Module 4: DC-DC Converters

Lec 9: DC-DC Converters for EV and HEV Applications

DC-DC Converters for EV and HEV Applications

Introduction

The topics covered in this chapter are as follows:

- EV and HEV configuration based on power converters
- Classification of converters
- Principle of Step Down Operation
- Buck Converter with RLE Load
- Buck Converter with RL Load and Filter

Electric Vehicle (EV) and Hybrid Electric Vehicle (HEV) Configurations

In **Figure 1** the general configuration of the EV and HEV is shown. Upon examination of the general configurations it can be seen that there are two major power electronic units

- DC-DC converter
- DC-AC inverter

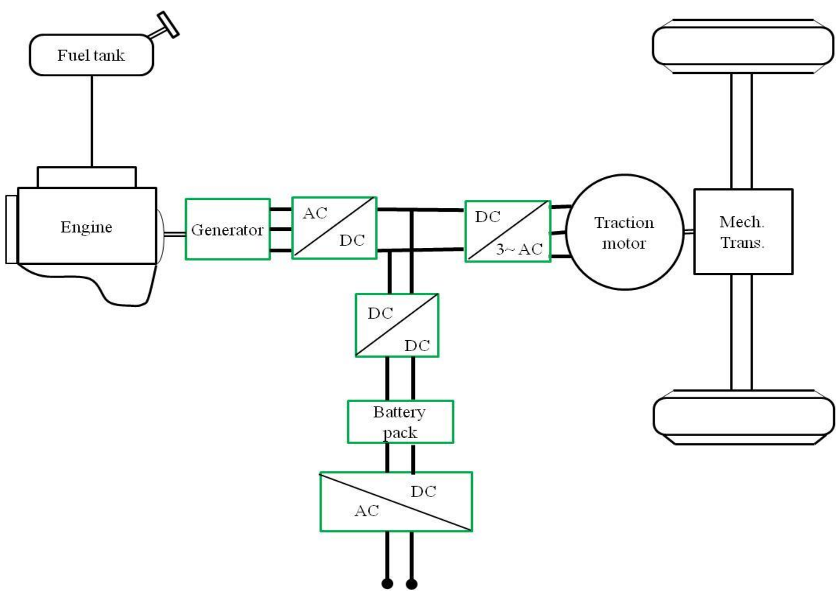


Figure 1:General Configuration of a Electric Vehicle [1]

Usually AC motors are used in HEVs or EVs for traction and they are fed by ***inverter*** and this inverter is fed by DC-DC converter (F**igure 1**). The most commonly DC-DC converters used in an HEV or an EV are:

- **Unidirectional Converters:** They cater to various onboard loads such as sensors, controls, entertainment, utility and safety equipments.
- **Bidirectional Converters:** They are used in places where battery charging and regenerative braking is required. The power flow in a bi-directional converter is usually from a low voltage end such as battery or a supercapacitor to a high voltage side and is referred to as ***boost operation***. During regenerative braking, the power flows back to the low voltage bus to recharge the batteries know as ***buck mode*** operation.

Both the unidirectional and bi-directional DC-DC converters are preferred to be ***isolated*** to provide safety for the lading devices. In this view, most of the DC-DC converters incorporate a high frequency transformer.

Classification of Converters

The converter topologies are classified as:

- **Buck Converter:** In **Figure 2a** a buck converter is shown. The buck converter is ***step down*** converter and produces a lower average output voltage than the dc input voltage.
- **Boost converter:** In **Figure 2b** a boost converter is shown. The output voltage is always greater than the input voltage.
- **Buck-Boost converter:** In **Figure 2c** a buck-boost converter is shown. The output voltage can be either higher or lower than the input voltage.

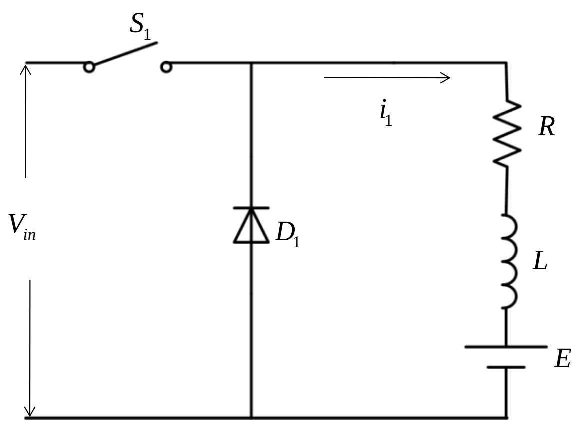


Figure 2a: General Configuration Buck Converter

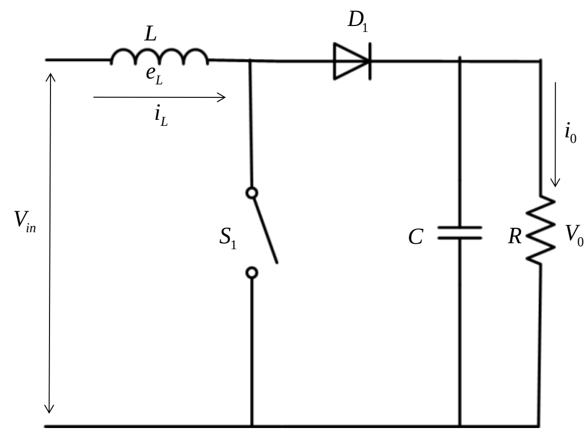


Figure 2b: General Configuration Boost Converter

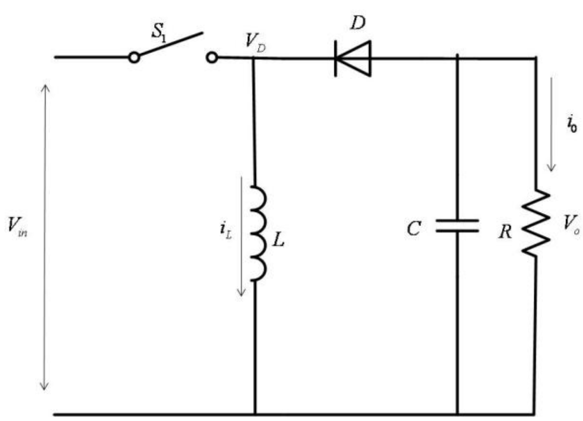


Figure 2c: General Configuration Buck-Boost Converter

Principle of Step Down Operation

The principle of step down operation of DC-DC converter is explained using the circuit shown in **Figure 3a**. When the switch S_1 is closed for time duration T_1, the input voltage V_{in} appears across the load. For the time duration T_2 is switch S_1 remains open and the voltage across the load is zero. The waveforms of the output voltage across the load are shown in **Figure 3b**.

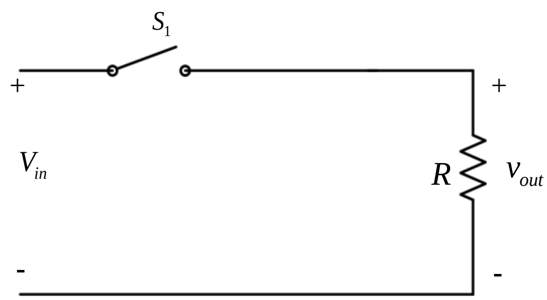


Figure 3a: Step down operation

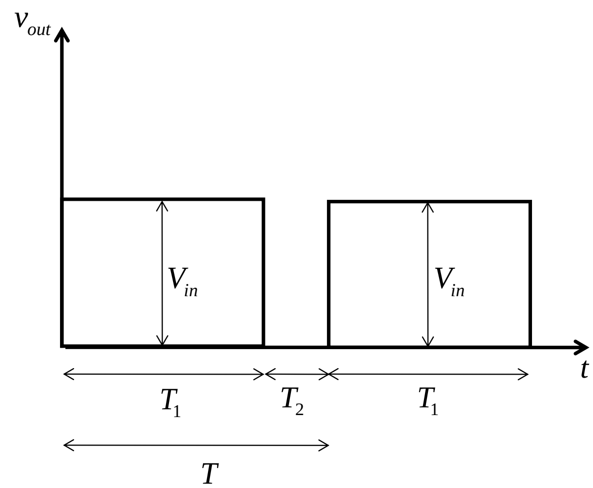


Figure 3b: Voltage across the load resistance

The average output voltage is given by

$$V_{oavg} = \frac{1}{T}\int_0^{T_1} v_{out}\,dt = \frac{T_1}{T}V_{in} = fT_1V_{in} = DV_{in} \tag{1}$$

The average load current is given by

$$I_{oavg} = \frac{V_{oavg}}{R} = \frac{DV_{in}}{R} \tag{2}$$

Where

T is the chopping period

$D = \frac{T_1}{T}$ is the duty cycle

f is the chopping frequency

The rms value of the output voltage is given by

$$V_{orms} = \left(\frac{1}{T}\int_0^{DT} v_{out}^2\,dt\right)^{1/2} = \sqrt{D}V_{in} \tag{3}$$

In case the converter is assumed to be lossless, the input power to the converter will be equal to the output power. Hence, the input power (P_{in}) is given by

$$P_{in} = \frac{1}{T}\int_0^{DT} v_{out} i_{out} dt = \frac{1}{T}\int_0^{DT} \frac{v_{out}^2}{R} dt = D\frac{V_{in}^2}{R} \tag{4}$$

The effective resistance seen by the source is (using **equation 2**)

$$R_{eff} = \frac{V_{in}}{I_{oavg}} = \frac{R}{D} \tag{5}$$

The duty cycle D can be varied from 0 to 1 by varying T_1, T or f. Thus, the output voltage V_{oavg} can be varied from 0 to V_{in} by controlling D and eventually the power flow can be controlled.

The Buck Converter with RLE Load

The buck converter is a voltage step down and current step up converter. The two modes in steady state operations are:

Mode 1 Operation

In this mode the switch S_1 is turned on and the diode D_1 is reversed biased, the current flows through the load. The time domain circuit is shown in **Figure**. The load current, in s domain, for ***mode 1*** can be found from

$$Ri_1(s) + sLi_1(s) + \frac{E}{s} = \frac{V_{in}}{s} + LI_{01} \tag{6}$$

Where

I_{01} is the initial value of the current and $I_{01} = I_1$.

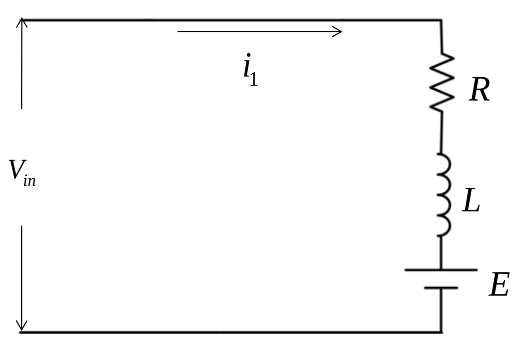


Figure 4: Time domain circuit of buck converter in mode 1

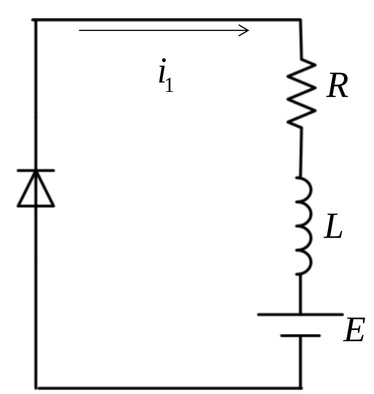


Figure 5: Time domain circuit of buck converter in mode 2

From **equation 6**, the current $i_1(s)$ is given by

$$i_1(s) = \frac{(V_{in} - E)}{s(R + sL)} + \frac{LI_1}{R + sL} \qquad (7)$$

In time domain the solution of **equation 7** is given by

$$i_1(t) = I_1 e^{-tR/L} + \frac{V_{in} - E}{R}\left(1 - e^{-tR/L}\right) \qquad (8)$$

The ***mode1*** is valid for the time duration $0 \le t \le T_1 \Rightarrow 0 \le t \le DT$. At the end of this mode, the load current becomes

$$i_1(t = T_1 = DT) = I_2 \qquad (9)$$

Mode 2 Operation

In this mode the switch S_1 is turned off and the diode D_1 is forward biased. The time domain circuit is shown in **Figure 5**. The load current, in s domain, can be found from

$$Ri_2(s) + sLi_2(s) + \frac{E}{s} = LI_{02} \qquad (10)$$

Where

I_{02} is the initial value of load current.

The current at the end of ***mode1*** is equal to the current at the beginning of ***mode 2***. Hence, from **equation 9** I_{02} is obtained as

$$I_{02} = I_2 \qquad (11)$$

Hence, the load current is time domain is obtained from **equation 10** as

$$i_2(t) = I_2 e^{-tR/L} - \frac{E}{R}\left(1 - e^{-tR/L}\right) \qquad (12)$$

Determination of I_1 and I_2

At the end of ***mode 2*** the load current becomes

$$i_2(t = T_2 = (1 - D)T) = I_3 \qquad (13)$$

At the end of ***mode 2***, the converter enters ***mode 1*** again. Hence, the initial value of current in ***mode 1*** is

$$I_{01} = I_3 = I_1 \qquad (14)$$

From **equation 8** and **equation 12** the following relation between I_1 and I_2 is obtained as

$$I_2 = I_1 e^{-DTR/L} + \frac{V_{in} - E}{R}\left(1 - e^{-DTR/L}\right) \qquad (15)$$

$$I_3 = I_1 = I_2 e^{-(1-D)TR/L} - \frac{E}{R}\left(1 - e^{-(1-D)TR/L}\right) \qquad (16)$$

Solving **equation 15** and **equation 16** for I_1 and I_2 gives

$$I_1 = \frac{V_{in}}{R}\left(\frac{e^{Da}-1}{e^{D}-1}\right) - \frac{E}{R} \tag{17}$$

$$I_2 = \frac{V_{in}}{R}\left(\frac{e^{-Da}-1}{e^{-D}-1}\right) - \frac{E}{R} \tag{18}$$

Where

$$a = \frac{TR}{L} = \frac{R}{fL} \tag{19}$$

where f is the chopping frequency.

Current Ripple

The peak to peak current ripple is given by

$$\Delta I = I_2 - I_1 = \frac{V_{in}}{R}\frac{1-e^{-Da}+e^{-a}-e^{-(1-D)a}}{1-e^{-a}} = \frac{V_{in}}{fL}\frac{1-e^{-Da}+e^{-a}-e^{-(1-D)a}}{a\left(1-e^{-a}\right)} \tag{20a}$$

In case $fL >> R$, $a \to 0$. Hence, for the limit $a \to 0$ **equation 20** becomes

$$\Delta I = \frac{V_{in}D(1-D)}{fL} \tag{20b}$$

To determine the maximum current ripple ($\Delta I_{\max}$), the **equation 20a** is differentiated w.r.t. D. The value of $\Delta I_{\max}$ is given by

$$\Delta I_{\max} = \frac{V_{in}}{R}\tanh\frac{R}{4fL} \tag{21}$$

For the condition $4fL >> R$,

$$\tanh\left(\frac{R}{4fL}\right) \approx \frac{R}{4fL} \tag{22}$$

Hence, the maximum current ripple is given by

$$\Delta I_{\max} = \frac{V_{in}}{4fL} \tag{23}$$

If **equation 20b** is used to determine the maximum current ripple, the same result is obtained.

Continuous and Discontinuous Conduction Modes

In case of large off time, particularly at low switching frequencies, the load current may be discontinuous, i.e. $i_2(t = T_2 = (1-D)T)$ will be zero. The necessary condition to ensure continuous conduction is given by

$$I_1 > 0 \Rightarrow \frac{V_{in}}{R}\left(\frac{e^{Da}-1}{e^{D}-1}\right) - \frac{E}{R} \geq 0$$
$$\Rightarrow \frac{E}{V_{in}} \leq \left(\frac{e^{Da}-1}{e^{D}-1}\right) \tag{24}$$

The Buck Converter with R Load and Filter

The output voltage and current of the converter contain harmonics due to the switching action. In order to remove the harmonics LC filters are used. The circuit diagram of the buck converter with LC filter is shown in **Figure 6**. There are two modes of operation as explained in the previous section.

