

## Module 2: Dynamics of Electric and Hybrid vehicles

### Lecture 3: Motion and dynamic equations for vehicles

#### Motion and dynamic equations for vehicles

##### Introduction

The fundamentals of vehicle design involve the basic principles of physics, specially the **Newton's second law of motion**. According to Newton's second law the **acceleration of an object is proportional to the net force exerted on it**. Hence, an object accelerates when **the net force acting on it is not zero**. In a vehicle several forces act on it and the net or resultant force governs the motion according to the Newton's second law. The propulsion unit of the vehicle delivers the force necessary to move the vehicle forward. This force of the propulsion unit helps the vehicle to overcome the resisting forces due **to gravity, air and tire resistance**. The **acceleration of the vehicle depends on:**

- the power delivered by the propulsion unit
- the road conditions
- the aerodynamics of the vehicle
- the composite mass of the vehicle

In this lecture the mathematical framework required for the analysis of vehicle mechanics based on Newton's second law of motion is presented. The following topics are covered in this lecture:

- General description of vehicle movement
- Vehicle resistance
- Dynamic equation
- Tire Ground Adhesion and maximum tractive effort

### General description of vehicle movement

The vehicle motion can be completely determined by analysing the forces acting on it in the direction of motion. The forces acting on a vehicle, moving up a grade, are shown in Figure 1. The tractive force ( $F_t$ ) in the contact area between the tires of the driven wheels and the road surface propels the vehicle forward. The tractive force ( $F_t$ ) is produced by the power plant and transferred to the driving wheels via the transmission and the final drive. When the vehicle moves, it encounters a resistive force that tries to retard its motion. The resistive forces are

- Rolling resistance
- Aerodynamic drag
- Uphill resistance

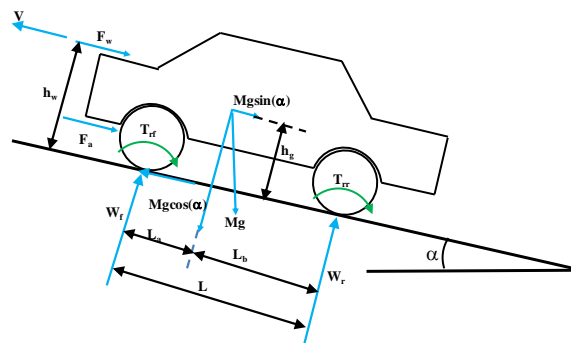


Figure 1: Forces acting on a vehicle going uphill [1]

Using the Newton's second law of motion, the vehicle acceleration can be expressed as

$$\frac{dV}{dt} = \frac{\sum F_t - \sum F_{resistac}}{\delta M}$$

where

$V$  = vehicle speed

$\sum F_t$  = total tractive effort [Nm]

$\sum F_{resistance}$  = total resistance [Nm]

$M$  = total mass of the vehicle [kg]

$\delta$  = mass factor for converting the rotational inertias of rotating components into translational mass

(1)

## Rolling resistance

The rolling resistance of tires on hard surfaces is **due to hysteresis in the tire material**. In **Figure 2** a tire at standstill is shown. On this tyre a force ( $\mathbf{P}$ ), is acting at its centre. The pressure in the contact area between the tire and the ground is distributed symmetrically to the centre line and the resulting reaction force ( $\mathbf{P}_z$ ) is aligned along  $\mathbf{P}$ .

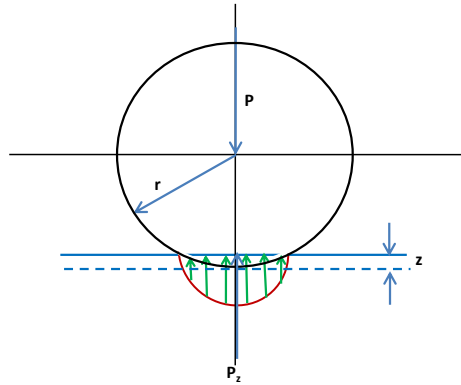


Figure 2: Pressure distribution in contact area [1]

The deformation,  $z$ , versus the load  $\mathbf{P}$ , in the loading and unloading process is shown in **Figure 3**. From this figure it can be seen that, due to the hysteresis, the force ( $\mathbf{P}$ ) for the same deformation ( $z$ ) of the tire material at loading is greater than at during unloading. Hence, the hysteresis causes an asymmetric distribution of the ground reaction forces.

