ in **equation 31** refers to torque angle.

Upon examination of **equation 28** and **equation 31** it can be seen that the torque has two terms. The first term gives the torque as a result of the interaction between the rotor's magnet and the q axis stator current. And this torque is usually referred to as ***synchronous torque*** (T_{syn}). The second term contains the reluctance variation and the torque due to that is known as ***reluctance torque*** (T_{reluc}). `For a typical machine, the air gap torque and its individual components are shown in **Figure 5** as a function of δ. The sum of synchronous and reluctance torques gives the total torque and its peak occurs at an angle greater than 90^o.

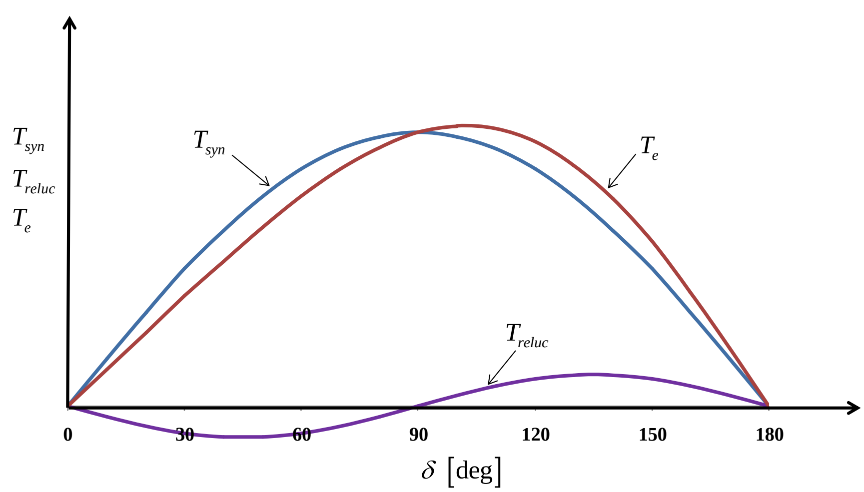

Figure 5: Air gap torque, reluctance torque and synchronous torque

This shift in angle occurs due to the reluctance torque. The reluctance torque enhances the air gap torque in the torque angle region between 90^o and 180^o and reduces it between 0^o and 90^o. From **Figure 6** it can be seen that the angle at which the maximum torque occurs changes for various stator current magnitudes. The loci of the torque angle versus the maximum torque as a function of stator current magnitude are important in the optimum torque per unit current operation of the machine.

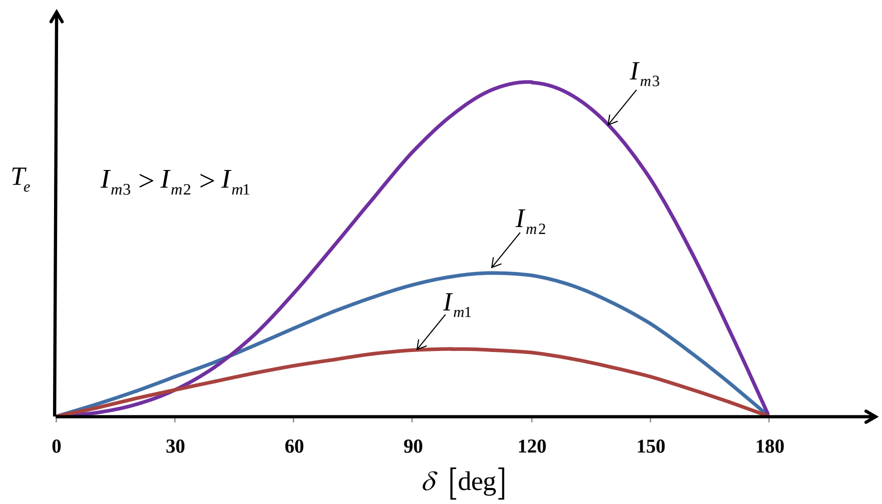

Figure 6: The variation of toque for different values of load angle and current

Models in Flux Linkages

The dynamic equations of the PM machine in rotor reference frames can be represented using the flux linkages as variables. The advantage of developing the model in terms of flux linkages is that even when the voltages and currents are discontinuous, the flux linkages are continuous. The continuity of flux linkages gives the advantage of differentiating these variables. In order to develop the model in terms of flux linkages, the rotor and stator flux linkages are written as

$$\lambda_{qs}^r = L_q i_{qs}^r \tag{32}$$

$$\lambda_{ds}^r = L_d i_{ds}^r + \lambda_{af} \tag{33}$$

Using **equations 32** and **33**, the q and d axis voltage (**equation 16**) can be written as

$$v_{qs}^r = \frac{R_s}{L_q}\lambda_{qs}^r + p\lambda_{qs}^r + \omega_r \lambda_{ds}^r \tag{34}$$

$$v_{ds}^r = \frac{R_s}{L_q}\left(\lambda_{ds}^r - \lambda_{af}\right) + p\lambda_{ds}^r - \omega_r \lambda_{ds}^r \tag{35}$$

Using **equation 32** and **eq33** in **equation 28**, the electromagnetic torque can be written as

$$T_e = \frac{3}{2}\frac{P}{2}\left[\lambda_{ds}^r i_{qs}^r - \lambda_{qs}^r i_{ds}^r\right] \qquad (36)$$

Suggested Reading:

[1] D. C. Hanselman, *Brushless Permanent Magnet Motor Design*, Magna Physics Pub, 2006

[2] P. C. Krause, O. Wasynczuk, S. D. Sudhoff, *Analysis of electric machinery*, IEEE Press, 1995

Lecture 25: Control of PM machines

Control of PM machines

Introduction

The topics covered in this chapter are as follows:

- Control Strategies of PM Machines
- Constant Torque Angle Control
- Constant Mutual Air gap Flux Linkage Control
- Optimum Torque per Ampere Control

Control Strategies of PM Machines

There are various control strategies and depending on the application a suitable strategy can be chosen. For example, a mutual flux air gap linkages control gives a smooth transition to flux weakening above the base speed. Similarly, a maximum efficiency control is suitable for applications where energy saving is important such as hybrid and electric vehicles. The most commonly used control strategies are:

- **Constant torque angle control**
- Unity power factor control
- **Constant mutual air gap flux linkages control**
- Angle control of air gap flux and current phasors
- **Optimum torque per ampere control**
- Constant loss based maximum torque speed boundary control
- Minim loss or maximum efficiency control.

The control strategies marked in bold are discussed in the following sections.

Constant Torque Angle Control

Consider that the PM motor is supplied three phase currents given as follows:

$$\begin{aligned} i_{as} &= I_m \sin\left(\omega_r t + \delta\right) \\ i_{bs} &= I_m \sin\left(\omega_r t + \delta - \frac{2\pi}{3}\right) \\ i_{cs} &= I_m \sin\left(\omega_r t + \delta - \frac{4\pi}{3}\right) \end{aligned} \quad (1)$$