The voltage drop across the inductor in ***mode 1*** is

$$e_{L_f} = V_{in} - V_o = L_f \frac{di_L}{dt} \text{ and } \ i_L = i_{sw} \tag{25}$$

where i_L is the current through the inductor L_f

i_{sw} is the current through the switch

The switching frequency of the converter is very high and hence, i_L changes linearly. Thus, **equation 25** can be written as

$$e_L = V_{in} - V_o = L_f \frac{\Delta i_L}{T_{on}} = L_f \frac{\Delta i_L}{DT} \tag{26}$$

where T_{on} is the duration for which the switch S remains on

T is the switching time period

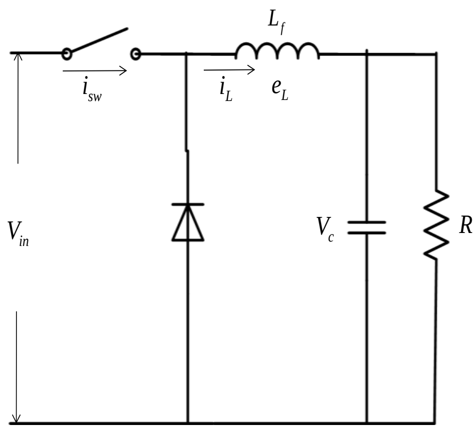


Figure 6: Buck converter with resistive load and filter

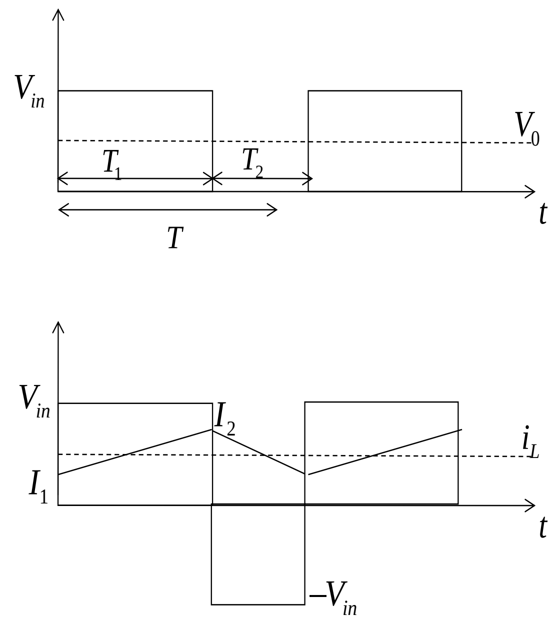


Figure 7: Voltage and current waveform

Hence, the current ripple Δi_L is given by

$$\Delta i_L = \frac{(V_{in} - V_o)}{L_f} DT \tag{27}$$

When the switch S is turned off, the current through the filter inductor decreases and the current through the switch S is zero. The voltage equation is

$$V_o = L_f \frac{di_L}{dt} = L_f \frac{di_D}{dt} \tag{28}$$

where i_D is the current through the diode D

Due to high switching frequency, the **equation 28** can be written as

$$V_o = L_f \frac{\Delta i_L}{T_{off}} = L_f \frac{\Delta i_L}{(1-D)T} \tag{29}$$

where T_{off} is the duration in which switch S remains off the diode D conducts

Neglecting the very small current in the capacitor C_f, it can be seen that

$i_o = i_{sw}$ for time duration in which switch S conducts

and

$i_o = i_D$ for the time duration in which the diode D conducts

The current ripple obtained from **equation 29** is

$$\Delta i_L = \frac{(1-D)T}{L} V_o \tag{30}$$

The voltage and current waveforms are shown in **Figure 7**.

From **equation 27** and **equation 30** the following relation is obtained for the current ripple

$$\Delta i_L = \frac{(V_{in} - V_o)}{L_f} DT = \frac{(1-D)T}{L_f} V_o \tag{31}$$

Hence, from **equation 31** the relation between input and output voltage is obtained as

$$V_o = DV_{in} \Rightarrow \frac{V_o}{V_{in}} = D \tag{32}$$

If the converter is assumed to be lossless, then

$$P_{in} = P_o \Rightarrow V_{in} i_{sw} = V_o i_o \Rightarrow V_{in} i_{sw} = DV_{in} i_o \Rightarrow i_{sw} = Di_o \tag{33}$$

The switching period T can be expressed as

$$T = \frac{1}{f} = T_{on} + T_{off} = L_f \frac{\Delta i_L}{V_{in} - V_o} + L_f \frac{\Delta i_L}{V_o} = L_f \frac{V_o \Delta i_L}{V_o (V_{in} - V_o)} \tag{34}$$

From **equation 34** the current ripple is given by

$$\Delta i_L = \frac{V_o (V_{in} - V_o)}{L_f V_o f} \tag{35}$$

Substituting the value of V_o from **equation 32** into **equation 35** gives

$$\Delta i_L = \frac{V_{in} D (1 - D_o)}{f L_f} \tag{36}$$

Using the Kirchhoff's current law, the inductor current i_L is expressed as

$$i_L = i_c + i_o \tag{37}$$

If the ripple in load current (i_o) is assumed to be small and negligible, then

$$\Delta i_L = \Delta i_c \tag{38}$$

The incremental voltage ΔV_c across the capacitor (C_f) is associated with incremental charge ΔQ by the relation

$$\Delta V_c = \frac{\Delta Q_f}{C_f} \tag{39}$$

The area of each of the isoceles triangles representing ΔQ in **Figure 7** is given by

$$\Delta Q_f = \frac{1}{2} \frac{T}{2} \frac{\Delta i_L}{2} = \frac{T \Delta i_L}{8} \tag{40}$$

Combining **equation 39** and **equation 40** gives

$$\Delta V_c = \frac{T \Delta i_L}{8 C_f} \tag{41}$$

Substituting the value of Δi_L from **equation 31** into **equation 41** gives

$$\Delta V_c = \frac{T}{8C_f}\frac{V_{in}D(1-D)}{fL_f} = \frac{V_{in}D(1-D)}{8L_f C_f f^2} \tag{42}$$

Boundary between Continuous and Discontinuous Conduction

The inductor (i_L) and the voltage drop across the inductor (e_L) are shown in **Figure 8**.

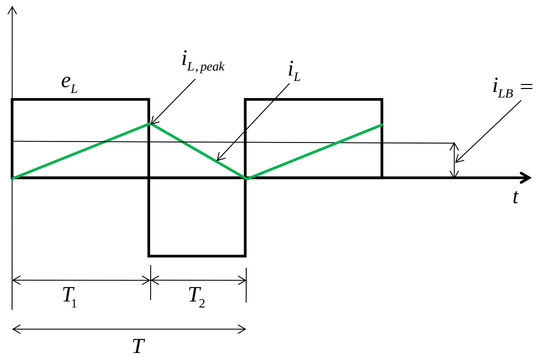


Figure 8: The inductor voltage and current waveforms for discontinuous operation

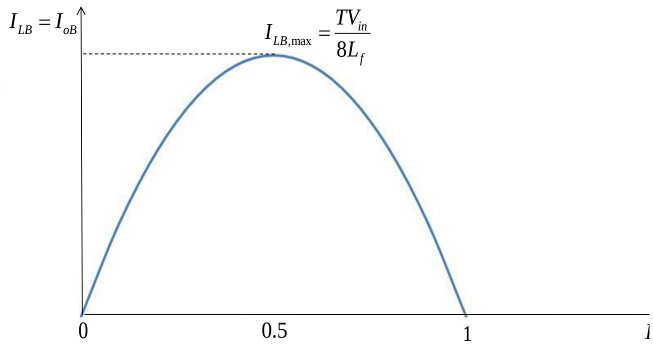


Figure 9: Current versus duty ratio keeping input voltage constant.

Being at the boundary between the continuous and the discontinuous mode, the inductor current i_L goes to zero at the end of the off period. At this boundary, the average inductor current is (**B** rferes to the boundary)

$$I_{LB} = \frac{1}{2}i_{L,peak} = \frac{T_{on}}{2L_f}(V_{in} - V_o) = \frac{DT}{2L_f}(V_{in} - V_o) = I_{oB} \tag{43}$$

Hence, during an operating condition, if the average output current (I_L) becomes less than I_{LB} , then I_L will become discontinuous.

Discontinuous Conduction Mode with ConstantInput Voltage V_{in}

In applications such as speed control of DC motors, the input voltage (V_{in}) remains constant and the output voltage (V_o) is controlled by varying the duty ratio D. Since $V_o = DV_{in}$, the average inductor current at the edge of continuous conduction mode is obtained from **equation 43** as

$$I_{LB} = \frac{TV_{in}}{2L_f}D(1-D) \tag{44}$$

In **Figure 9** the plot of I_{LB} as a function of D, keeping all other parameters constant, is shown. The output current required for a continuous conduction mode is maximum at $D = 0.5$ and by substituting this value of duty ration in **equation 44** the maximum current ($I_{LB,\max}$) is obtained as

$$I_{LB,\max} = \frac{TV_{in}}{8L} \tag{45}$$

From **equation 44** and **equation 45**, the relation between I_{LB} and $I_{LB,\max}$ is obtained as

$$I_{LB} = 4I_{LB,\max} D(1-D) \tag{46}$$

To understand the ratio of output voltage to input voltage (V_o / V_{in}) in the discontinuous mode, it is assumed that initially the converter is operating at the edge of the continuous conduction (**Figure 7**), for given values of T, L, V_d and D. Keeping these parameters constant, if the load power is decreased (i.e., the load resistance is increased), then the average inductor current will decrease. As is shown in **Figure 10**, this dictates a higher value of V_o than before and results in a discontinuous inductor current.

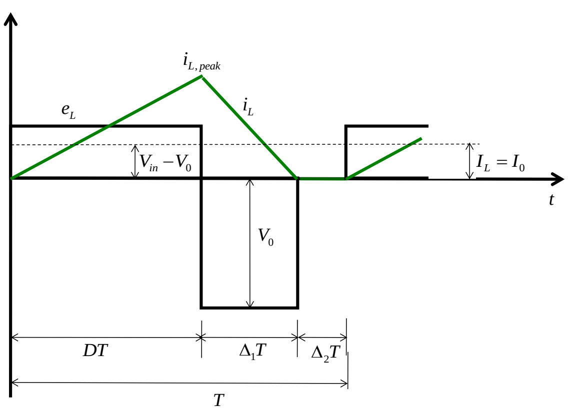


Figure 10: Discontinuous operation is buck converter

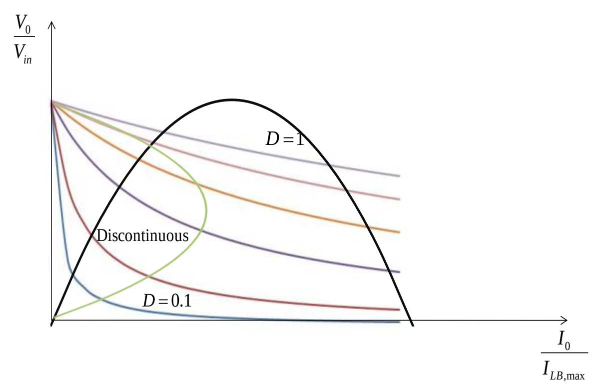


Figure 11: Buck converter characteristics for constant input current

In the time interval $\Delta_2 T$ the current in the inductor L_f is zero and the power to the load resistance is supplied by the filter capacitor alone. The inductor voltage e_L during this time interval is zero. The integral of the inductor voltage over one time period is zero and in this case is given by

$$\left(V_{in} - V_o\right) DT + \left(-V_o\right)\Delta_1 T_s = 0 \Rightarrow \frac{V_o}{V_{in}} = \frac{D}{D+\Delta_1} \tag{47}$$

In the interval $0 \le t \le \Delta_1 T_s$ (**Figure 10**) the current ripple in L_f is

$$e_L = L_f \frac{di_L}{dt} \Rightarrow e_L = L_f \frac{\Delta i_L}{\Delta_1 T} \tag{48}$$

From **Figure 10** it can be seen that

$$\Delta i_L = -i_{L,peak} \quad \text{(since the current falls)} \tag{49}$$

$$e_L = -V_o \tag{50}$$

Substituting the values of Δi_L and e_L from **euqation 49** and **equation 50** into **equation 48** gives

$$V_o = L_f \frac{i_{L,peak}}{\Delta_1 T} \Rightarrow i_{L,peak} = \frac{V_o}{L_f} \Delta_1 T \tag{51}$$

$$\begin{aligned} \therefore I_o &= i_{L,peak} \frac{D+\Delta_1}{2} \\ &= \frac{V_o T_s}{2L_f}\left(D+\Delta_1\right)\Delta_1 \text{ (from eq.51)} \\ &= \frac{V_{in} T}{2L_f} D\Delta_1 \text{ (from eq.47)} \\ &= 4L_{LB,\max} D\Delta_1 \text{ (from eq.45)} \end{aligned} \tag{52}$$

$$\text{Hence, } \Delta_1 = \frac{I_o}{4L_{LB,\max} D} \tag{53}$$

From **equation 47** and **equation 53** the ratio V_o / V_{in} is obtained as

$$\frac{V_o}{V_{in}} = \frac{D^2}{D^2 + \frac{1}{4}\left(I_o / I_{LB,\max}\right)} \tag{54}$$

In **Figure 11** the step down characteristics in ***continuous*** and ***discontinuous*** modes of operation is shown. In this figure the voltage ratio (V_o / V_{in}) is plotted as a function of $I_o / I_{LB,\max}$ for various duty ratios using **equation 32** and **equation 54**. The boundary between the continuous and the discontinuous mode, shown by dashed line in **Figure 11**, is obtained using **equation 32** and **equation 48**.

Discontinuous-Conduction Mode with Constant V_o

In some applications such as regulated dc power supplies, V_{in} may vary but V_o is kept constant by adjusting the duty ratio. From **equation 44** the average inductor current at the boundary of continuous conduction is obtained as

$$I_{LB} = \frac{TV_o}{2L_f}\left(1-D\right) \tag{56}$$

From **equation 56** it can be seen that, for a given value of V_o the maximum value of I_{LB} occurs at $D=0$ and is given by

$$I_{LB,\max} = \frac{TV_o}{2L_f} \tag{57}$$

From **equation 56** and **equation 57** the relation between I_{LB} and $I_{LB,\max}$ is

$$I_{LB} = (1-D)I_{LB,\max} \tag{58}$$

From **equation 52**, the output current is obtained as

$$\begin{aligned} I_o &= \frac{V_o T}{2L_f}\left(D+\Delta_1\right)\Delta_1 \\ &= I_{LB,\max}\left(D+\Delta_1\right)\Delta_1 \text{ (from eq.57)} \end{aligned} \tag{59}$$

Solving the **equation 59** for Δ_1 and substituting its value in **equation 47** gives

$$D = \frac{V_o}{V_{in}}\left(\frac{I_o / I_{LB,\max}}{1 - V_o / V_{in}}\right)^{\frac{1}{2}} \tag{60}$$

References:

[1] M. Ehsani, *Modern Electric, Hybrid Electric and Fuel Cell Vehicles: Fundamentals, Theory and Design*, CRC Press, 2005

Suggested Reading:

[1] M. H. Rashid, *Power Electronics: Circuits, Devices and Applications*, 3rd edition, Pearson, 2004

Lecture 10: Boost and Buck-Boost Converters

Boost and Buck-Boost Converters

Introduction

The topics covered in this chapter are as follows:

- Principle of Step-Up Operation
- Boost Converter with Resistive Load and EMF Source
- Boost Converter with Filter and Resistive Load
- Buck-Boost Converter

Principle of Step-Up Operation (Boost Converter)

The circuit diagram of a step up operation of DC-DC converter is shown in **Figure 1**. When the switch S_1 is closed for time duration t_1, the inductor current rises and the energy is stored in the inductor. If the switch S_1 is openerd for time duration t_2, the energy stored in the inductor is transferred to the load via the didode D_1 and the inductor current falls. The waveform of the inductor current is shown in **Figure 2**.

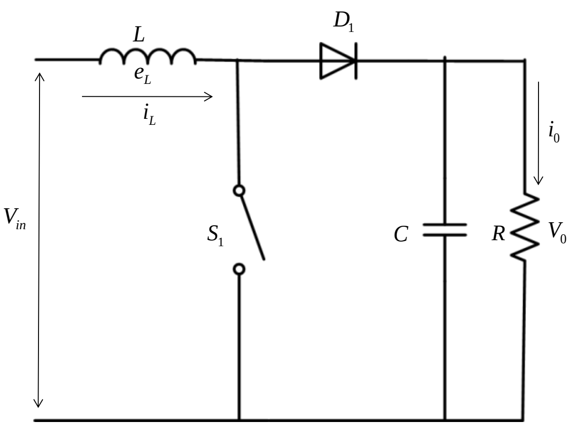


Figure 1:General Configuration of a Boost Converter

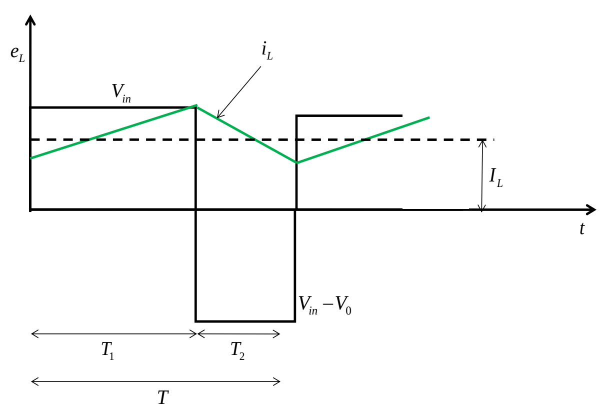


Figure 2: Inductor current waveform

When the switch S_1 is turned on, the voltage across the inductor is

$$v_L = L\frac{di}{dt} \tag{1}$$

The peak to peak ripple current in the inductor is given by

$$\Delta I = \frac{V_s}{L}T_1 \tag{2}$$

The average output voltage is

$$v_0 = V_s + L\frac{\Delta I}{T_2} = V_s\left(1+\frac{T_1}{T_2}\right) = V_s\frac{1}{1-D} \tag{3}$$

From **Equation 3** the following observations can be made:

- The voltage across the load can be stepped up by varying the duty ratio D
- The minimum output voltage is V_s and is obtained when $D=0$
- The converter cannot be switched on continupusly such that $D=1$. For values of D tending to unity, the output becomes very sensitive to changes in D

For values of D tending to unity, the output becomes very sensitive to changes in (**Fig.3**).

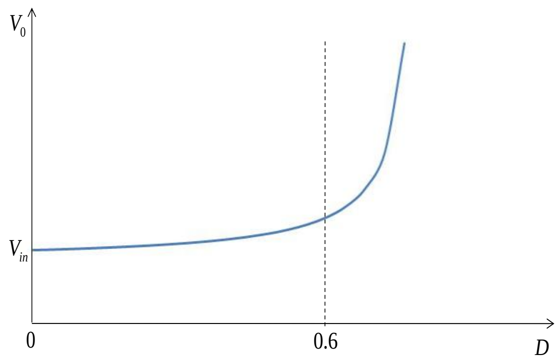


Figure 3: Output voltage vs. Duty ration for Boost Converter

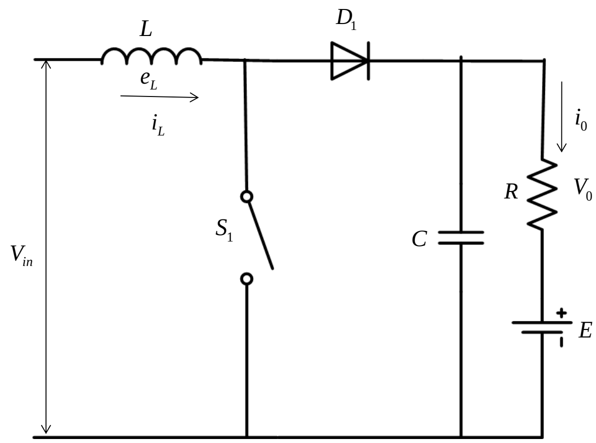


Figure 4: Boost converter with resistive load and emf source

Boost Converter with Resistive Load and EMF Source

A boost converter with resistive load is shown in **Figure 4**. The two modes of operation are:

Mode 1: This mode is valid for the time duration

$$0 \le t \le DT \tag{4}$$

where D is the ***duty ratio*** and T is the ***switching period***.