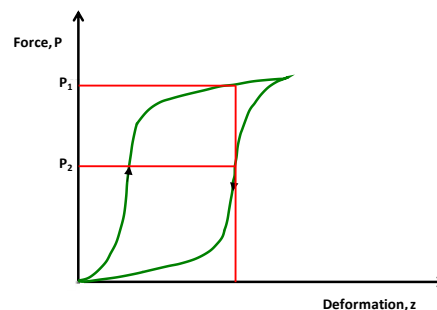


Figure 3: Force acting on a tyre vs. deformation in loading and unloading [1]

The scenario of a rolling tire is shown in **Figure 4**. When the tire rolls, the leading half of the contact area is loading and the trailing half is unloading. Thus, the pressure on the leading half is greater than the pressure on the trailing half (**Figure 4a**). **This phenomenon results in the ground reaction force shifting forward.** The shift in the ground reaction force creates a moment that opposes rolling of the wheels. On soft surfaces, the rolling resistance is mainly caused by deformation of the ground surface, (**Figure 4b**). In this case the ground reaction force almost completely shifts to the leading half.

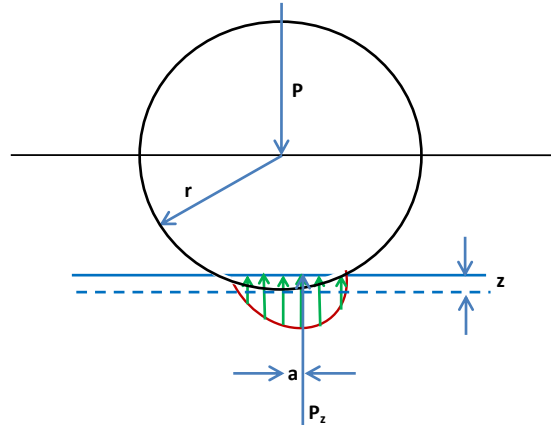


Figure 4a: Force acting on a tyre vs. deformation in loading and unloading on a hard surface [1]

The moment produced by forward shift of the resultant ground reaction force is called **rolling resistance moment (Figure 4a)** and can be expressed as

$$T_r = Pa = Mga$$

where

$T_r$  = rolling resistance [ $Nm$ ]

$P$  = Normal load acting on

the centre of the rolling wheel [ $N$ ]

$M$  = mass of the vehicle [ $kg$ ]

$g$  = acceleration constant [ $m/s^2$ ]

$a$  = deformation of the tyre [ $m$ ]

(2)

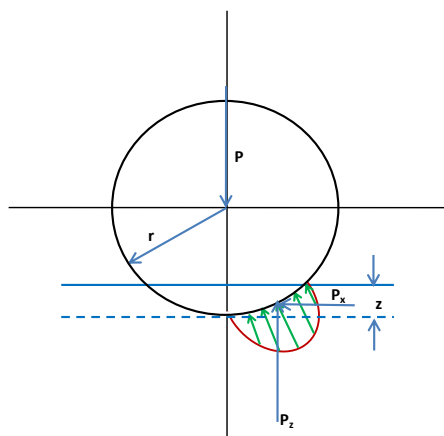


Figure 4a: Force acting on a tyre vs. deformation in loading and unloading on a soft surface [1]

To keep the wheel rolling, a force  $\mathbf{F}_r$ , acting on the centre of the wheel is required to balance this rolling resistant moment. This force is expressed as

$$F_r = \frac{T_r}{r_{dyn}} = \frac{Pa}{r_{dyn}} = Pf_r$$

where

$$T_r = \text{rolling resistance [Nm]} \quad (3)$$

$P$  = Normal load acting on the centre of the rolling wheel [N]

$r_{dyn}$  = dynamic radius of the tyre [m]

$f_r$  = rolling resistance coefficient

The rolling resistance moment can be equivalently replaced by horizontal force acting on the wheel centre in the direction opposite to the movement of the wheel. This equivalent force is called **the rolling resistance** and its magnitude is given by

$$F_r = Pf_r$$

where

(4)

$P$  = Normal load acting on the centre of the rolling wheel [N]