The q and d axes stator currents in the rotor reference frames are obtained through the transformation matrix as:

$$\begin{bmatrix} i_{qs}^r \\ i_{ds}^r \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \cos\omega_r t & \cos\left(\omega_r t - \frac{2\pi}{3}\right) & \cos\left(\omega_r t - \frac{4\pi}{3}\right) \\ \sin\omega_r t & \sin\left(\omega_r t - \frac{2\pi}{3}\right) & \sin\left(\omega_r t - \frac{4\pi}{3}\right) \end{bmatrix}\begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix}$$

$$= \frac{2}{3}\begin{bmatrix} \cos\omega_r t & \cos\left(\omega_r t - \frac{2\pi}{3}\right) & \cos\left(\omega_r t - \frac{4\pi}{3}\right) \\ \sin\omega_r t & \sin\left(\omega_r t - \frac{2\pi}{3}\right) & \sin\left(\omega_r t - \frac{4\pi}{3}\right) \end{bmatrix}\begin{bmatrix} I_m \sin\left(\omega_r t + \delta\right) \\ I_m \sin\left(\omega_r t + \delta - \frac{2\pi}{3}\right) \\ I_m \sin\left(\omega_r t + \delta - \frac{4\pi}{3}\right) \end{bmatrix} \quad (2)$$

$$= I_m \begin{bmatrix} \sin\delta \\ \cos\delta \end{bmatrix}$$

$$T_e = \frac{3}{2}\frac{N_p}{2}\left[\lambda_{af} i_{qs}^r + \left(L_d - L_q\right) i_{qs}^r i_{ds}^r\right] \quad (3a)$$

Substituting the values of i_{qs}^r and i_{ds}^r from **equation 2** into **equation 3a** gives

$$T_e = \frac{3}{2}\frac{N_p}{2}\left[\lambda_{af} I_m \sin\delta + \frac{1}{2}\left(L_d - L_q\right) I_m^2 \sin(2\delta)\right] \quad (3b)$$

Having developed the basic equations, we now focus on the ***Constant Torque Angle Control***. In this strategy the torque angle δ is maintained at 90^{o}. Hence, the above torque equation becomes:

$$T_e = \frac{3}{2}\frac{N_p}{2}\lambda_{af} I_m \quad (4)$$

The q and d axis voltage for the PM machine (***refer Lecture 24 equation 16***) is given by

$$\begin{bmatrix} v_{qs}^r \\ v_{ds}^r \end{bmatrix} = \begin{bmatrix} R_s + L_q p & \omega_r L_d \\ -\omega_r L_q & R_s + L_d p \end{bmatrix}\begin{bmatrix} i_{qs}^r \\ i_{ds}^r \end{bmatrix} + \begin{bmatrix} \omega_r \lambda_{af} \\ 0 \end{bmatrix} \quad (5)$$

Since the load angle $\delta = 90^o$, from **equation 2**, $i_{ds}^r = 0$ and $i_{qs}^r = I_m$ and **equation 5** can be written as:

$$\begin{bmatrix} v_{qs}^r \\ v_{ds}^r \end{bmatrix} = \begin{bmatrix} R_s + L_q p & \omega_r L_d \\ -\omega_r L_q & R_s + L_d p \end{bmatrix} \begin{bmatrix} I_m \\ 0 \end{bmatrix} + \begin{bmatrix} \omega_r \lambda_{af} \\ 0 \end{bmatrix}$$

$$v_{qs}^r = \left(R_s + L_q p\right) I_m + \omega_r \lambda_{af} \qquad (6)$$

$$v_{ds}^r = -\omega_r L_q I_m$$

For the analysis of the control strategy, it is convenient to convert **equation 4** and **equation 6** into per unit (p.u) values. The base values chosen are:

I_b base value of stator current

λ_{af} base value of magnet flux

ω_b base speed

V_b base voltage=$\lambda_{af}\omega_b$

R_b base resistance (7)

L_b base inductance

T_b base value of torque $= \frac{3}{2}\frac{N_p}{2}\lambda_{af}\omega_b$

$X_b$base value of reactance=$\omega_b L_b$

Using the base values given in **equation 7** the normalized can be written as

$$T_{en} = \frac{T_e}{T_b} = \frac{\frac{3}{2}\frac{N_p}{2}\lambda_{af} I_m}{\frac{3}{2}\frac{N_p}{2}\lambda_{af} I_b} = I_{mn} \qquad (8)$$

From **equation 8** it can be seen that the normalized torque (T_{en}) is equal to the normalized stator current I_{sn}. The voltage equation for steady state analysis can be obtained by making $p = 0$ (because in steady state the time variation is zero) in **equation 6** and is written as

$$v_{qs}^r = R_s I_m + \omega_r \lambda_{af};\ \ v_{ds}^r = -\omega_r L_q I_m \qquad (9)$$

$$v_{qs}^r = R_s I_m + \omega_r \lambda_{af};\ \ v_{ds}^r = -\omega_r L_q I_m$$

The magnitude of the stator voltage is given by

$$V_s = \sqrt{\left(v_{qs}^r\right)^2 + \left(v_{ds}^r\right)^2} \qquad (10)$$

The normalized stator voltage is obtained as

$$V_{sn} = \frac{V_s}{V_b} = \frac{V_s}{\omega_b \lambda_{af}} = \sqrt{\left(R_{sn} I_{mn} + \omega_{rn}\right)^2 + \left(\omega_{rn} L_{qn} I_{mn}\right)^2} \quad (11)$$

The phasor diagram for this control strategy is shown in **Figure 1**. From this figure the power factor is obtained as

$$\cos\phi = \frac{v_{qs}^r}{V_s} = \frac{v_{qs}^r}{\sqrt{\left(v_{qs}^r\right)^2 + \left(v_{ds}^r\right)^2}} = \frac{1}{\sqrt{1+\left(\frac{v_{ds}^r}{v_{qs}^r}\right)^2}} = \frac{1}{\sqrt{1+\left(\frac{L_{qn} I_{sn}}{1+\frac{R_{sn} I_{sn}}{\omega_{rn}}}\right)^2}} \quad (12)$$

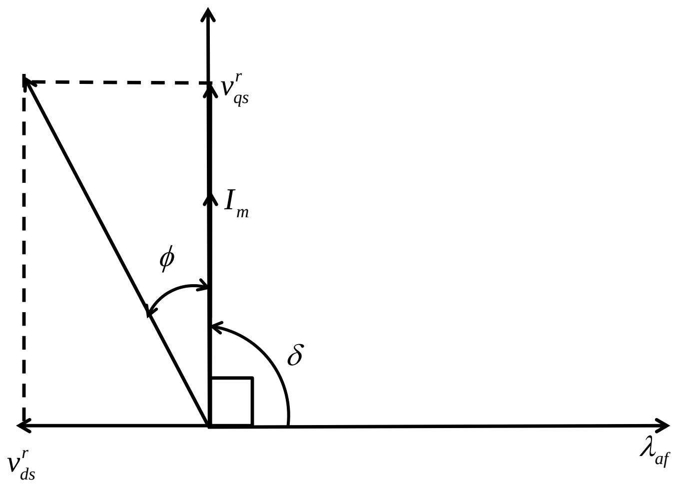

Fig 1: The Phasor Diagram of PM machine with Constant Torque Angle Control

The **equation 12** shows that the power factor deteriorates as the rotor speed goes up. The maximum rotor speed with this control strategy can be obtained by solving **equation 11** for ω_{rn}, neglecting the stator resistive drop ($R_{sn} I_{mn} \approx 0$), and is given as

$$\omega_{rn(\max)} = \frac{V_{sn(\max)}}{\sqrt{1+\left(L_{qn} I_{sn}\right)^2}} \quad (13)$$

Assuming that the motor is driven by a three phase DC-AC converter, the maximum voltage is given by (refer **Lecture 15**):

$$V_{sn(\max)} = \frac{\sqrt{2} \times 0.45 V_{dc}}{V_b} \quad (14)$$

The performance characteristics of the PM machine are shown in **Figure 2**. The parameters of the machine for a speed of 1p.u. (ω_{rn} =1) used to plot the curves are given in **Table 1**. From the **Figure 2** the following can be observed:

- The power factor falls as the current rises.
- The torque is proportional to the current as is evident from **equation 8**.
- The normalized increases with the increase in current. The impedance of the machine remains constant because its speed is constant at 1p.u. hence, when the current through the machine has to increase the applied voltage also has to increase (**equation 11**).