The mode 1 ends at $t = DT$.

In this mode the switch S_1 is closed and the equivalent circuit is shown in **Figure 5**. The current rises throught the inductor L and switch S_1. The current in this mode is given by

$$V_s = L\frac{di}{dt}i_1 \tag{5}$$

Since the time instants involved are very small, the term $dt \approx t$. Hence, the solution of **Equation 5** is

$$i_1(t) = \frac{V_s}{L}t + I_1 \tag{6}$$

where I_1 is the initial value of the current. Assuming the current at the end of mode 1($t = DT$) to be I_2 ($i_1(t = DT) = I_2$), the **Equation 6** can be written as

$$I_2 = \frac{V_s}{L}DT + I_1 \tag{7}$$

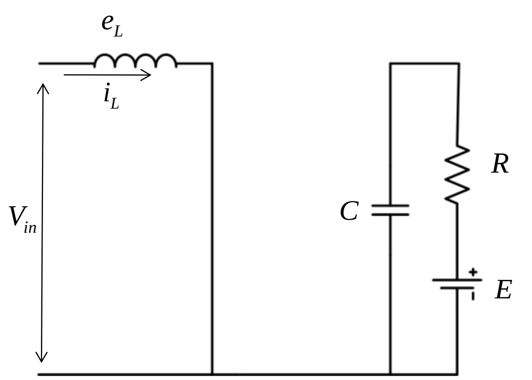


Figure 5: Configuration of a Boost Converter in *mode 1*

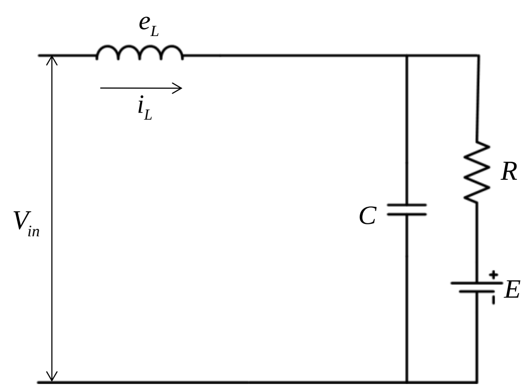


Figure 6: Configuration of a Boost Converter in *mode 2*

Mode2: This mode is valid for the time duration

$$DT \le t \le T \tag{8}$$

In this mode the switch S_1 is open and the inductor current flows through the RL load and the equivalent circuit is shown in **Figure 6**. The voltage equation in this mode is given by

$$V_s = Ri_2 + L\frac{di_2}{dt} + E \tag{9}$$

For an initial current of I_2, the solution of **Equation 9** is given by

$$i_2(t) = \frac{V_s - E}{L}\left(1 - e^{-\frac{R}{L}t}\right) + I_2 e^{-\frac{R}{L}t} \tag{10}$$

The current at the end of ***mode 2*** is equal to I_1:

$$i_2\left(t = (1-D)t\right) = I_2 = \frac{V_s - E}{L}\left(1 - e^{-(1-D)z}\right) + I_2 e^{-(1-D)z} \tag{11}$$

where $z = TR/L$

Solving **Equation 7** and **Equation 11** gives the values of I_1 and I_2 as

$$I_1 = \frac{V_s Dz}{R} \frac{e^{-(1-D)z}}{1-e^{-(1-D)z}} + \frac{V_s - E}{R} \tag{12}$$

$$I_2 = \frac{V_s Dz}{R} \frac{1}{1-e^{-(1-D)z}} + \frac{V_s - E}{R} \tag{13}$$

The ripple current is given by

$$\Delta I = I_2 - I_1 = \frac{V_s}{L} DT \tag{14}$$

The above equations are valid if $E \le V_s$. In case $E \ge V_s$, the converter works in discontinuous mode.

Boost Converter with Filter and Resistive Load

A circuit diagram of a Buck with filter is shown in **Figure 7**. Assuming that the inductor current rises linearly from I_1 to I_2 in time t_1

$$V_{in} = L\frac{I_2 - I_1}{t_1} = L\frac{\Delta I}{t_1} \Rightarrow t_1 = \frac{\Delta I}{V_{in}} L \tag{15}$$

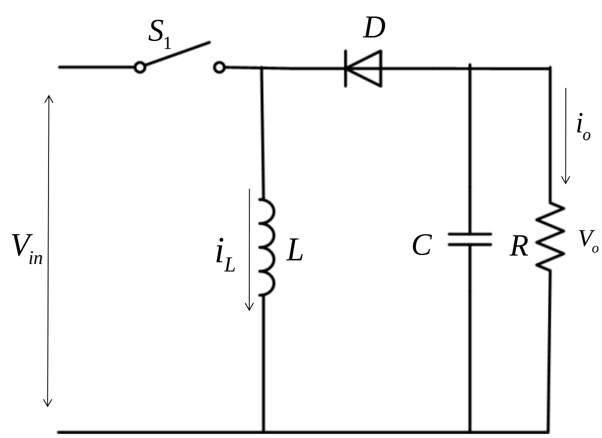


Figure 7: Configuration of a Buck Boost Converter

The inductor current falls linearly from I_2 to I_1 in time t_2

$$V_{in} - V_o = -L\frac{\Delta I}{t_2} \Rightarrow t_2 = L\frac{\Delta I}{V_o - V_{in}} \tag{16}$$

where $\Delta I = I_2 - I_1$ is the peak to peak ripple current of inductor L. From **equation 15** and **equation 16** it can be seen that

$$\Delta I = \frac{V_{in} t_1}{L} = \frac{(V_o - V_{in}) t_2}{L} \tag{17}$$

Substituting $t_1 = DT$ and $t_2 = (1-D)T$ gives the average output voltage

$$V_o = V_{in}\frac{T}{t_2} = \frac{V_{in}}{1-D} \Rightarrow (1-D) = \frac{V_{in}}{V_o} \quad (18)$$

Substituting $D = \frac{t_1}{T} = t_1 f$ into **equation 18** gives

$$t_1 = \frac{V_o - V_{in}}{V_o f} \quad (19)$$

If the boost converter is assumed to be lossless then

$$V_{in}I_{in} = V_o I_o = V_{in}I_o/(1-D) \quad (20)$$

$$I_{in} = \frac{I_a}{1-D} \quad (21)$$

The switching period T is given by

$$T = \frac{1}{f} = t_1 + t_2 = L\frac{\Delta I}{V_{in}} + L\frac{\Delta I}{V_o - V_{in}} = \frac{\Delta I L V_o}{V_{in}\left(V_o - V_{in}\right)} \quad (22)$$

From **equation 22** the peak to peak ripple current is given by

$$\Delta I = \frac{V_{in}\left(V_o - V_{in}\right)}{fLV_o} \Rightarrow \Delta I = \frac{V_{in}D}{fL} \quad (23)$$

When the switch S is on, the capacitor supplies the load current for $t = t_1$. The average capacitor current during time t_1 is $I_c = I_o$ and the peak to peak ripple voltage of the capacitor is

$$\Delta V_c = v_c - v_c(t=0) = \frac{1}{C}\int_0^{t_1} I_c dt = \frac{1}{C}\int_0^{t_1} I_o dt = \frac{I_a t_1}{C} \quad (24)$$

Substituting the value of t_1 from **equation 19** into **equation 24** gives

$$\Delta V_c = \frac{I_o\left(V_o - V_s\right)}{V_o fC} \Rightarrow \Delta V_c = \frac{I_o D}{fC} \quad (25)$$

Condition for Continuous Inductor Current and Capacitor Voltage

If I_L is the average inductor current, the inductor ripple current is $\Delta I = 2I_L$. Hence, from **equation 18** and **equation 23** the following expression is obtained

$$\frac{DV_{in}}{fL} = 2I_L = 2I_o = \frac{2V_{in}}{(1-D)R} \quad (26)$$

The critical value of the inductor is obtained from **equation 26** as

$$L = \frac{D(1-D)R}{2f} \quad (27)$$

If V_c is the averag capacitor voltage, the capacitor ripple voltage $\Delta V_c = 2V_a$. Using **equation 25** the following expression is obtained

$$\frac{I_o D}{C_f f} = 2V_a = 2I_o R \tag{28}$$

Hence, from **equation 28** the critical value of capacitance is obtained as

$$C = \frac{D}{2fR} \tag{29}$$

Buck-Boost Converter

The general configuration of Buck-Boost converter is shown **Figure 7**. A *buck-boost* converter can be obtained by cascade connection of the two basic converters:

- the step down converter
- the step up converter

The circuit operation can be divided into two modes:

- During **mode 1 (Figure 8a)**, the switch S_1 is turned on and the diode D is reversed biased. In ***mode 1*** the input current, ***which rises***, flows through inductor L and switch S_1.
- In ***mode 2*** (**Figure 8b**), the switch S_1 is ***off*** and the current, ***which was flowing through the inductor***, would flow through L, C, D and ***load***. In this mode the energy stored in the inductor (L) is transferred to the load and the inductor current (i_L) falls until the switch S_1 is turned on again in the next cycle.

The waveforms for the steady-state voltage and current are shown in **Figure 9**.

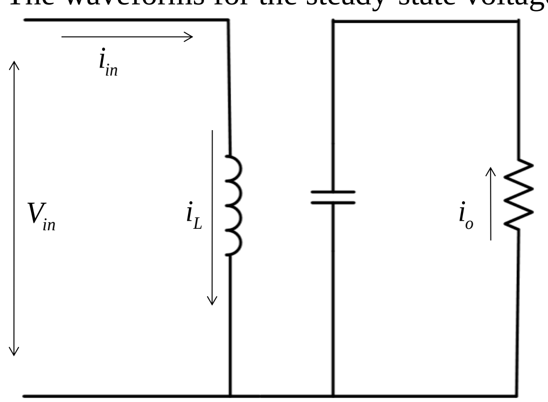


Figure 8a: Buck Boost Converter in mode 1

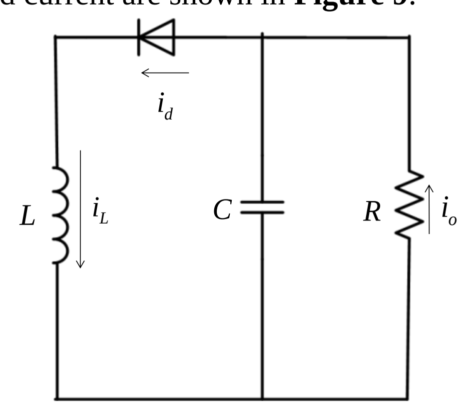
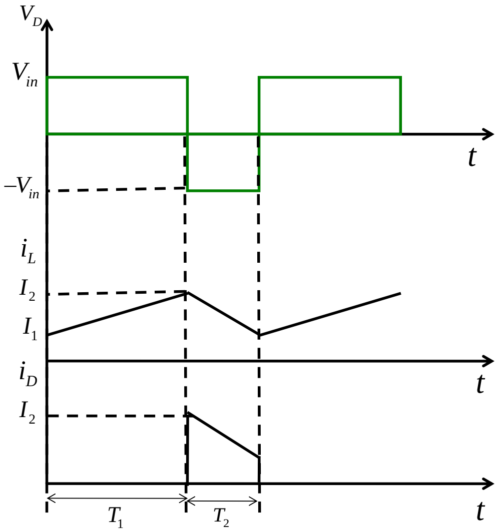


Figure 8b: Buck Boost Converter in mode 2



V_d is the voltage across the diode

i_d is the current through the diode

i_L is the current through the inductor

Figure 9: Current and voltage waveforms of Buck Boost Converter

Buck-Boost Converter Continuous Mode of Operation

Since the switching frequency is considered to be very high, it is assumed that the current through the inductor (L) rises linearly. Hence, the relation of the voltage and current in ***mode 1*** is given by

$$V_{in} = L\frac{I_2 - I_1}{T_1} = L\frac{\Delta I}{T_1}$$
$$\Rightarrow T_1 = L\frac{\Delta I}{V_{in}} \tag{29}$$

The inductor current falls linearly from I_2 to I_1 in ***mode 2*** time T_2 and is given by

$$V_o = -L\frac{\Delta I}{T_2}$$
$$\Rightarrow T_2 = -L\frac{\Delta I}{V_o} \tag{30}$$

The term $\Delta I(= I_2 - I_1)$, in ***mode 1*** and ***mode 2***, is the peak to peak ripple current through the inductor L. From **equation 29** and **equation 30** the relation between the input and output voltage is obtained as

$$\Delta I = \frac{V_{in}T_1}{L} = -\frac{V_o T_2}{L} \tag{31}$$

The relation between the ***on*** and off time, of the switch S_1, and the total time duration is given in terms of duty ratio (D) as:

$$T_1 = DT \tag{32a}$$

$$T_2 = (1-D)T \tag{32b}$$

Substituting the values of T_1 and T_2 from **equation 32a** and **equation 32b** into **equation 31** gives:

$$V_o = -\frac{V_{in} D}{1-D} \tag{33}$$

If the converter is assumed to be lossless, then

$$\begin{aligned} V_{in} I_{in} &= -V_o I_o \\ V_{in} I_{in} &= \frac{V_{in} D}{1-D} I_o \Rightarrow I_{in} = \frac{I_o D}{1-D} \end{aligned} \tag{34}$$

The switching period T obtained from **equation 29** and **equation 30** as:

$$T = T_1 + T_2 = L\frac{\Delta I}{V_o} - L\frac{\Delta I}{V_{in}} = L\Delta I \frac{\left(V_{in} - V_o\right)}{V_{in} V_o} \tag{35}$$

The peak to peak ripple current ΔI is obtained from **equation 35** as

$$\begin{aligned} &\Delta I = \frac{T V_{in} V_o}{L\left(V_o - V_{in}\right)} = \frac{DT}{L} V_{in} = \frac{V_{in} D}{fL} \\ &\textit{where} \\ &f = \text{switching frequency} \end{aligned} \tag{36}$$

When the switch S_1 is turned ***on***, the filter capacitor supplies the load current for the time duration T_1. The average discharge current of the capacitor $I_{cap} = I_{out}$ and the peak to peak ripple current of the capacitor are:

$$\Delta V_{cap} = \frac{1}{C}\int_0^{T_1} I_{cap} dt = \frac{1}{C}\int_0^{T_1} I_o dt = \frac{I_o T_1}{C} = \frac{I_o D}{fC} \tag{37}$$

Buck-Boost Converter Boundary between Continuous and Discontinuous Conduction

In **Figure 10** the voltage and load current waveforms of at the edge of continuous conduction is shown. In this mode of operation, the inductor current (i_L) goes to zero at the end of the ***off interval*** (T_2). From **Figure 10**, it can be seen that the average value of the inductor current is given by

$$I_{LB} = \frac{1}{2} I_2 = \frac{1}{2}\Delta I \tag{38}$$

Substituting the value of ΔI from **equation 36** into **equation 38** gives:

$$I_{LB} = \frac{1}{2}\frac{DT}{L} V_{in} \tag{39}$$

In terms of output voltage, **equation 39** can be written as

$$I_{LB} = \frac{1}{2}\frac{T}{L} V_o \left(1-D\right) \tag{40}$$

The average value of the output current is obained substituting the value of input current from **equation 34** into **equation 40** as:

$$I_{OB} = \frac{1}{2}\frac{T}{L}V_o\left(1-D\right)^2 \tag{41}$$

Most applications in which a buck-boost converter may be used require that V_{out} be kept constant. From **equation 40** and **equation 41** it can be seen that I_{LB} and I_{OB} result in their maximum values at $D=0$ as

$$\begin{aligned} I_{LB,\max} &= \frac{TV_{out}}{2L} \\ I_{OB,\max} &= \frac{TV_{out}}{2L} \end{aligned} \tag{42}$$

From **equation 38** it can be seen that peak-to-peak ripple current is given by

$$\Delta I = 2I_{LB} \tag{43}$$

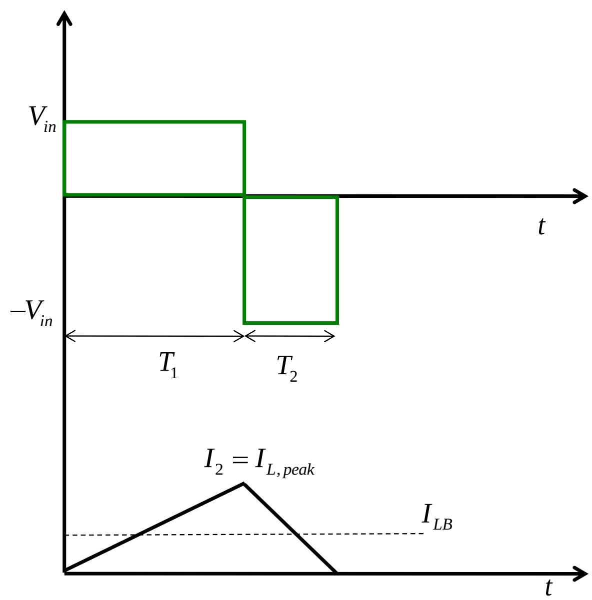


Figure 10: Current and voltage waveforms of Buck Boost Converter in boundary between continuous and discontinuous mode

Suggested Reading:

[1] M. H. Rashid, *Power Electronics: Circuits, Devices and Applications*, 3rd edition, Pearson, 2004

Lecture 11: Multi Quadrant DC-DC Converters I

Multi Quadrant DC-DC Converters I

Introduction

The topics covered in this chapter are as follows:

- Converter classification
- Two Quadrant Converters

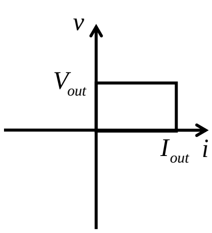
Converter Classification

DC-DC converters in an EV may be classified into ***unidirectional*** and ***bidirectional*** converters. Unidirectional converters are used to supply power to various onboard loads such as sensors, controls, entertainment and safety equipments. Bidirectional DC-DC converters are used where regenerative braking is required. During regenerative braking the power flows back to the voltage bus to recharge the batteries.