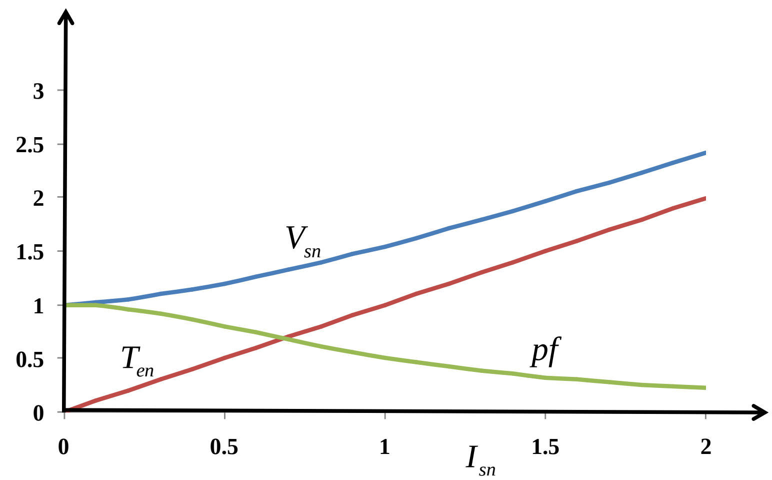

Fig 2: The characteristics curves of PM machine with Constant Torque Angle Control

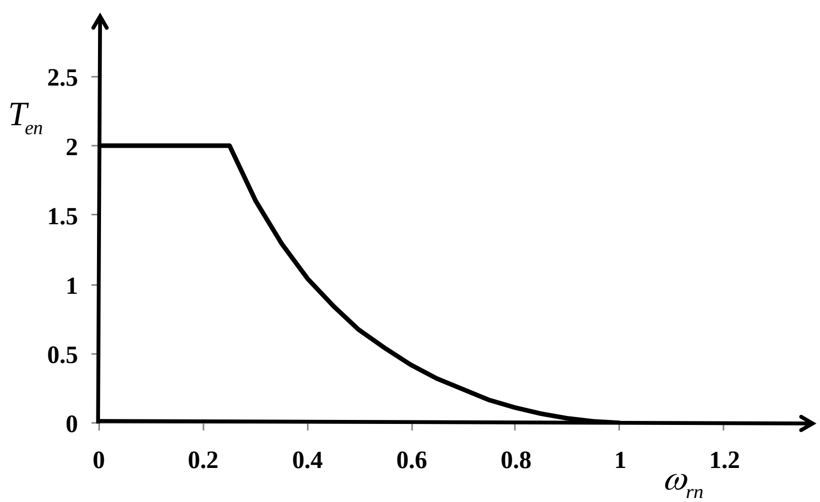

Fig 3: The torque vs. speed characteristic curves of PM machine with Constant Torque Angle Control

Table 1: The parameters of a salient pole PM machine

Base Voltage (V_b)	Base Current (I_b)	Base Inductance (L_b)	Base Speed (ω_b)	Base Flux Linkage (λ_b)	Resistance (R)	d-axis inductance (L_d)	q-axis inductance (L_q)
100 V	10 A	0.02 H	600 rad/s	0.167 Vs/rad	1.8 Ω	0.011 H	0.022 H

The torque vs. speed curve for the machine, whose parameters are given in **Table 1**, is shown in **Figure 3**. In determining the curve it has been assumed that the magnitude of the normalized stator voltage (V_{sn}) is 1 p.u. and the maximum value of normalized stator current (I_{mn}) is fixed to 2 p.u. From the **Figure 3** the following can be observed:

- Through this control strategy, the PM machine is able to produce 2 p.u. torque up to a speed of 0.25 p.u.
- The machine is able to produce 1 p.u. torque up to a speed of 0.4 p.u.

Constant Mutual Flux Linkage Control

In this control strategy, the resultant flux linkage of the stator q and d axis and rotor is maintained constant. The main advantage of this control strategy is that it keeps the stator voltage requirement is kept low. To start with the analyses consider the flux linkage expression (***Lecture 24***, ***equations 32*** and ***33***) for the q and d axis:

$$\lambda_{qs}^r = L_q i_{qs}^r \tag{15}$$

$$\lambda_{ds}^r = L_d i_{ds}^r + \lambda_{af} \tag{16}$$

The magnitude of the flux linkage is given by

$$\lambda_m = \sqrt{\left(\lambda_{qs}^r\right)^2 + \left(\lambda_{ds}^r\right)^2} = \sqrt{\left(L_q i_{qs}^r\right)^2 + \left(L_d i_{ds}^r + \lambda_{af}\right)^2} \tag{17}$$

In this strategy, the mutual flux linkage given by **equation 17** is held constant and its magnitude is made equal to λ_{af}. Substituting the values of i_{ds}^r and i_{qs}^r from **equation 2** into **equation 17** gives

$$\lambda_m = \lambda_{af} = \sqrt{\left(L_q I_m \sin\delta\right)^2 + \left(L_d I_m \cos\delta + \lambda_{af}\right)^2} \tag{18}$$

Solving **equation 18** for I_m gives

$$I_m = -\frac{2\lambda_{af}}{L_d}\left(\frac{\cos\delta}{\cos^2\delta + \rho^2 \sin^2\delta}\right) \tag{19}$$

where $\rho = \frac{L_q}{L_d}$

The normalized current is given bu

$$I_{mn} = \frac{I_m}{I_b} = -\frac{2}{L_{dn}}\left(\frac{\cos\delta}{\cos^2\delta + \rho^2 \sin^2\delta}\right) \tag{20}$$

The stator voltage is given

$$V_s = \sqrt{\left(v_{qs}^r\right)^2 + \left(v_{ds}^r\right)^2} = \sqrt{\left(R_s i_{qs}^r + \omega_r L_d i_{ds}^r + \omega_r \lambda_{af}\right)^2 + \left(-\omega_r L_q i_{qs}^r + R_s i_{ds}^r\right)^2} \tag{21}$$

The normalized values of the stator voltage is

$$V_{sn} = \frac{V_s}{V_b} = \frac{\sqrt{\left(R_s i_{qs}^r + \omega_r L_d i_{ds}^r + \omega_r \lambda_{af}\right)^2 + \left(-\omega_r L_q i_{qs}^r + R_s i_{ds}^r\right)^2}}{\omega_b \lambda_{af}}$$

$$= \sqrt{\left(R_{sn} i_{qsn}^r + \omega_{rn} L_{dn} i_{dsn}^r + \omega_{rn}\right)^2 + \left(-\omega_{rn} L_{qn} i_{qsn}^r + R_{sn} i_{dsn}^r\right)^2} \tag{22}$$

Using **equation 2**, the normalized voltage given by **equation 25** can be written as

$$V_{sn} = \sqrt{\left(R_{sn} I_{mn} \sin\delta + \omega_{rn} L_{dn} I_{mn} \cos\delta + \omega_{rn}\right)^2 + \left(\omega_{rn} L_{qn} I_{mn} \sin\delta + R_{sn} I_{mn} \cos\delta\right)^2} \tag{23}$$

In order to determine the value of angle δ, two distinct cases have to be considered: when $\rho = 1$ and $\rho \neq 1$. Once the angle δ is known, the torque can be obtained from **equation 3**.

Each of these cases are explained in the following subsections.

Case when $\rho = 1$

Substituting $\rho = 1$ into **equation 20** and solving for δ gives

$$\delta = \cos^{-1}\left(\frac{-L_{dn} I_{mn}}{2}\right) \tag{24}$$

The torque produced by the machine is given by

$$T_e = \frac{3}{2}\frac{N_p}{2}\left[\lambda_{af} I_m \sin\delta\right] \tag{25}$$

The normalized torque is given by

$$T_{en} = \frac{T_e}{T_b} = I_{mn} \sin\delta \tag{26}$$

The performance characteristics of a PM machine at a speed of 1 p.u. are shown in **Figure 4a** and the parameters of this machine are given in **Table 2**. The torque versus the speed characteristics of the PM Machine are shown in **Figure 4b**.

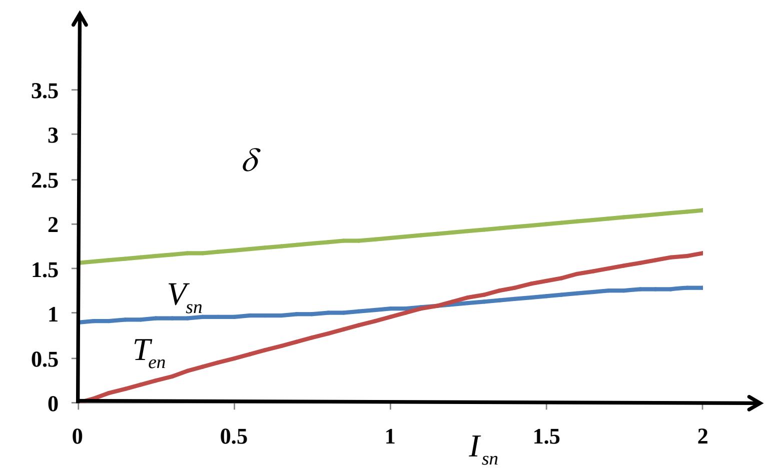

Fig 4a: The characteristics curves of PM machine ($\rho = 1$) with Constant Mutual Flux Linkage Control

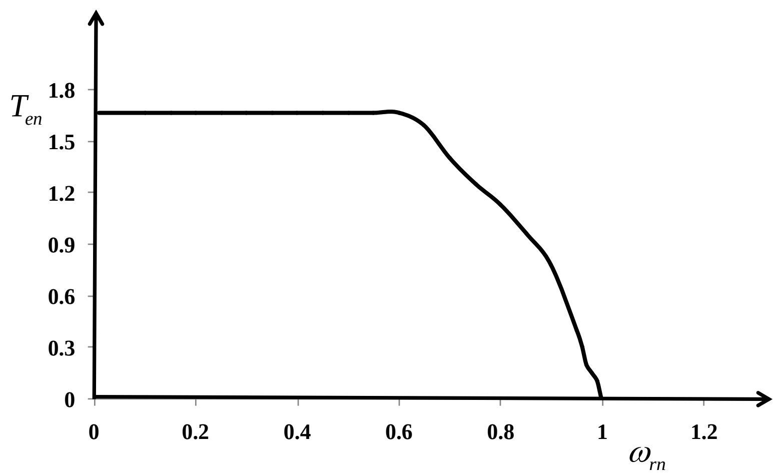

Fig 4b: The torque vs. speed characteristic curves of PM machine ($\rho = 1$) with Constant Mutual Flux Linkage Control

Table 2: The parameters of a non salient pole PM machine

Base Voltage (V_b)	**Base Current (I_b)**	**Base Inductance (L_b)**	**Base Speed (ω_b)**	**Base Flux Linkage (λ_b)**	**Resistance (R)**	**d-axis inductance (L_d)**	**q-axis inductance (L_q)**
100 V	10 A	0.02 H	600 rad/s	0.167 Vs/rad	1.8 Ω	0.011 H	0.011 H

Case when $\rho \neq 1$

When $\rho \neq 1$, using **equation 20** the expression for δ is obtained as

$$\delta = \cos^{-1}\left[\frac{-1}{L_{dn}I_{mn}(1-\rho)^2} \pm \sqrt{\left\{\frac{1}{L_{dn}\left(1-\rho^2\right)I_{sn}}\right\}^2 - \frac{\rho^2}{\left(1-\rho^2\right)}}\right] \quad (27)$$

The performance characteristics of the machine, whose parameters are given in **Table 1**, are shown in **Figure 5a** and the torque versus speed characteristics is shown in **Figure 5b**.

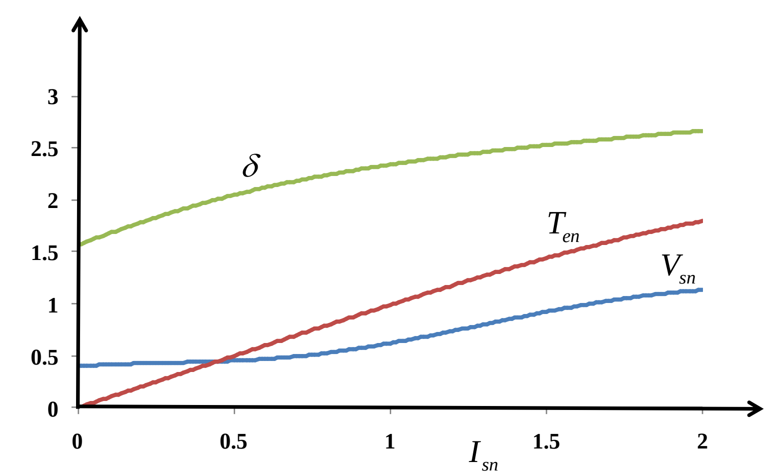

Fig 5a: The characteristics curves of PM machine ($\rho \neq 1$) with Constant Mutual Flux Linkage Control

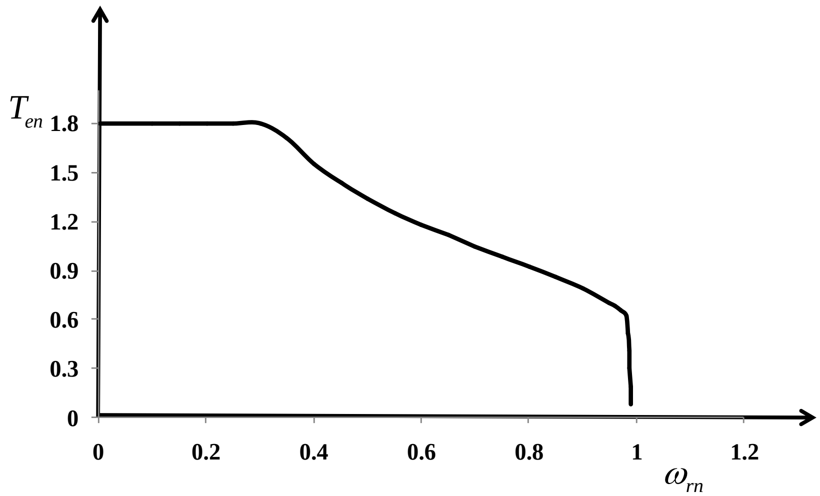

Fig 5b: The torque vs. speed characteristic curves of PM machine ($\rho \neq 1$) with Constant Mutual Flux Linkage Control

Optimum Torque per Unit Current Control

The aim of this control strategy is to maximize electromagnetic torque for a unit stator current. By using this strategy the PM machine will produce maximum torque for a given magnitude of current. To develop the mathematical models of this strategy, consider the torque equation of the PM machine given in **equation 3** and normalize it into p.u. system. The normalized torque expression is

$$T_{en} = \frac{T_e}{T_b} = I_{mn}\left[\sin(\delta) + \frac{1}{2}\left(L_{dn} - L_{qn}\right) I_{mn} \sin(2\delta)\right] \tag{28}$$