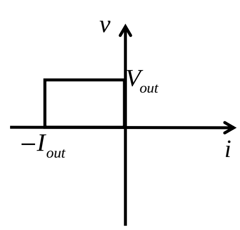
The buck, boost and the buck-boost converters discussed so far allow power to flow from the supply to load and hence are ***unidirectional*** converters. Depending on the directions of current and voltage flows, dc converters can be classified into **five types:**

- First quadrant converter
- Second quadrant converter
- First and second quadrant converter
- Third and fourth quadrant converter
- Four quadrant converter

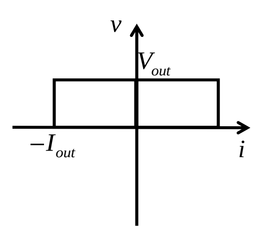
Among the above five converters, the ***first*** and ***second quadrant*** converrters are ***unidirectional*** where as the ***first and second***, ***third and fourth***and ***four quadrant*** converters are ***bidirectional converters***. In **Figure 1** the relation between the load or output voltage (V_{out}) and load or output current (I_{out}) for the five types of converters is shown.



First Quadrant



Second Quadrant



First and Second Quadrant

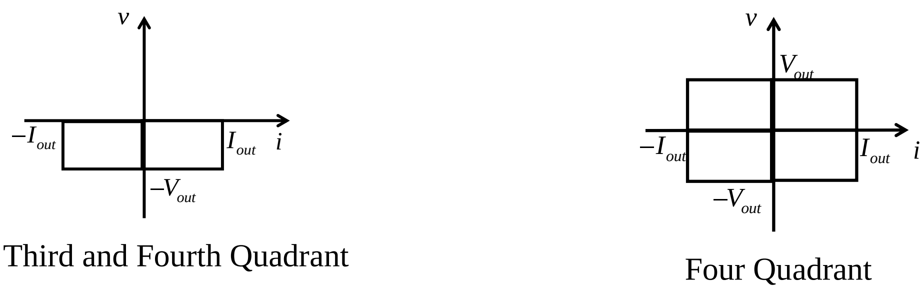


Figure 1: Possible converter operation quadrants.

Second Quadrant Converter

The second quadrant chopper gets its name from the fact that the flow of current is from the load to the source, the voltage remaining positive throughout the range of operation. Such a reversal of power can take place only if the load is active, i.e., the load is capable of providing continuous power output. In **Figure 2** the general configuration of the second quadrant converter consisting of a emf source in the load side is shown. The ***emf source can be a separately excited dc motor with a back emf of*** E and armature resistace and inductance of R and L respectively.

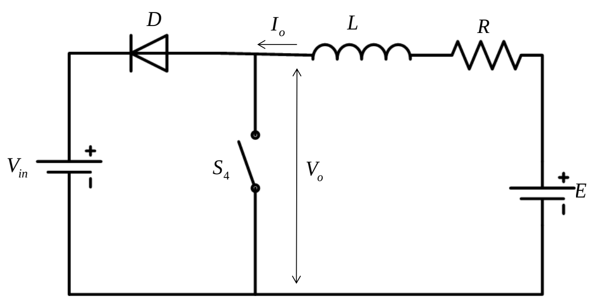


Figure 2: Second Quadrant DC-DC Converter

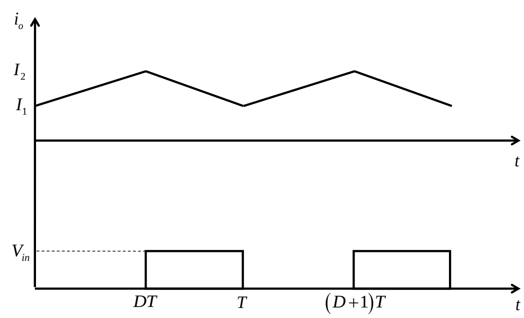


Figure 3: Current and voltage waveform

The load current flows out of the load. The load voltage is positive but the load current is negative as shown in **Figure 2**. This is a single quadrant converter but operates in the second quadrant. In **Figure 2** it can be seen that switch S_4 is turned on, the voltage E drives current through inductor L and the output voltage is zero. The instantaneous output current and output voltage are shown in **Figure 3**. The system equation when the switch S_4 is ***on*** (***mode 1***) is given by

$$0 = L\frac{di_o}{dt} + Ri_o + E \tag{1}$$

With initial condition $i_o(t=0)=I_1$, gives

$$i_o = I_1 e^{-\left(\frac{R}{L}\right)t} - \frac{E}{R}\left(1-e^{-\left(\frac{R}{L}\right)t}\right) \text{ for } 0 \le t \le DT \tag{2}$$

At time $t=DT$ the output current is given by reaches a value of I_2, i.e., $i_o(t=DT)=I_2$. When the switch S_4 is turned ***off*** (***mode 2***), a magnitude of the energy stored in the inductor L is returned to the input voltage V_{in} via the diode D_1 and the output current I_o falls. Redefining the time origin $t=0$, the load current is described as

$$V_{in} = L\frac{di_{out}}{dt} + Ri_{out} + E \tag{3}$$

At the beginning of ***mode 2*** the initial value of the current is same as the final value of current at the end of ***mode 1***. Hence, the initial condition at the beginning of ***mode 2*** is I_2. With this initial condition, the solution of **equation 3** is

$$i_o = I_2 e^{-\left(\frac{R}{L}\right)t} + \frac{V_{in}-E}{R}\left(1-e^{-\left(\frac{R}{L}\right)t}\right) \text{ for } DT \le t \le T \tag{4}$$

At the end of ***mode 2*** the load current becomes

$$i_2(t=T_2=(1-D)T)=I_3 \tag{5}$$

However, at the end of ***mode 2***, the converter enters ***mode 1*** again. Hence, the initial value of current in ***mode 1*** is $I_3=I_1$.

From **equation 2** and **equation 4** the values of I_1 and I_2 is obtained as

$$I_1 = \frac{V_{in}}{R}\left[\frac{1-e^{-(1-D)z}}{1-e^{-z}}\right] - \frac{E}{R}$$

$$I_2 = \frac{V_{in}}{R}\left[\frac{e^{-Dz}-e^{-z}}{1-e^{-z}}\right] - \frac{E}{R} \tag{6}$$

where

$$z = \frac{TR}{L}$$

Two Quadrant Converters

This converter is a combination of the first and second quadrant converters. Two such converters are discussed here:

- operating in first and second quadrant
- operating in first and fourth quadrant

The following assumptions are made for ease of analysis:

- The input voltage is greater than the load voltage $V_{in} > E$
- The positive direction of the current is taken to be the direction from source to load.

First and Second Quadrant Converter

In **Fugure 4a** the configuration of a two quadrant converter providing operation in first and second quadrants is shown.

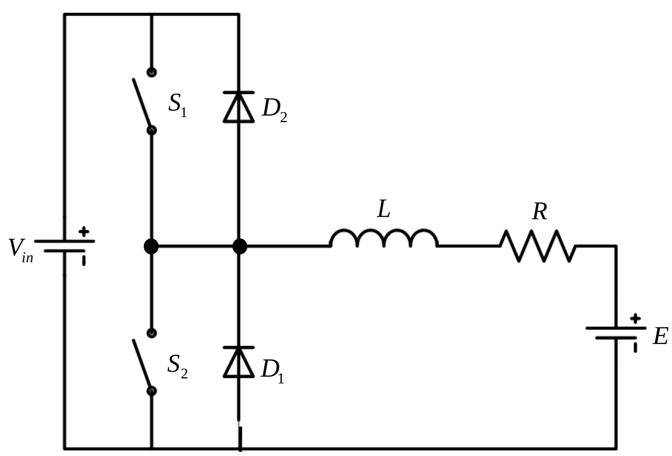


Figure 4: First and Second Quadrant Converter

The converter works in ***first quadrant*** when S_2 is ***off***, diode D_2 is not conducting and S_1 is on. If the switch S_1 is off, S_2 is on and diode D_1 is not forward biased, then the converter operates in ***second quadrant***. There are four possible modes of operation of this converter. These four possibilities are:

i. ***The minimum current*** $I_1 > 0$ ***and minimum*** (I_1) ***and maximum*** (I_2) ***currents are positive:*** In this mode, only the switch S_1 and the diode D_1 operate. When S_1 is switched ***on*** at time $t = 0$ (**Figure 5a**), current flows from the source to the motor and the inductor L gains energy. At time $t = T_1$ S_1 is turned ***off*** but the current continuous to flow in the same direction and finds a closed path through the load, the freewheeling diode D_1 (**Figure 5b**). Hence, the instantaneous output current i_o is positive throughout and hence the average output current I_o is also positive. Therefore, the converter operates in ***first quadrant***. The waveforms in this condition are shown in **Figure 5c**.

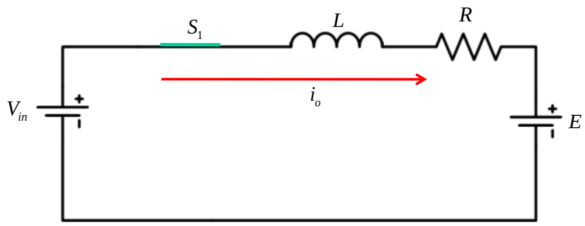


Figure 5a: The switch S_1 is on

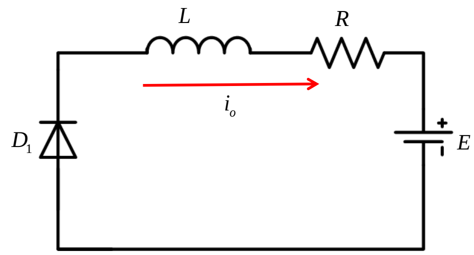


Figure 5b: The diode D_1 is conducts

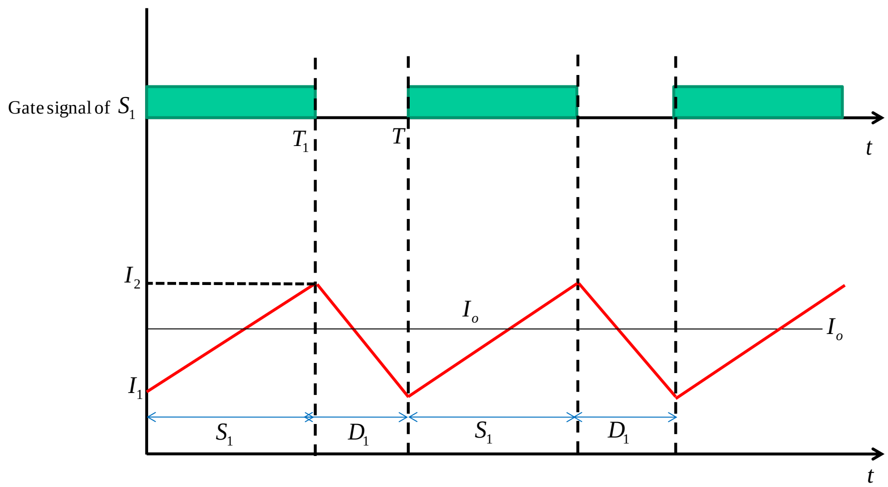


Figure 5c: The current waveform

ii. ***The minimum current*** $I_1 < 0$, ***maximum current*** $I_2 > 0$ ***and average load current*** I_o ***is positive:*** In this case the instantaneous load current i_o can be positive or negative but its profile is such that the average load current I_o is positive. In order to analyse the operation of the converter it is assumed that the converter is in steady state. The S_1 is turned ***on*** at $t = 0$, the instantaneous load current is negative $(i_o < 0)$ and D_2 conducts it (**Figure 6a**). The drop across the D_2 reverse biases S_1 thus preventing conduction. The input voltage V_{in} is greater than the load voltage E, hence, di_o/dt is positive. When $i_o = 0$, the switch S_1 starts conduction and continuous to do so till T_1 (**Figure 6b**). At time T_1 the switch S_1 is turned off and the switch S_2 is turned on. At this instant the switch S_2 cannot conduct because the current is in positive direction. Since the source is isolated, D_1 freewheels the inductive current (**Figure 6c**). The slope di_o/dt being negative, i_o becomes zero after some time and D_1 stops conduction. When i_o becomes negative, S_2 starts conduction (**Figure 6d**). This condition remains till time T at which instant S_1 is turned on again. The quantities T_1 and T are such that the average load current (I_o) is positive. The presence of the S_2 and D_2 facilitate continuous flow of current irrespective of its direction. The current waveforms for this mode of operation are shown in **Figure 6e**.

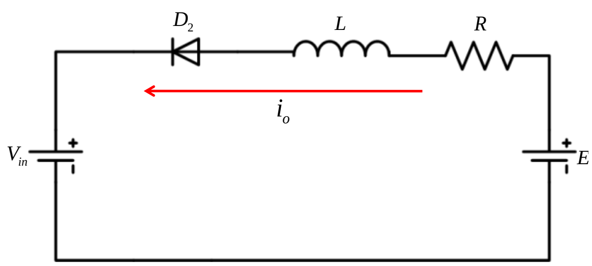


Figure 6a: Load current –ve and D_2 conducts

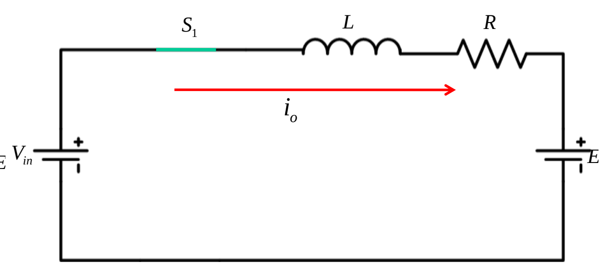


Figure 6b: Load current +ve and S_1 conducts

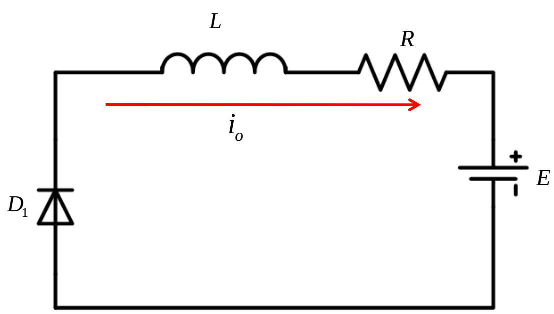


Figure 6c: Load current +ve and D_1 conducts

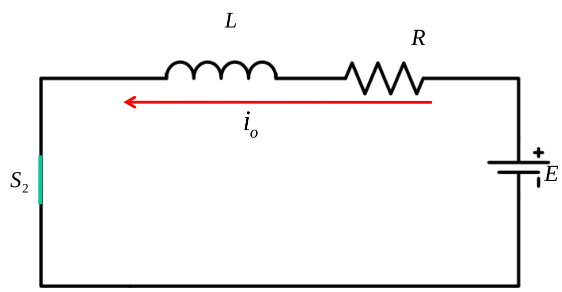


Figure 6d: Load current -ve and S_2 conducts

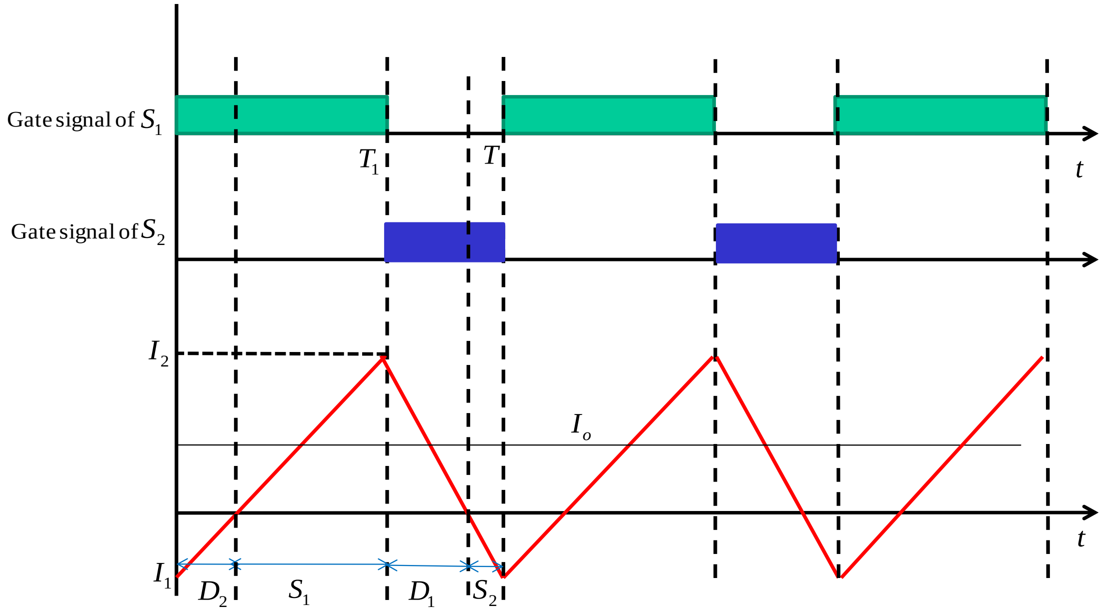


Figure 6e: Current waveforms

iii. ***The minimum current*** $I_1 < 0$***, maximum current*** $I_2 > 0$ ***and average load current*** I_o ***is negative:*** The sequence of events for this case is same as case ***ii*** except that T_1 and T are such that the average load current I_o is negative. Hence, the converter operates in second quadrant. The current waveforms are shown in **Figure 7**.

iv. $I_2 < 0$***:*** In this case the instantaneous load current is always negative. Hence, the average load current is also negative and the converter operates in the second quadrant. The diode D_2 conducts till time T_1, di_o/dt being positive. The current rises from I_1 to I_2 at T_1. The switch S_2 starts conduction at T_1 and this conduction continuous till T, from which moment onwards the sequence repeats. The waveforms are shown in **Figure 8**.