The torque per unit stator current is defined as

$$\frac{T_{en}}{I_{mn}} = \left[\sin(\delta) + \frac{1}{2}\left(L_{dn} - L_{qn}\right) I_{mn} \sin(2\delta)\right] \tag{29}$$

The condition under which the machine produces maximum torque per unit stator current is obtained by differentiating **equation 29** with respect to δ and equating it to zero, that is

$$\frac{d\left[\sin(\delta) + \frac{1}{2}\left(L_{dn} - L_{qn}\right) I_{mn} \sin(2\delta)\right]}{d\delta} = 0 \tag{30}$$

$$\Rightarrow \cos(\delta) + \left(L_{dn} - L_{qn}\right) I_{mn} \cos(2\delta) = 0$$

Using the trigonometric identity $\cos(2\delta) = 2\cos^2(\delta) - 1$ in **equation 30** gives

$$\cos(\delta) + \left(L_{dn} - L_{qn}\right) I_{mn} \left[2\cos^2(\delta) - 1\right] = 0 \tag{31}$$

The solution of **equation 31** gives

$$\delta = \cos^{-1}\left[-\frac{1}{4K} \pm \sqrt{\left(\frac{1}{4K}\right)^2 + \frac{1}{2}}\right] \tag{32}$$

where $K = \dfrac{1}{\left(L_{dn} - L_{qn}\right) I_{mn}}$

In **equation 32**, only the value of δ greater than 90^o is considered so as to reduce the field in the air gap. The performance characteristics of the PM machine (parameters of the machine are given in **Table 1**) for this control strategy are shown in **Figure 6a** and the torque versus speed characteristics are shown in **Figure 6b**.

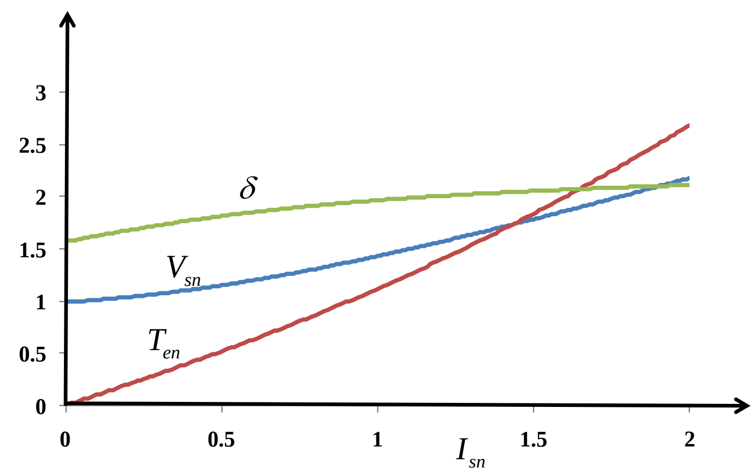

Fig 6a: The characteristics curves of PM machine with Optimum Torque per Unit Current Control

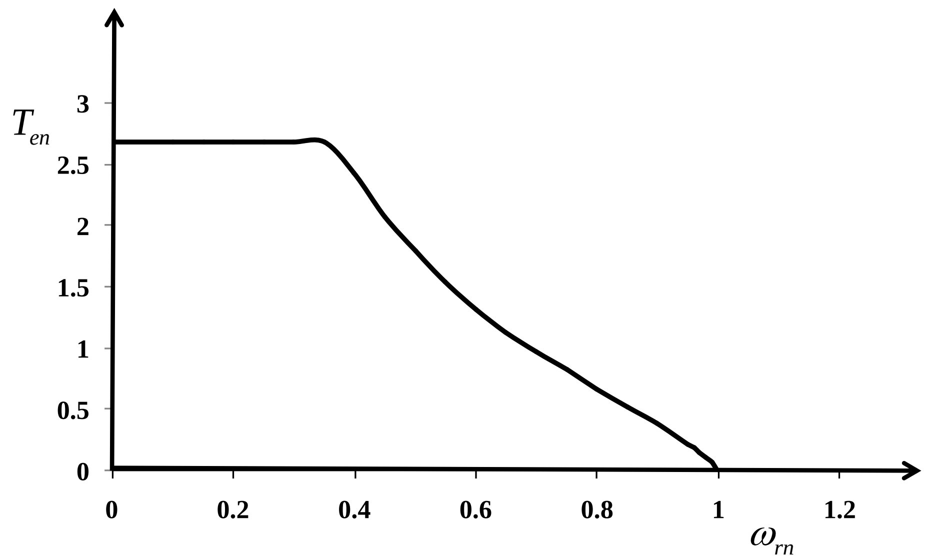

Fig 6b: The torque vs. speed characteristic of PM machine with Optimum Torque per Unit Current Control

Suggested Reading:

[1] R. Krishnan, *Electric motor drives: modeling, analysis, and control*, Prentice Hall, 2001

[2] P. C. Krause, O. Wasynczuk, S. D. Sudhoff, *Analysis of electric machinery*, IEEE Press, 1995

Lecture 26: Flux Weakening Control of PM machines

Flux Weakening Control of PM machines

Introduction

The topics covered in this chapter are as follows:

- Need for Flux Weakening Control of PM Machine
- Maximum Speed of PM Machine
- The Flux Weakening Algorithm
- The Implementation of Flux Weakening

Need for Flux Weakening Control of PM Machine

The PM machines are driven by the DC-AC converter and the converter is fed by a DC source such as a battery. This DC source has a fixed voltage, hence the motor input voltage and current ratings are limited. The voltage and current limits influence the maximum speed with rated torque capability and the maximum torque producing capability of the motor. In EV and HEV applications, it is required to produce rated power at as high a speed as possible. For a given DC source voltage the machine produces torque up to a speed known as ***base speed***. Above base speed, the induced emf in the machine will exceed the maximum input voltage and no currents will flow of into machine phases. To overcome this situation, the induced e.m.f is constrained to be less than the applied voltage by weakening the air gap flux linkages. In principle, the flux weakening is made to be inversely proportional to the stator frequency so that the induced e.m.f is constant and will not increase with the increasing speed. Such a control strategy is known as ***flux weakening control***.

Similarly, a maximum efficiency control is suitable for applications where energy saving is important

Maximum Speed of PM Machine

In order to determine the maximum speed of the PM, consider the q and d axis stator voltages in p.u.:

$$\begin{aligned} v_{qsn}^{r} &= R_{sn} i_{qsn}^{r} + L_{qn} p i_{qsn}^{r} + \omega_{rn} L_{dn} i_{dsn}^{r} + \omega_{rn} \\ v_{dsn}^{r} &= -\omega_{rn} L_{qn} i_{qsn}^{r} + R_{sn} i_{dsn}^{r} + L_{dn} p i_{dsn}^{r} \end{aligned} \quad (1)$$

In steady state operation the derivative of the currents is zero, hence the voltage equation becomes:

$$\begin{aligned} v_{qsn}^r &= R_{sn} i_{qsn}^r + \omega_{rn} L_{dn} i_{dsn}^r + \omega_{rn} \\ v_{dsn}^r &= -\omega_{rn} L_{qn} i_{qsn}^r + R_{sn} i_{dsn}^r \end{aligned} \qquad (2)$$

The maximum speed is attained by reducing the air gap field drastically and all the stator current is utilized for that purpose. This is done by making the q axis current equal to zero, hence the stator voltage is given by

$$\begin{aligned} v_{qsn}^r &= \omega_{rn} L_{dn} i_{dsn}^r + \omega_{rn} \\ v_{dsn}^r &= R_{sn} i_{dsn}^r \end{aligned} \qquad (3)$$

and the stator voltage phasor is given by

$$V_{sn}^2 = \left(v_{qsn}^r\right)^2 + \left(v_{dsn}^r\right)^2 = \omega_{rn}^2 \left(L_{dn} i_{dsn}^r + 1\right)^2 + \left(R_{sn} i_{dsn}^r\right)^2 \qquad (4)$$