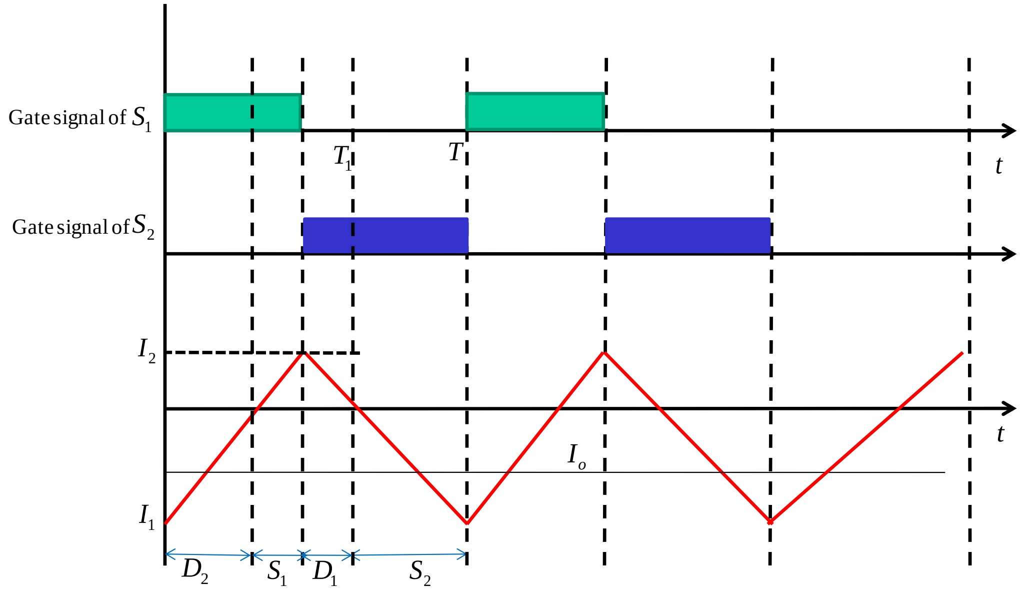


Figure 7: Current waveforms for case iii

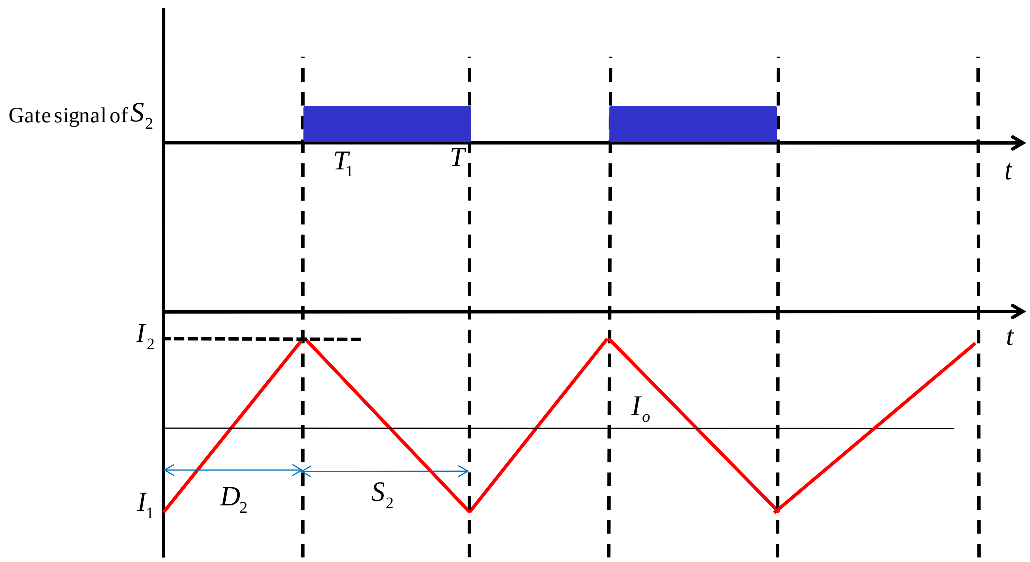


Figure 8: Current waveforms for case iv

The following can be observed from the four cases discussed above:

a. For the ***cases i*** and ***iv***, during the conduction of D_2, $i_o < 0$ but the load voltage $E > 0$ and hence, the load power is negative. This can be interpreted as that the kinetic energy of the motor gets converted into electrical energy and fed back to the source, thereby implying that the motor operates in regenerative braking mode.

b. The switches S_1 and S_2 can conduct only when their respective triggering signals are present and the instantaneous current through them is positive.

c. The average current through the load is given by

$$I_o = \frac{V_{in}\left(\frac{T_1}{T}\right) - E}{R}$$

(7)

This current is either positve or negative, respectively, depending on whether $V_{in}\left(\frac{T_1}{T}\right) > E$ or $V_{in}\left(\frac{T_1}{T}\right) < E$.

Suggested Reading:

[1] M. H. Rashid, *Power Electronics: Circuits, Devices and Applications*, 3rd edition, Pearson, 2004

[2] V. R. Moorthi, *Power Electronics: Devices, Circuits and Industrial Applications*, Oxford University Press, 2007

Lecture 12: Multi Quadrant DC-DC Converters II

Multi Quadrant DC-DC Converters II

Introduction

The following topics are covered in this lecture:

- First and Second Quadrant Converter
- Four Quadrant Chopper

First and Fourth Quadrant Converter

In **Figure 1a** the configuration of two quandrant converter capable of operating in ***first*** and ***fourth*** quadrants is shown. Both the switches S_1 and S_2 are turned ***on*** for a duration $t=0$ to $t=T_1$ (**Figure 1b**) and ***off*** for a duration $t=T_1$ to $t=T$. The instantaneous output voltage appearing across the load v_{out} is:

$$\begin{aligned} v_{out} &= V_{in} \quad 0 \le t \le T_1 \\ v_{out} &= -V_{in} \quad T_1 \le t \le T \end{aligned} \tag{1}$$

When the switches S_1 and S_2 are turned ***off***, the current throught the inductor L continues to flow in the same direction, making the diodes D_1 and D_2 conduct thus feeding the load energy back to the dc source (**Figure 1c**). The average load voltage V_{out} is obtained as

$$\begin{aligned} V_{out} &= \frac{1}{T}\left[\int_0^{T_1} V_{in}dt + \int_{T_1}^{T}(-V_{in})dt\right] = \frac{V_{in}}{T}\left(T_1 - T_{off}\right) \\ &where \\ T_{off} &= T - T_1 \end{aligned} \tag{2}$$

From **equation 2** it can be seen that for $T_1 > T_{off}$, V_{out} is positive and the current flows from the DC source to load. Both the average load voltage V_{out} and load current I_o being positive, the operation of the converter is in ***first quadrant*** (**Figure 1d**). When $T_1 < T_{off}$, V_{out} is negative but I_o is positive and the converter operates in ***fourth quadrant*** (**Figure 1e**).

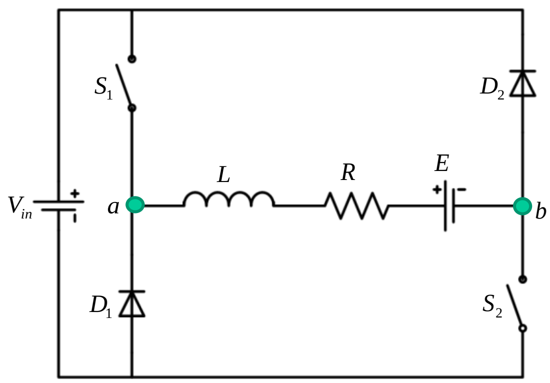


Figure 1a: First and Fourth quadrant converter

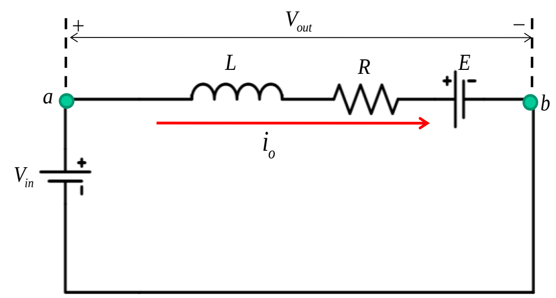


Figure 1b: When switches are on (First quadrant operation)

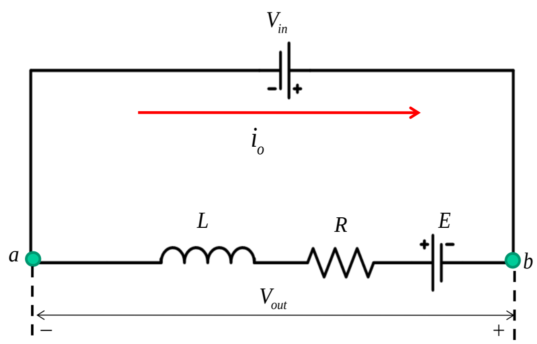


Figure 1c: When freewheeling diodes operate (Fourth quadrant operation)

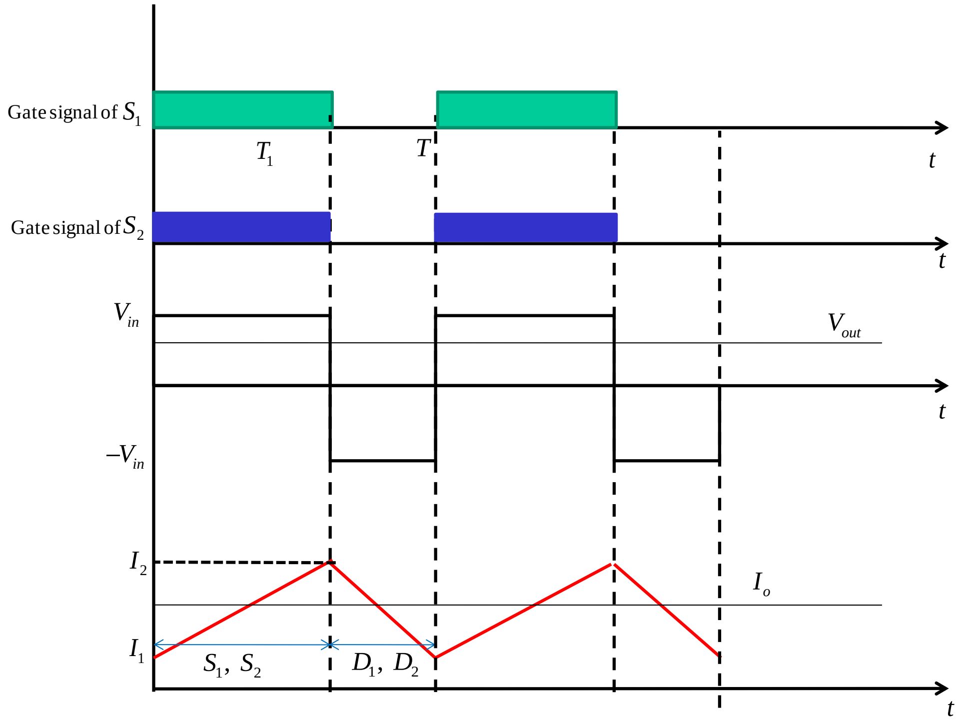


Figure 1d: Waveforms when $T_1 > T_{off}$

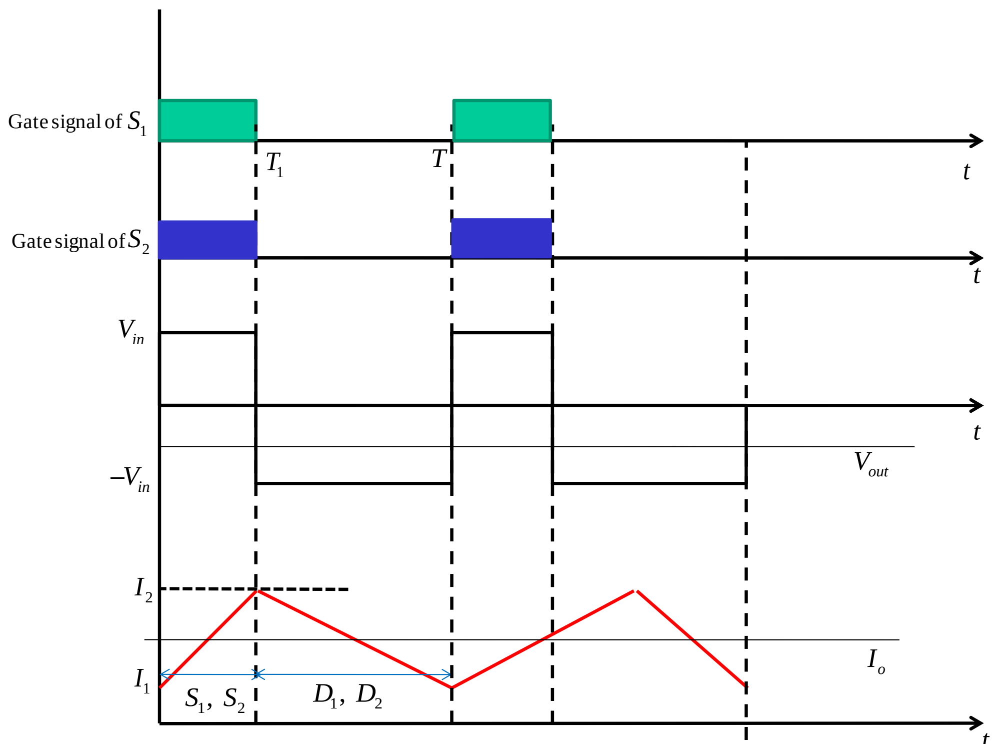


Figure 1e: Waveforms when $T_1 < T_{off}$

Four Quadrant Converters

A ***four quadrant*** converter is shown in **Figure 2**. The circuit is operated as a two quadrant converter to obtain:

a. ***Sequence 1:*** First and second quadrant operation
b. ***Sequence 2:*** Third and fourth quadrant operation

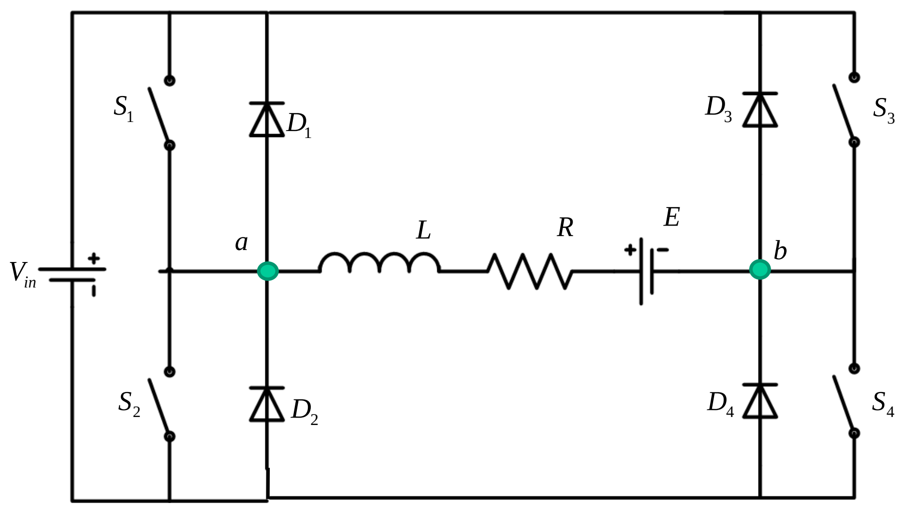


Figure 2: Four quadrant converter

Sequence 1 Operation

In this mode S_4 is kept permanently ***on***. The switches S_1 and S_2 are controlled as per the following four steps:

- ***Mode 1:*** If S_1 and S_4 are turned ***on***, the input voltage V_{in} is applied across the load and current flows in the positve direction from a to b **Figure 3a**. The instantaneous output voltage across the load is $v_{out} = V_{in}$.
- ***Mode 2:***When S_1 is turned ***off*** at time $t = T_1$, the current due to the stored $\left(\frac{1}{2}\right) L i_o^2$ energy of the inductor L drives through D_2 and S_4 as shown in **Figure 3b**. The switch S_2 is turned ***on*** at $t = T_1$ but it doesnot conduct because it is shorted by D_2.
- ***Mode 3:*** The switch S_2 conducts when the cureent reverses its direction (**Figure 3c**).

- ***Mode 4:*** Finally when S_2 is turned ***off*** at $t = T$, current flows in the negative direction (**Figure 3d**). The converter operates in the ***fourth quadrant*** and the power flows from load to source.

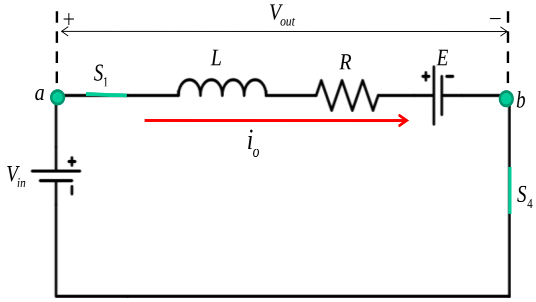


Figure 3a: Mode 1 operation of sequence 1

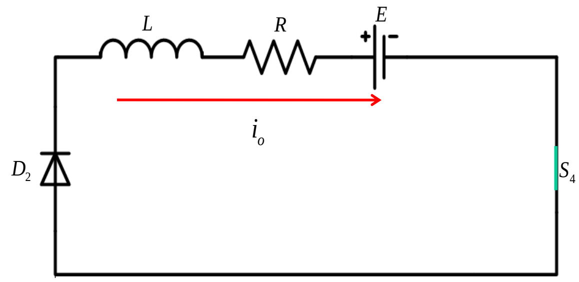


Figure 3b: Mode 2 operation of sequence 1

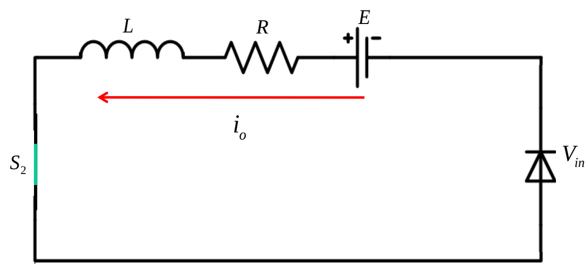


Figure 3c: Mode 3 operation of sequence 1

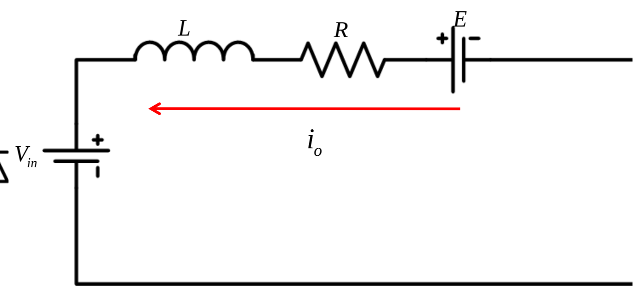


Figure 3d: Mode 4 operation of sequence 1

The wavforms for ***sequence 1*** are shown in **Figure 4**.

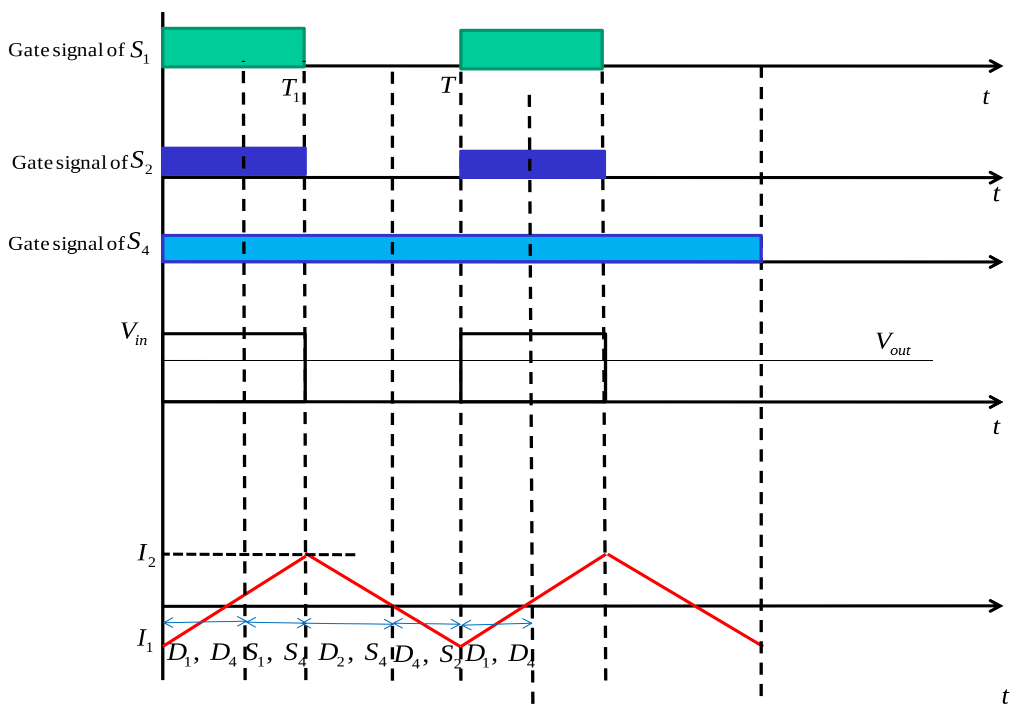


Figure 4: Waveforms for sequence 1

Sequence 2Operation

In this sequence, the converter operates in **third** *and* **fourth** quadrant and the switch S_3 is permanently kept ***on***. The switches S_1 and S_2 are controlled as per the following four steps:

- ***Mode 1:*** S_2 is turned ***on*** at $t=0$ but starts conduction only when the current changes sign. The diodes D_2 and D_3 conduct (**Figure 5a**) till the current changes itrs sign. The instantaneous output voltage across the load is $v_{out} = -V_{in}$.
- ***Mode 2:*** When S_2 is turned ***off*** at $t=T_1$, the inductor continuous to drive the current in the reverse direction through S_3 and D_1 (**Figure 5b**).
- ***Mode 3:*** The switch S_1 is turned ***on*** at $t=T_1$ but does not conduct because the current flows in the negative direction and D_1 and S_3 conduct. Once the current changes the sign S_1 and D_3 conduct D_1 (**Figure 5c**).
- ***Mode 4:*** When S_1 is turned off at $t=T$, $V_{out} = -V_{in}$ but positive current flows, hence, D_2 and D_3 conduct D_1 (**Figure 5d**).

The waveforms for this sequence are shown in **Figure 6**.

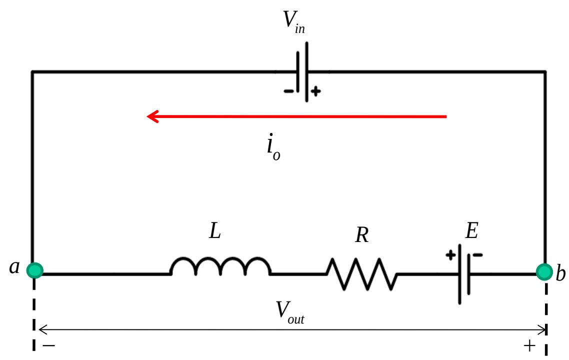


Figure 5a: Mode 1 operation of sequence 2

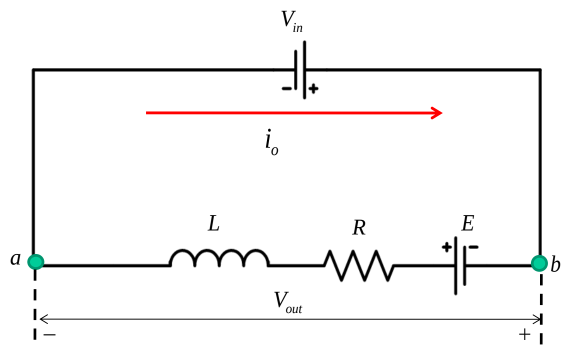


Figure 5b: Mode 2 operation of sequence 2

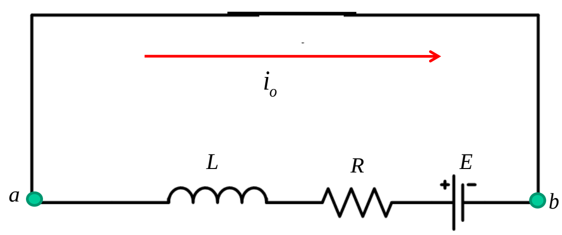


Figure 5c: Mode 3 operation of sequence 2

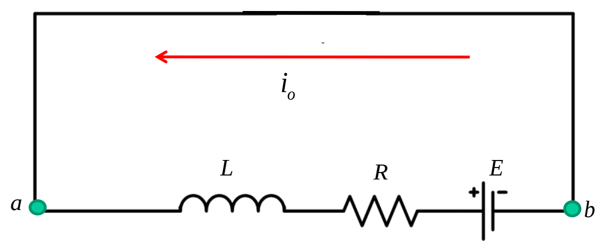


Figure 5d: Mode 4 operation of sequence 2

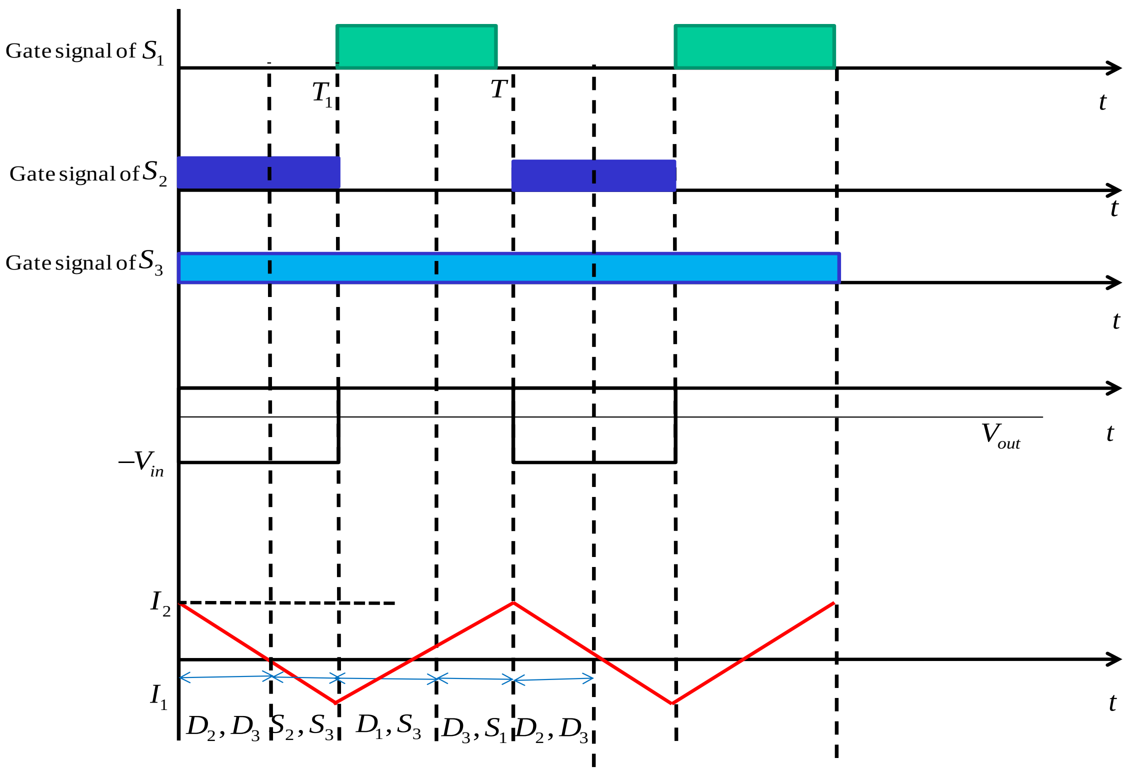


Figure 6: Waveforms for sequence 2

Suggested Reading:

[1] M. H. Rashid, *Power Electronics: Circuits, Devices and Applications*, 3rd edition, Pearson, 2004

[2] V. R. Moorthi, *Power Electronics: Devices, Circuits and Industrial Applications*, Oxford University Press, 2007

Lecture 13: DC-DC Converters for EV and HEV Applications

DC-DC Converters for EV and HEV Applications

Introduction

The topics covered in this chapter are as follows:

- Multi-input DC-DC Converters
- Multi-input converter Using High/Low Voltage Sources
- Flux Additive DC-DC Converter

Multi-input DC-DC Converters

The rechargable batteries are the common energy sources for EVs. In order to achive performance comparable to internal combustion engine vehicle (ICEV), the EVs are powered by an energy source consisting of batteryand ultracapacitors. The battery pack supplies the main power and the high power requirements, such as during acceleration, is suplied by supercapacitor bank. Combination of battery and supercapacitor bank enables use of smaller battery pack. In **Figure 1** a configuration of an EV with a battery bank and Ultracapacitor bank is shown.

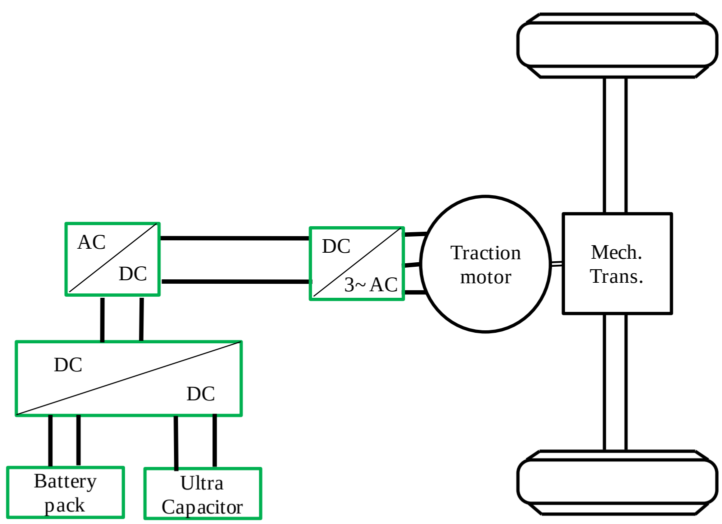


Figure 1: An EV with two input sources [1]

In order to suplly the traction motor with two sources, multi-input configuration of D-DC converters are used. The multi input DC-DC converters are calssified into following two categories:

- Multi-input Converter Using High/Low Voltage Sources
- Flux additive dc-dc converter.

Multi-input Converter Using High/Low Voltage Sources

The Multi input converters can be classified into follwoing types:

- ***Type 1: Buck-Buck Converter* (Figure 2a)**
- ***Type 2: BuckBoost-BuckBoost Converter* (Figure 2b)**
- ***Type 3: Buck-BuckBoost Converter* (Figure 2c)**
- ***Type 4: Boost-Boost Converter* (Figure 2d)**
- ***Type 5:Bidirectional BuckBoost-BuckBoost Converter* (Figure 2e)**

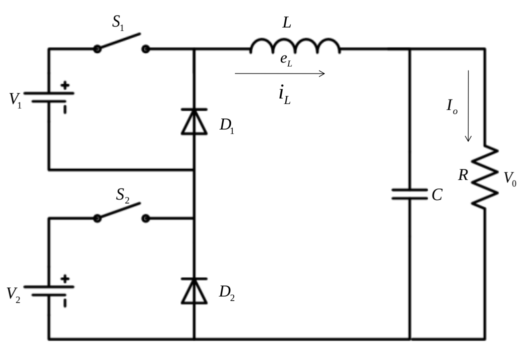


Figure 2a: Type 1: DC DC converter with two input voltages

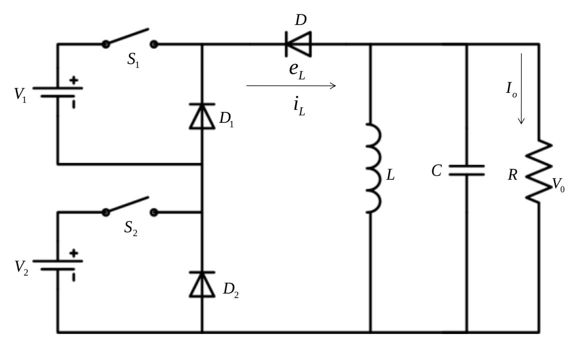


Figure 2b: Type 2: DC DC converter with two input voltages

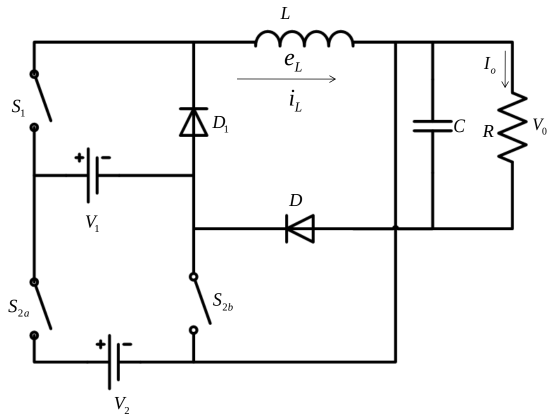


Figure 2c: Type 3: DC DC converter with two input voltages

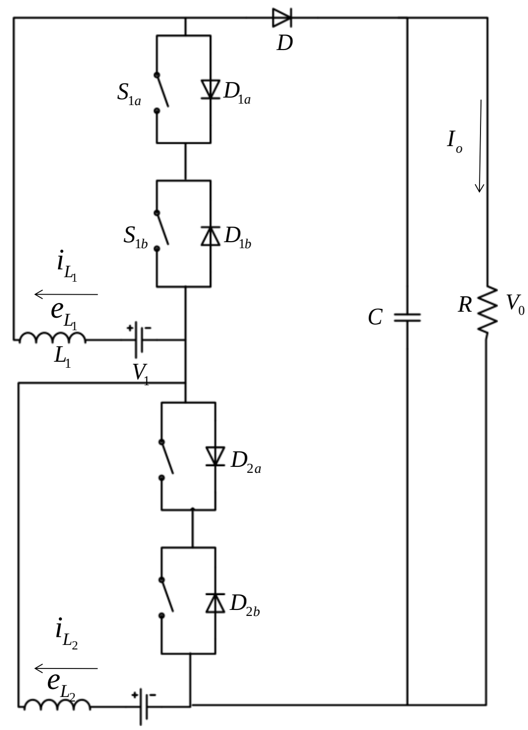


Figure 2d: Type 4: DC DC converter with two input voltages

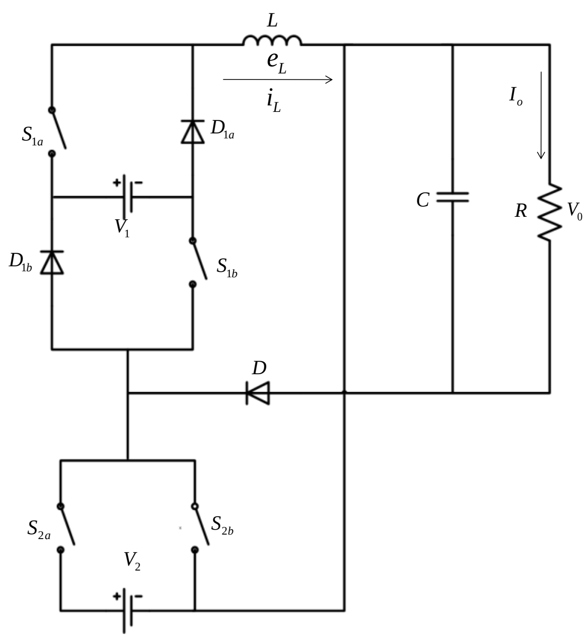


Figure 2e: Type 5: DC DC converter with two input voltages

In this chapter the functioning of ***Type 1*** Converter is shown. In the analysis given below, the following assumptions have been made:

- The converter has two input voltage sources V_1 and V_2.
- The two voltage sources have different magnitude and $V_1 > V_2$.