From **equation 4**, the maximum speed for a given stator current magnitude of i_{dsn}^r is

$$\omega_{rn\_\max} = \frac{\sqrt{V_{sn}^2 - R_{sn}^2 \left(i_{dsn}^r\right)^2}}{\left(1 + L_{dn} i_{dsn}^r\right)} \qquad (5)$$

From **equation 5** it can be seen that the denominator has to be greater than zero. This sets the condition for the maximum stator current and it is equal to

$$i_{dsn}^r < -\frac{1}{L_{dn}} \qquad (6)$$

The Flux Weakening Algorithm

The voltage and current phasors for the PM machine under steady state operation is given by

$$V_{sn}^2 = \left(v_{qsn}^r\right)^2 + \left(v_{dsn}^r\right)^2 = \left(R_{sn} i_{qsn}^r + \omega_{rn} L_{dn} i_{dsn}^r + \omega_{rn}\right)^2 + \left(-\omega_{rn} L_{qn} i_{qsn}^r + R_{sn} i_{dsn}^r\right)^2 \qquad (7)$$

$$I_{mn}^2 = \left(i_{qsn}^r\right)^2 + \left(i_{dsn}^r\right)^2 \qquad (8)$$

Substituting the value of i_{qsn}^r from **equation 8** into **equation 7** gives

$$V_{sn}^2 = \left(v_{qsn}^r\right)^2 + \left(v_{dsn}^r\right)^2 = \left(R_{sn} \left(I_{mn}^2 - (i_{dsn}^r)^2\right) + \omega_{rn} L_{dn} i_{dsn}^r + \omega_{rn}\right)^2 + \left(-\omega_{rn} L_{qn} \left(I_{mn}^2 - (i_{dsn}^r)^2\right) + R_{sn} i_{dsn}^r\right)^2 \qquad (9)$$

Rewriting the voltage phasor as a polynomial of i_{dsn}^r and neglecting the stator resistance gives

$$V_{sn}^2 = \omega_{rn}\left(K_1\left(i_{dsn}^r\right)^2 + K_2 i_{dsn}^r + K_3\right)$$
$$\text{where} \qquad (10)$$
$$K_1 = L_{dn}^2 - L_{qn}^2;\ K_2 = 2L_{dn};\ K_3 = 1 + L_{qn}^2 I_{mn}^2$$

Solving **equation 10** for i_{dsn}^r gives

$$i_{dsn}^r = -\frac{L_{dn}}{L_{dn}^2 - L_{qn}^2} \pm \sqrt{\frac{V_{sn}^2}{\left(L_{dn}^2 - L_{qn}^2\right)\omega_{rn}^2} + \frac{L_{qn}^2 I_{mn}^2}{\left(L_{dn}^2 - L_{qn}^2\right)} - \frac{L_{qn}^2}{\left(L_{dn}^2 - L_{qn}^2\right)}} \qquad (11)$$

In case the rotor speed is given, using **equation 11**, the $d-$axis current (i_{dsn}) can be obtained which would satisfy the constraints of maximum stator current (I_{mn}) **and** stator voltage (V_{sn}) Once the value of i_{dsn}^r is known then for the given maximum I_{mn} the value of i_{qsn}^r can be determined using **equation 8**. Once the values of i_{dsn}^r and i_{qsn}^r are known, the three phase currents $i_{asn}, i_{bsn}, i_{csn}$ can be determined using the inverse transformation as

$$\begin{bmatrix} i_{asn} \\ i_{bsn} \\ i_{csn} \end{bmatrix} = \begin{bmatrix} \cos\theta_r & \sin\theta_r \\ \cos\left(\theta_r - \frac{2\pi}{3}\right) & \sin\left(\theta_r - \frac{2\pi}{3}\right) \\ \cos\left(\theta_r + \frac{2\pi}{3}\right) & \sin\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix} \begin{bmatrix} i_{qsn}^r \\ i_{dsn}^r \end{bmatrix} \qquad (12)$$

Using the relations $i_{qsn}^r = I_{mn}\sin\delta$ and $i_{dsn}^r = I_{mn}\cos\delta$ (*refer **equation 2** in **Lecture 25***) in **equation 12** results in

$$\begin{bmatrix} i_{asn} \\ i_{bsn} \\ i_{csn} \end{bmatrix} = \begin{bmatrix} \sin\left(\theta_r + \delta\right) \\ \sin\left(\theta_r + \delta - \frac{2\pi}{3}\right) \\ \sin\left(\theta_r + \delta - \frac{4\pi}{3}\right) \end{bmatrix} I_{mn} \qquad (13)$$

and the torque angle is given by

$$\delta = \tan^{-1}\left(\frac{i_{qsn}^r}{i_{dsn}^r}\right) \qquad (14)$$

The torque produced by the machine is given by (*refer **equation 3a** in **Lecture 25***)

$$T_e = \frac{3}{2}\frac{N_p}{2}\left[\lambda_{af} i_{qs}^r + \left(L_d - L_q\right) i_{qs}^r i_{ds}^r\right] \tag{15a}$$

The torque in p.u. is given by

$$T_{en} = \left[i_{qsn}^r + \left(L_{dn} - L_{qn}\right) i_{qsn}^r i_{dsn}^r\right] \tag{15b}$$

Having established all the relevant, the basic philosophy of field weakening current can be explained as:

i. For a given limit on the maximum stator current I_{mn} and the maximum stator voltage V_{sn}, i_{dsn}^r can be calculated using **equation 11**.

ii. The value of i_{qsn}^r is determined using the **equation 8** and the value of i_{dsn}^r obtained in ***step i***.

iii. Once the values of i_{qsn}^r and i_{dsn}^r are known, the torque that could be produced by the PM ($T_{e\max}$) machine can be calculated using **equation 15**.

iv. In case the requested torque (T_{ereq}) is greater than $T_{e\max}$, then the final requested torque (T_e^*) is made equal to $T_{e\max}$ else it is equal to T_{ereq}. Mathematically this can be written as

$$\begin{aligned} &if\ \ T_{ereq} > T_{e\max} \\ &then\ \ T_e^* = T_{e\max} \\ &else\ \ T_e^* = T_{ereq} \end{aligned}$$

v. After having determined the value of final requested torque, the $q-$axis current (i_{qsn}^r) is calculated using **equation 15** as:

$$i_{qsn}^r = \frac{T_e^*}{1 + \left(L_{dn} - L_{qn}\right) i_{dsn}^r} \tag{16}$$

The Implementation of Flux Weakening

An implementation of the flux weakening control strategy is shown in **Figure 1**. From **Figure 1** it can be seen that there are two distinct blocks namely:

i. Constant torque mode controller
ii. Flux weakening controller

The working of the control strategy is as follows:

i. Based on the error between the reference speed (ω_{rn}^*) and the measured speed (ω_{rn}), the required torque T_{ereq} is calculated in the ***Speed Controller***.
ii. As long as the measured speed of the motor is less than the ***base speed*** (ω_{base}), the ***Constant Torque Mode Controller*** operates. Once the measured speed exceeds the base speed, the ***Flux Weakening Controller*** comes into action.
iii. The output of the two controllers explained in ***step ii*** is the required magnitude of the stator current (I_{mn}^*) and the load angle (δ^*).
iv. Having known the values of I_{mn}^* and δ^*, the q and d axis currents ($i_{qsn}^{r*}, i_{dsn}^{r*}$) are calculated using ***equation 2*** *in* ***Lecture 25***. From the calculated values of i_{qsn}^{r*} and i_{dsn}^{r*} the values of three phase currents ($i_{asn}^*, i_{bsn}^*, i_{csn}^*$) are calculated using **equation 12** or **13**. These thee phase currents are in turn multiplied by base value of the current (I_b) and the required values of the currents ($i_{as}^*, i_{bs}^*, i_{cs}^*$) are obtained. All these calculations are done in the ***Transformation Block***.
v. The error between $i_{as}^*, i_{bs}^*, i_{cs}^*$ and the measured three phase currents (i_{as}, i_{bs}, i_{cs}) is then fed to the ***DC-AC Controller*** and the switching logics for the gates are calculated using PWM technique (refer ***Lecture 16***).
vi. These switching logic are fed to the DC-AC converter and this produced the required voltage to motor.

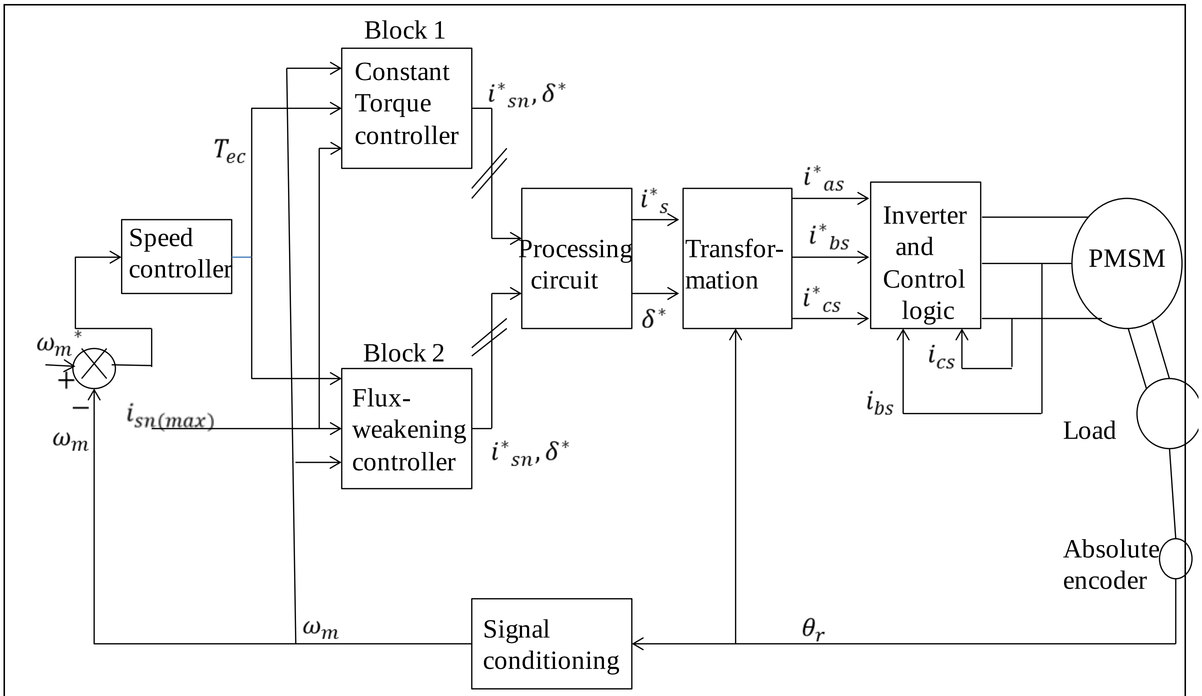

Figure 1: Field weakening control of PMSM [1]

The detailed explanation of the ***Constant Torque Mode Controller*** and ***Flux Weakening Controller*** are given in the subsequent sections.

The Constant Torque Mode Controller

In this block the **Optimum torque per ampere control** (refer ***Lecture 25***) is implemented. Here, based on T_{ereq} the values of I^*_{mn} and δ^* are calculated. This is done by using curve fitting the characteristics for online computation or programming it in the memory of the microcontroller. To understand this consider the PM machine whose parameters are given in **Table I.** For this machine the Torque (T_e) versus I_{mn} and δ characteristics are calculated using ***equation 3a*** and ***equation 32*** given in ***Lecture 26*** and in **Figure 2**.The characteristics curves are approximated by third order polynomial and are written as

$$I_{mn} = 0.015T_e^3 - 0.153T_e^2 + 1.05T_e - 0.003 \quad (17)$$

$$\delta = 0.038T_e^3 - 0.242T_e^2 + 0.578T_e + 1.572 \quad (18)$$

The **equations 17** and **18** are implemented in form of look-up table in the microcontrollers and their implementation is shown **Figure 3**.

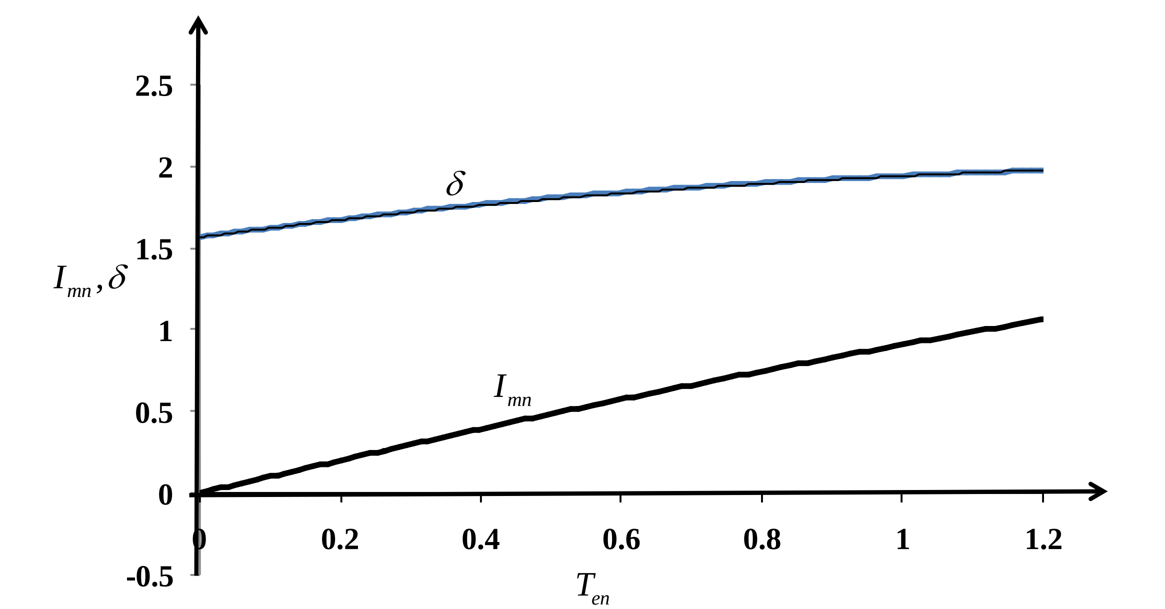

Figure 2: Stator current magnitude and torque angle versus torque for PMSM

Table1: Parameters of the motor

Resistance	0.18	p.u.
q-axis inductance	1.1	p.u.
d-axis inductance	0.55	p.u.
e.m.f constant (λ)	0.167	V/rad/s

The details of the controller shown in **Figure 3** are:

i. The maximum stator current limit ($I_{mn(\max)}$) is given to the ***maximum torque*** block. In this block the Torque versus Current characteristic of the motor is implemented as lookup table or as a polynomial expression. The Torque versus Current characteristic of the PM machine parameters given in **Table I** is shown in **Figure 4**. The output of this block is the maximum possible torque ($T_{en(\max)}$) that the motor can produce for the given value of $I_{mn(\max)}$.

ii. The measured speed of the motor (ω_{rn}) is required to determine whether the motor operates is clockwise or anticlockwise direction.

iii. In the comparator block the torques $T_{en(\max)}$ and T_{ereq} are compared and output of this block is the minimum of the two values. This torque is referred to as T_{ereq}^{*} and it is equal to $\min\left(T_{ereq}, T_{en(\max)}\right)$.

iv. The value of T^*_{ereq} goes to the ***current block*** and ***angle block***. In these blocks the **equations 17** and **18** are implemented as polynomials or lookup tables. The output of these blocks are I^*_{mn} and δ^*.

v. The values of I^*_{mn} and δ^* go to the transformation block and the further operation of the control strategy is already explained in the previous section.