Type 1: Buck-Buck Converter

A DC-DC converter with multi input is shown in **Figure 2a**. By controling the switching of S_1 and switch S_2 the power can be extracted form two voltage sources (V_1, V_2) individually or simultaneously.

Based on the switching of S_1 and S_2, the converter's operation can be divided into four distinct modes, namely:

Mode 1: In this mode of operation, switch S_1 is turned ***on*** and S_2 is turned ***off***. The equivalent circuit for this mode is shown in **Figure 3a**. In this mode the voltage source V_1 provides power to the load resistor R. The potential across the inductor is

$$e_L = V_1 - V_o \tag{1a}$$

Mode 2: In this mode the switch S_1 is turned ***off*** and the S_2 turned ***on***. The equivalent circuit for this mode is shown in **Figure 3b**. In this mode of operation the voltage source V_2 charges the inductor L and supplies the load. The potential across the inductor is

$$e_L = V_2 - V_o \tag{1b}$$

Mode 3: Both the switches S_1 and S_2 are turned ***off***. The diodes D_1 and D_2 provide the current path for the inductor current (**Figure 3c**). The energy stored in L and C is released to the load. The potential across the inductor is

$$e_L = -V_o \tag{1c}$$

Mode 4: The switches S_1 and S_2 are turned ***on*** and both the voltage sources V_1 and V_2 are connected in series and chrge the inductor L and supply to the load. The configuration of the circuit in this mode is shown in **Figure 3d**. The voltage across the inductor is

$$e_L = V_1 + V_2 - V_o \tag{1d}$$

The waveforms for the different modes of operation are shown in **Figure 4**.

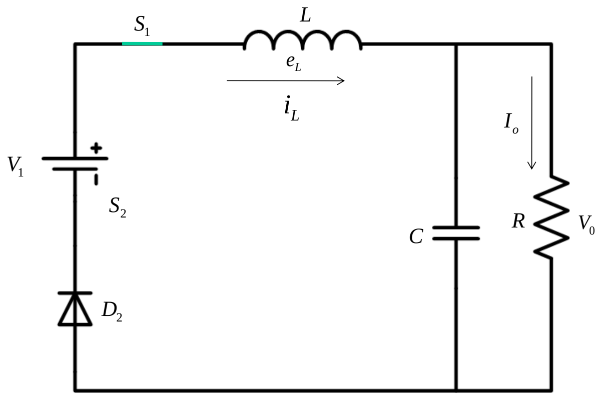


Figure 3a: Mode 1 operation

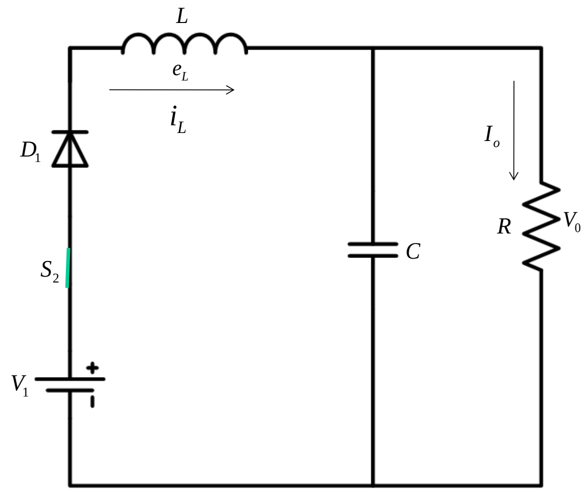


Figure 3b: Mode 2 operation

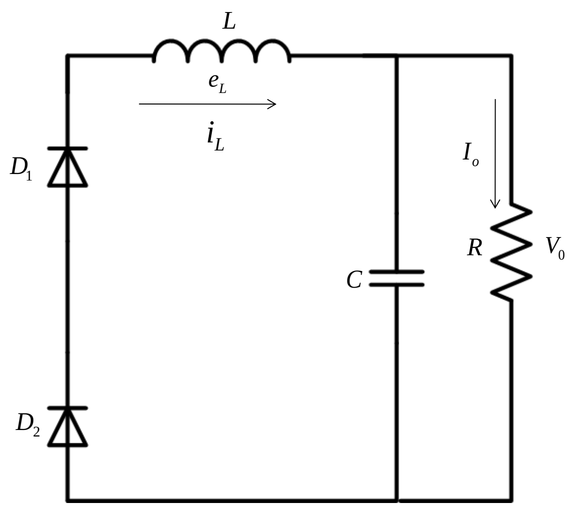


Figure 3c: Mode 3 operation

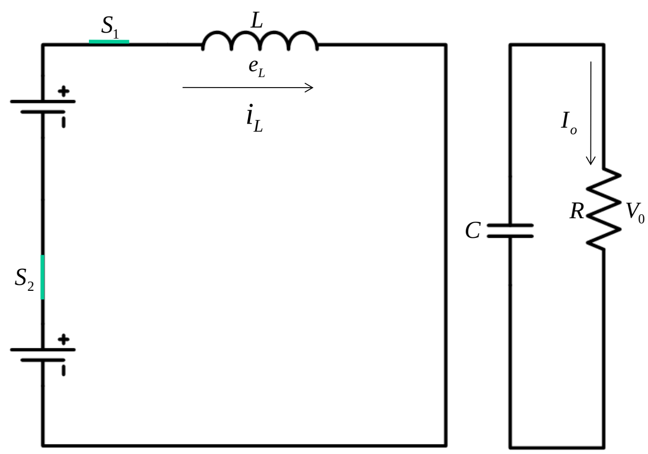


Figure 3d: Mode 4 operation

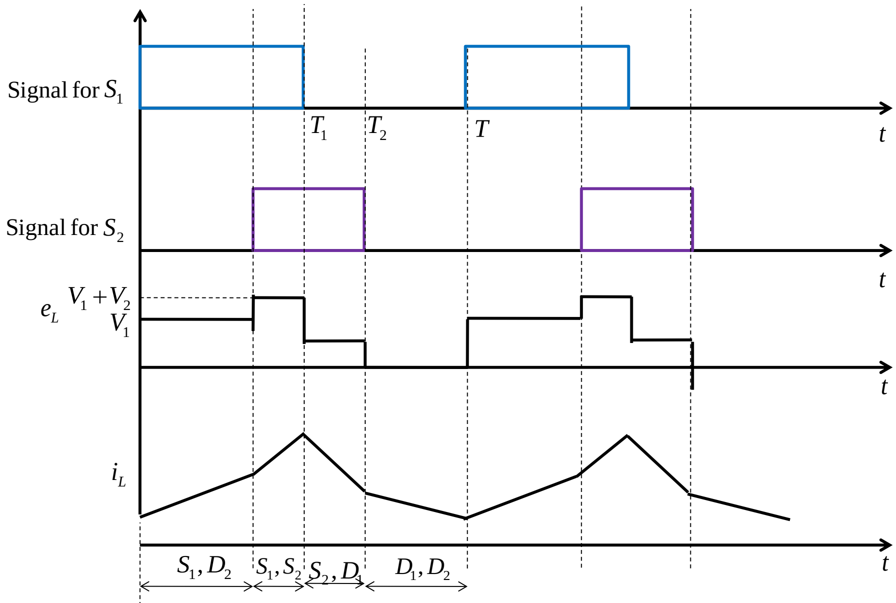


Figure 4: The voltage and current waveforms

Having discussed the ***Type*** 1 ***Multi-input Converter Using High/Low Voltage Sources***, the next section deals with the ***Flux additive dc-dc converter***.

Flux Additive DC-DC Converter.

A schematic diagram of a ***flux additive dc-dc converter*** is shown in **Figure 5** [2]. The converter consists of:

- Two voltage sources
- Three winding coupled transformer
- Common output stage circuit

The converter is fundamentally composed of the buck-boost type dc-dc converter. Based on the switching scheme of the switches, the operation of the converter is divided into 12 modes:

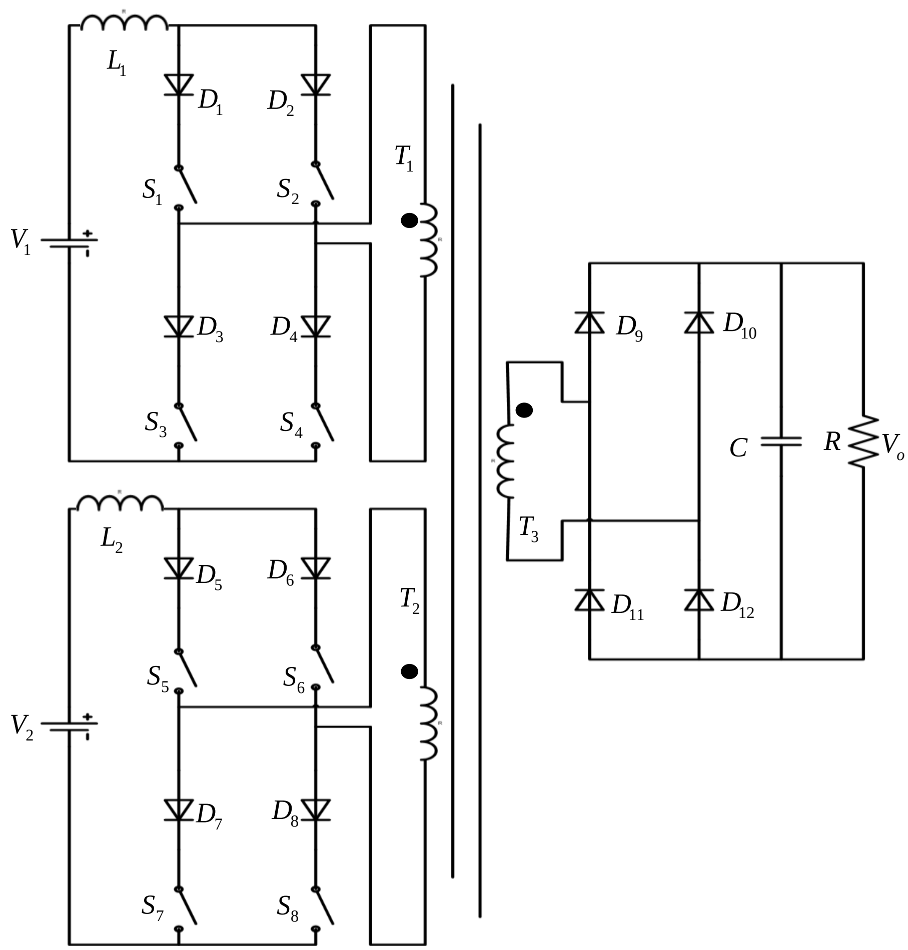


Figure 5: Flux additive multi input DC-DC converter [2]

Mode 1: From time_$0 \le t < t_1$, the switches S_2 and S_3 are turned ***off****i* and the switches S_1 and S_4 are turned ***on***. The power flows from the first input input stage supplied by voltage source V_1. The input current from the first stage (i_{in1}) flows through the transformer T_1, D_1, S_1, D_4 and S_4. The input current of the second stage (i_{in2}) freehwheels. The magnetic flux produced by i_{in1} induces emf in the other transformer windings. Due to this induced emf, the current through the output transformer is i'_{in3}. The magnitude of the current i'_{in2} is ***zero*** because no closed path is available for the current. Due to the direction of the current i'_{in3} the diodes D_9 and D_{12} in the output stage circuit turned on. The equivalent circuit for this mode of operation is shown in **Figure 6a**.

Mode 2: In the time interval $t_1 \le t < t_2$, the switches S_1, S_4, S_5, S_7 and S_8 are ***on***. The equivalent circuit for this mode is shown in **Figure 6b**. The switch S_8 is on but it doesnot conduct. The input current of the second stage (i_{in2}) still freehwheels thorugh D_5, S_5, D_7 and S_7. The operations of the first input stage and the output stage circuits remain unchanged.

Mode 3: This mode lasts for the time interval $t_2 \le t < t_3$. At time $t = t_2$, the switch S_7 is turned ***off***. The equivalent circuit is shown in **Figure 6c**. The current i_{in2} does not freewheel anymore and flows through D_5, S_5, D_8 and S_8. Operation of first input stage remains unchanged. In this mode, both the input stages transfer power to the output stage. The contribution of bothe the sources can be explained as follows: since both the currents (i_{in1}, i_{in2}) flow through the windings of transformets T_1 and T_2 respectively, the flux linked by the output stage transfor T_3 increases and hence, the current through T_3 is increased resulting in more power flow to the load.

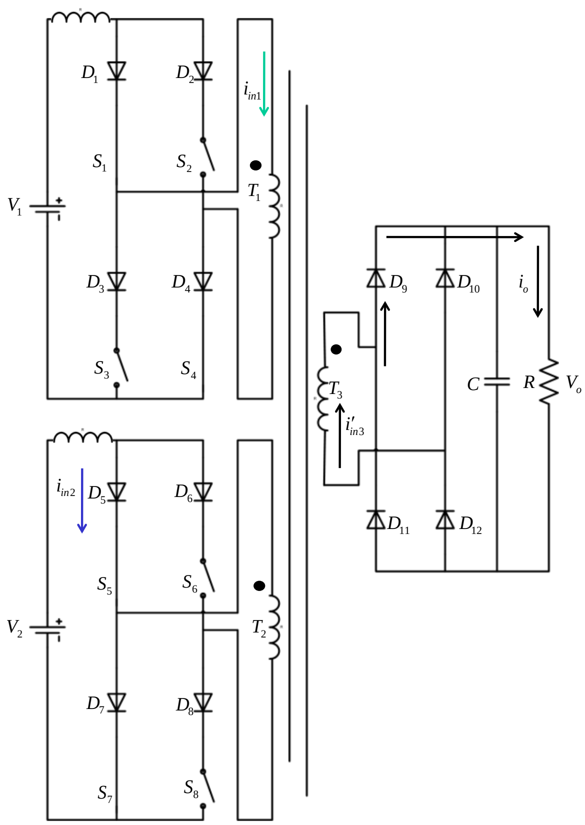


Figure 6a: Mode 1 operation [2]

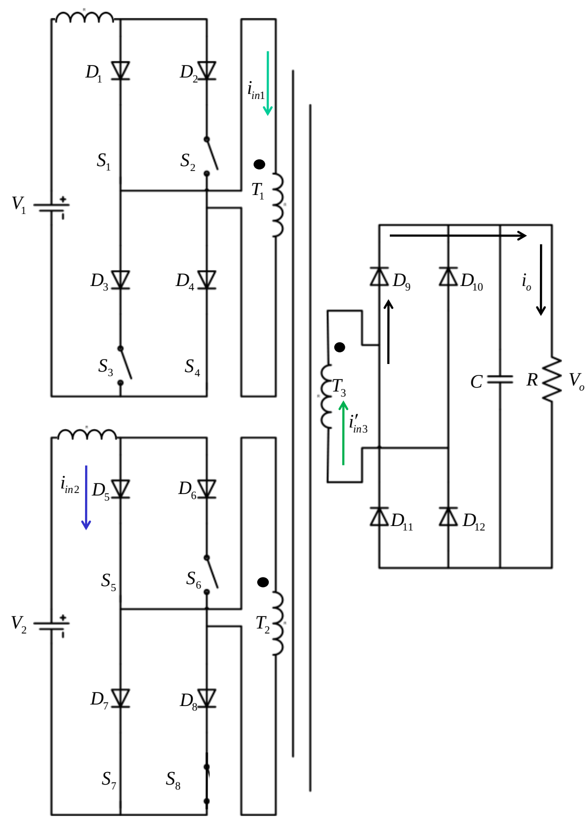


Figure 6b: Mode 2 operation [2]

Mode 4: This modes lasts for the time duration $t_3 \le t < t_4$. At time $t = t_3$ the switches S_2 and S_6 are turned ***on***. The switches S_1 and S_5 are still ***on*** but do not conduct any current (**Figure 6d**). The current i_{in1} freewhells through D_2, S_2, D_4 and S_4 , whereas the current i_{in2} freewheels through D_6, S_6, D_8 and S_8 and no current flows through the transformers T_1 and T_2. As a result of this the no emf is induced in the transformer T_3 and the diodes in the output side ($D_9, D_{10}, D_{11}, D_{12}$) are reverse biased. Hence, no power is transferred from any input stage to the output stage. The power demanded by the load is supplied by the output capacitor C.