Once the PM machine reaches the base speed, the flux weakening controller comes into action. The next section deals with the flux weakening controller.

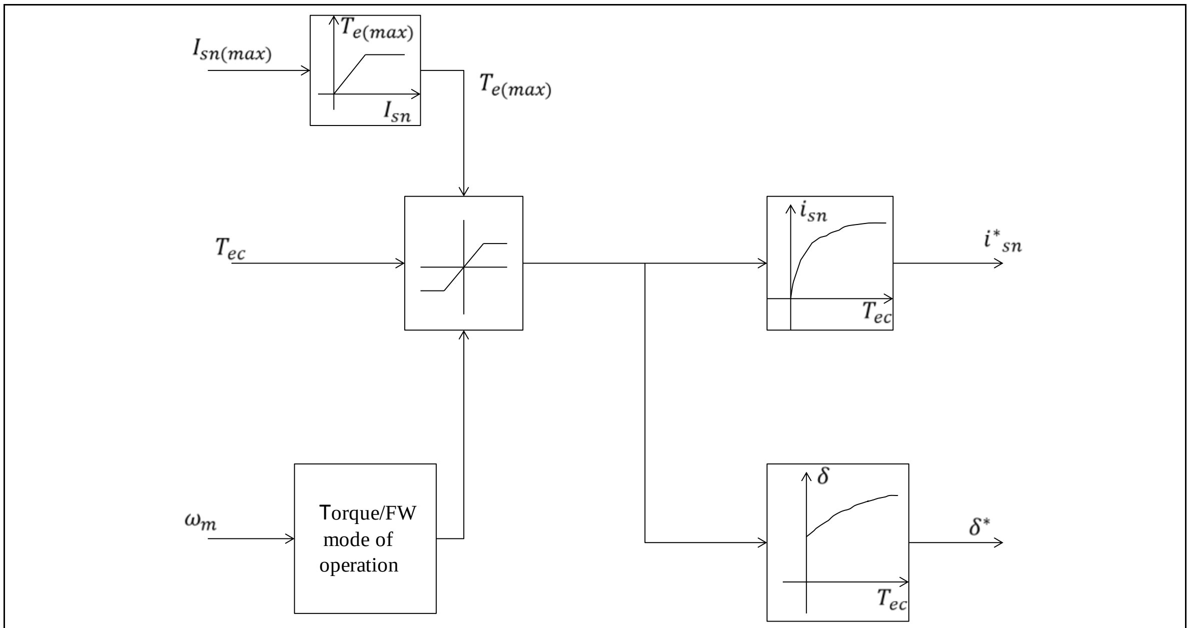

Figure 3: Constant torque mode controller [1]

The Flux Weakening Controller

The schematic of the Flux Weakening Controller is shown in **Figure 4**. There are three inputs to the controller namely:

- The requested torque (T_{ereq})
- The measured speed (ω_{rn})
- The maximum possible stator current ($I_{mn(\max)}$)

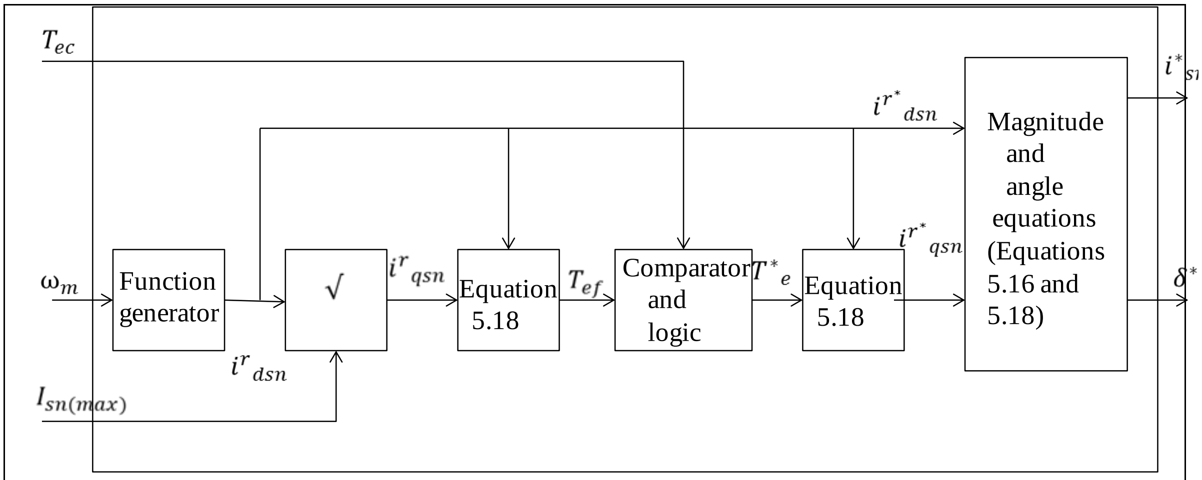

Figure 4: Schematic diagram of flux weakening controller [1]

Based on these inputs, the steps involved in the operation of the controller are as follows:

i. The three inputs are used to calculate the values of i^r_{dsn}. In the ***Function 1*** block the **equation 11** is implemented. In this block the values of L_{dn} and L_{qn} have to be known in advance and can be obtained from measurements. The value of V_{sn} is set to the maximum value of voltage (in p.u.) that the power supply can deliver.

ii. After the value of i^r_{dsn} is determined, the value of i^r_{qsn} is determined using $i^r_{qsn} = \sqrt{\left(I_{mn}\right)^2 - \left(i^r_{dsn}\right)^2}$ This is implemented in the ***Function 2*** block.

iii. Once the values of i^r_{qsn} and i^r_{dsn} are known, the maximum torque ($T_{en(\max)}$) that the PM machine can produce is determined using **equation 15b**.

iv. In the comparator block the $T_{en(\max)}$ and T_{ereq} are compared and the final torque value (T^*_{ereq}) that the machine has to produce is generated. The comparison is done using the following logic:

$$if\ \ T_{req} \geq T_{en(\max)}\ \ then\ T^*_{ereq} = T_{en(\max)}$$
$$else\ T^*_{ereq} = T_{req}$$

v. Using the value of T^*_{ereq} the actual value of i^{r*}_{qsn} is calculated using **equation 16**. In **equation 16,** the value of i^{r*}_{dsn} used is same as that calculated in ***step i***.

vi. The magnitude of stator current I^*_{mn} and the load angle δ^* are determined using **equations** 8 and **14**.

vii. The values of I^*_{mn} and δ^* go to the transformation block and the further operation of the control strategy is already explained in the previous section.

References:

[1] R. Krishnan, *Electric motor drives: modeling, analysis, and control*, Prentice Hall, 2001

Suggested Reading:

[1] P. C. Krause, O. Wasynczuk, S. D. Sudhoff, *Analysis of electric machinery*, IEEE Press, 1995