Mode 5: The duration of this mode is $t_4 \le t < t_5$. At time $t = t_4$, the switches S_1 and S_5 are turned ***off***. The current i_{in1} and i_{in2} freewheel and no power is transferred from the sources to the load. The equivalent circuit for this mode is shown in **Figure 6e**.

Mode 6: This mode lasts for time duration $t_5 \le t < t_6$. At time $t = t_5$,the switch S_3 is turned on. The rest of the circuit behaves as in ***mode 5*** and no power is transferred from the input stage to the output stage. The equivalent circuit is shown in **Figure 6f**.

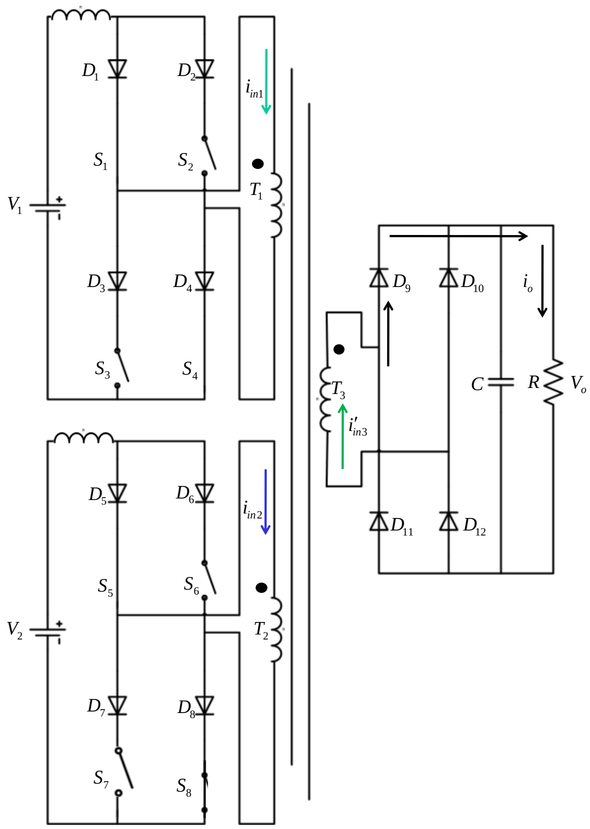


Figure 6c: Mode 3 operation [2]

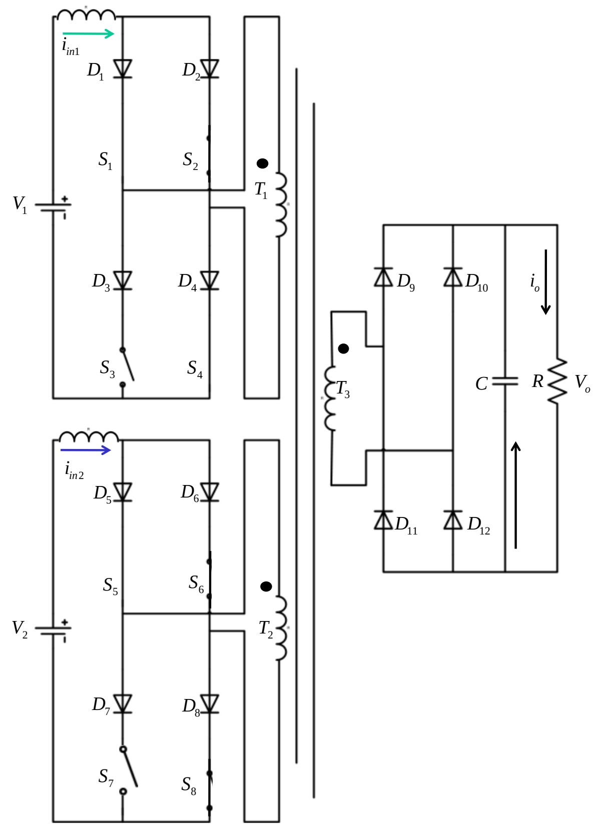


Figure 6d: Mode 4 operation [2]

Mode 7: This mode begins at time $t = t_6$ and the switch S_4 is turned on. The equivalent circuit is shown in **Figure 6g**. The circuit of **Figure 6g** is similar to that of **Figure 6a** except that the polarity of the transformer emfs and currents are opoosite. Consequently, ***mode 8*** to ***mode 12*** are symmetric to ***mode 2*** to ***6***. The equivalent circuits of ***mode 8*** to ***mode 12*** are shown in **Figure 6h** to **Figure 6l**.

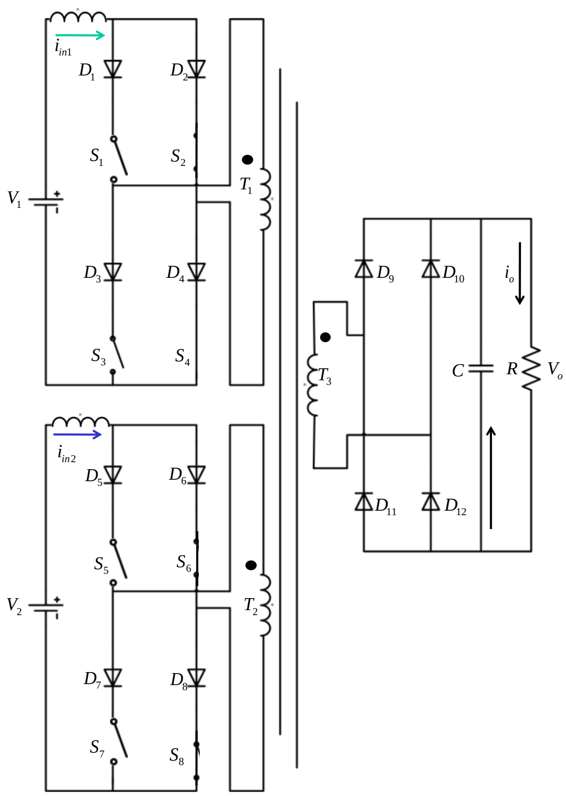


Figure 6e: Mode 5 operation [2]

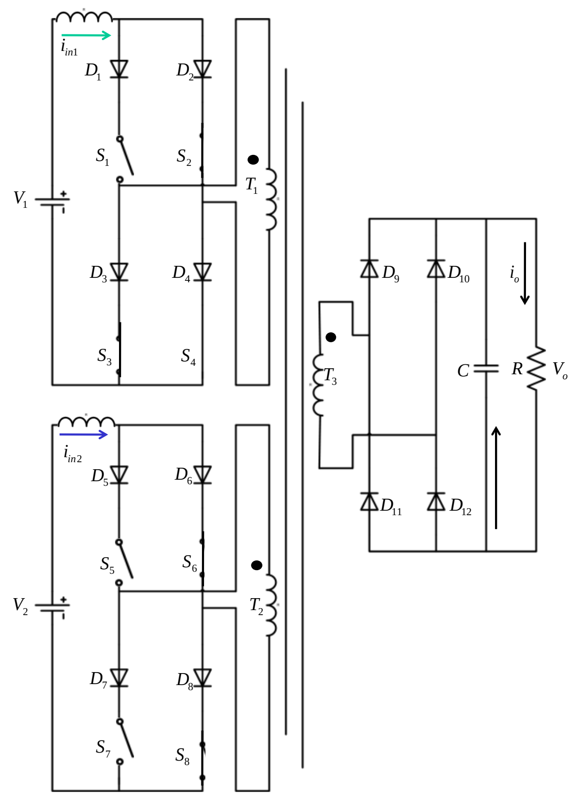


Figure 6f: Mode 6 operation [2]

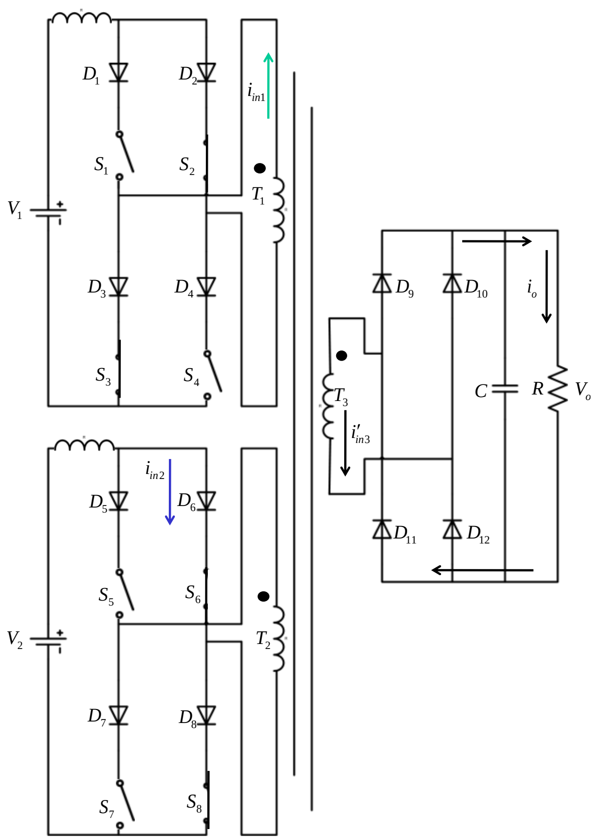


Figure 6g: Mode 7 operation [2]

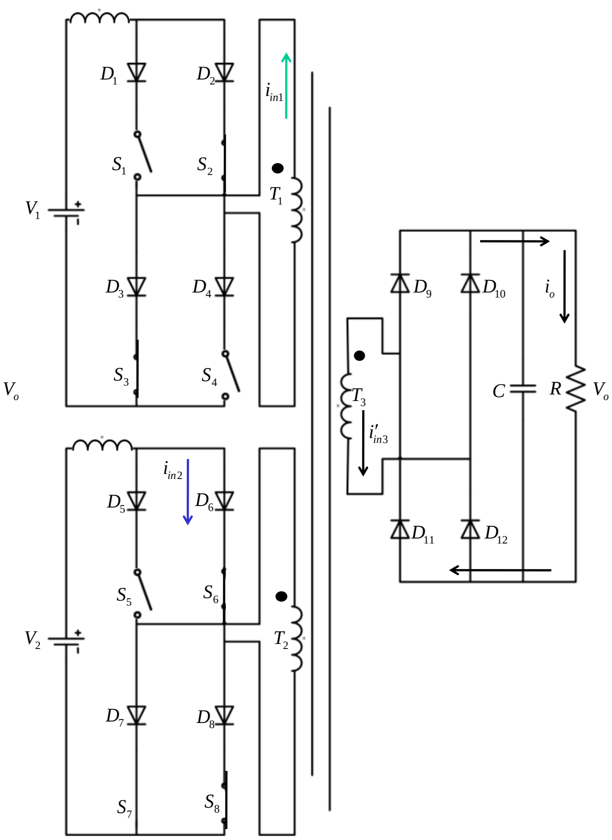


Figure 6h: Mode 8 operation [2]

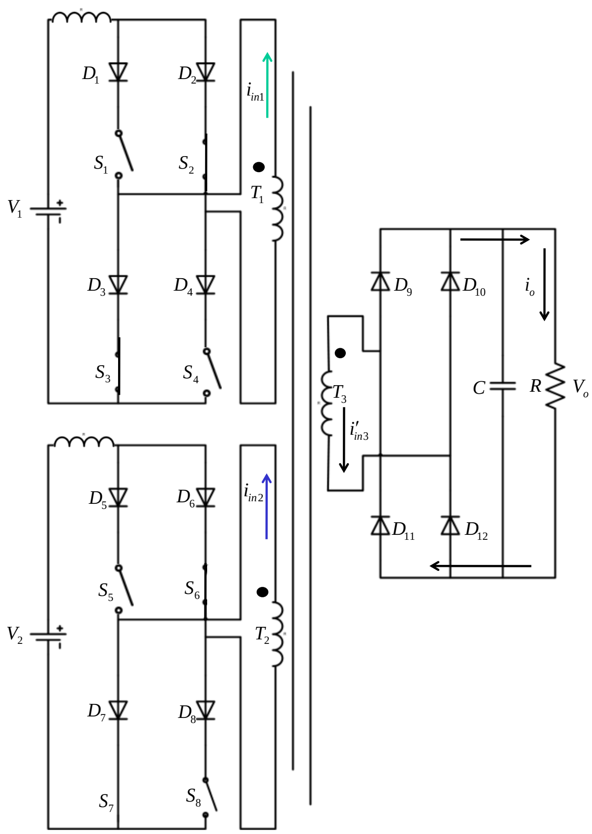


Figure 6i: Mode 9 operation [2]

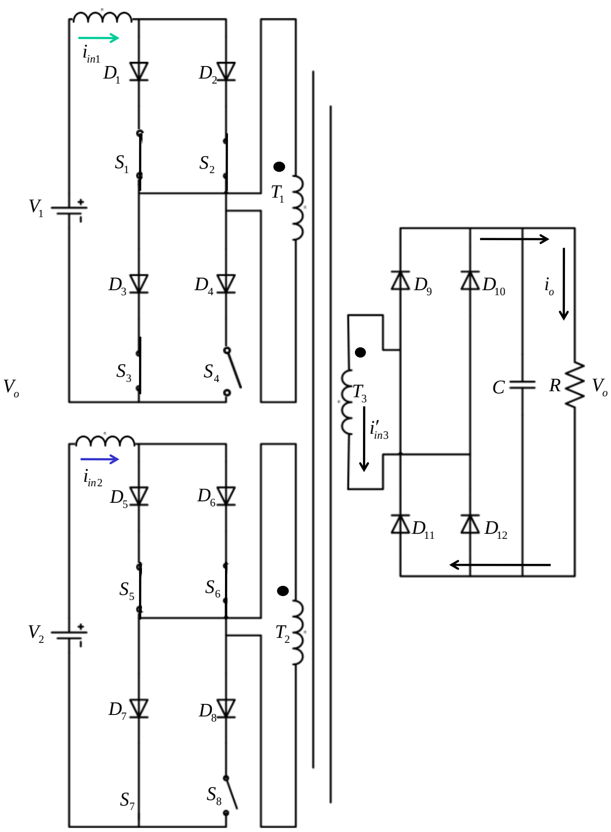


Figure 6j: Mode 10 operation [2]

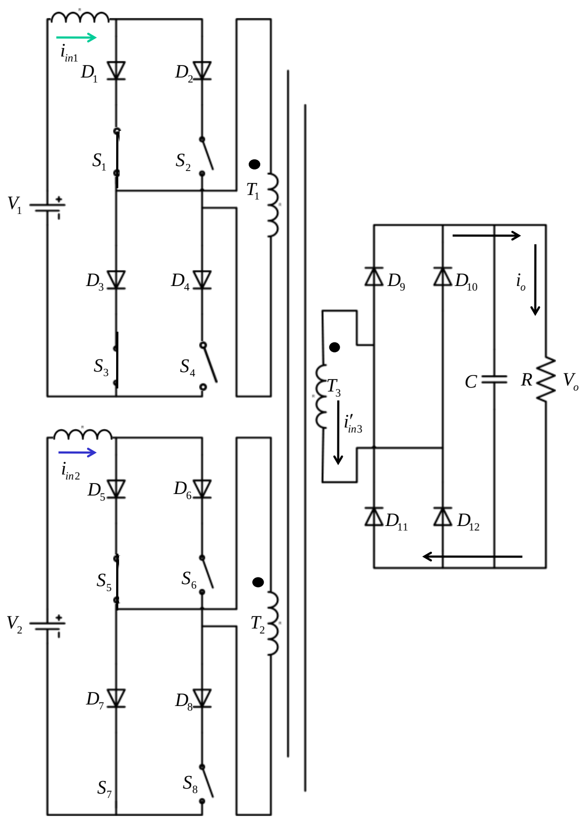


Figure 6k: Mode 11 operation [2]

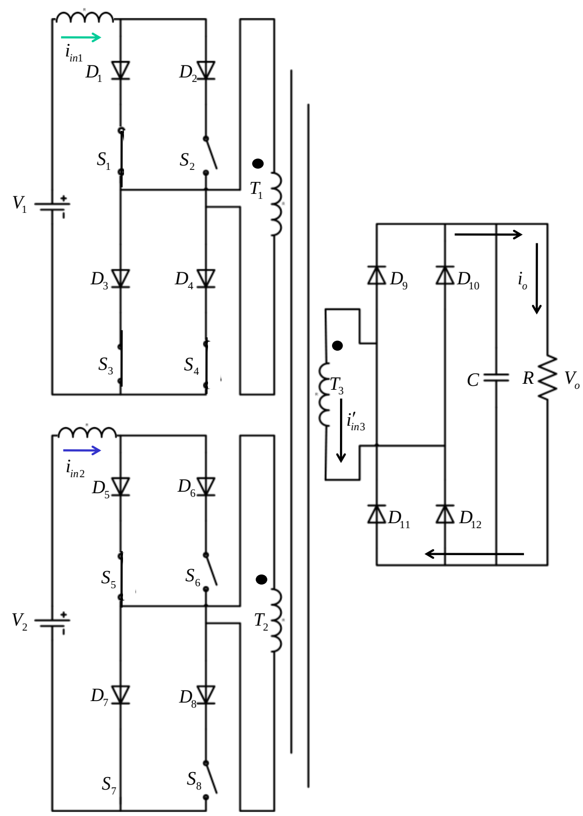


Figure 6l: Mode 12 operation [2]

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[1] M. Ehsani, *Modern Electric, Hybrid Electric and Fuel Cell Vehicles: Fundamentals, Theory and Design*, CRC Press, 2005

[2] Yaow-Ming Chen, et. al. „Double Input PWM DC/DC Converter for Hig/Low Voltage Sources“, IEEE Transactions on Industrial Electronics, Vol.53, No.5, pp.1538-1545.

Suggested Reading:

[1] M. H. Rashid, *Power Electronics: Circuits, Devices and Applications*, 3rd edition, Pearson, 2004

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