$f_r$  = rolling resistance coefficient

When a vehicle is moving up a gradient, the normal force ( $\mathbf{P}$ ), in **equation 4**, is replaced by the component that is perpendicular to the road surface. Hence, **equation 4** is rewritten as

$$F_r = Pf_r \cos(\alpha) = Mgf_r \cos(\alpha)$$

where

(5)

$P$  = Normal load acting on the centre of the rolling wheel [N]

$f_r$  = rolling resistance coefficient

$\alpha$  = road angle [radians]

The rolling resistance coefficient,  $f_r$ , is a function of:

- tire material
- tire structure
- tire temperature
- tire inflation pressure
- tread geometry
- road roughness
- road material
- presence of absence of liquids on the road

The typical values of the rolling resistance coefficient ( $f_r$ ) are given in **Table 1**.

**Table 1: Reference values for the rolling resistance coefficient ( $f_r$ )**

Conditions	Rolling resistance coefficient ( $f_r$ )
Car tire on smooth tarmac road	0.01
Car tire on concrete road	0.011
Car tire on a rolled gravel road	0.02
Tar macadam road	0.025
Unpaved road	0.05
Bad earth tracks	0.16
Loose sand	0.15-0.3
Truck tire on concrete or asphalt road	0.006-0.01
Wheel on iron rail	0.001-0.002

The values given in table 1 do not take into account the variation of  $f_r$  with speed. Based on experimental results, many empirical formulas have been proposed for calculating the rolling resistance on a hard surface. For example, the rolling resistance coefficient of a passenger car on a concrete road may be calculated as:

$$f_r = f_0 + f_s \left( \frac{V}{100} \right)^{2.5}$$

where (6)

$V$  = vehicle speed [ $km/h$ ]

In vehicle performance calculation, it is sufficient to consider the rolling resistance coefficient as **a linear function of speed**. For most common range of inflation pressure, the following equation can be used for a passenger car on a concrete road

$$f_r = 0.01 \left( 1 + \frac{V}{160} \right) \quad (7)$$

where

$V$  = vehicle speed [ $km/h$ ]

The **equation 7** can predict the values of  $f_r$  with acceptable accuracy for speed up to 128km/h.

### **Aerodynamic drag**

A vehicle traveling at a particular speed in air encounters **a force resisting its motion**. This force is known as aerodynamic drag. The main causes of aerodynamic drag are:

- shape drag
- skin effect

The shape drag is due to **the shape of the vehicle**. The forward motion of the vehicle pushes the air in front of it. However, the air cannot instantaneously move out of the way and its pressure is thus increased. This results in high air pressure in the front of the vehicle. The air behind the vehicle cannot instantaneously fill the space left by the forward motion of the vehicle. This creates a zone of low air pressure. Hence, the motion of the vehicle creates two zones of pressure. The high pressure zone in the front of the vehicle opposes its movement by pushing. On the other hand, the low pressure zone developed at the rear of the vehicle opposes its motion by pulling it backwards.

The air close to the skin of the vehicle moves almost at the speed of the vehicle while the air away from the vehicle remains still. Between these two layers (the air layer moving at the vehicle speed and the static layer) the molecules move at a wide range of speeds. The difference in speed between two air molecules produces friction. This friction results in the second component of aerodynamic drag and it is known as skin effect.

The aerodynamic drag is expressed as

$$F_w = \frac{1}{2} \rho A_f C_D V^2 \quad (8)$$

where

$\rho$  = density of air [ $kg/m^3$ ]

$A_f$  = vehicle frontal area [ $m^2$ ]

$V$  = vehicle speed [ $m/s$ ]

$C_D$  = drag coefficient

The aerodynamic drag coefficients and the frontal area for different vehicle types are given in **Table 2**.

**Table 2: Reference values for drag coefficient ( $C_D$ ) and the frontal area ( $A_f$  in  $m^2$ ) for some vehicle types**

Vehicle	$C_D$	$A_f$
Motorcycle with rider	0.5-0.7	0.7-0.9
Open convertible	0.5-0.7	1.7-2.0
Limousine	0.22-0.4	1.7-2.3
Coach	0.4-0.8	6-10
Truck without trailer	0.45-0.8	6.0-10.0
Truck with trailer	0.55-1.0	6.0-10.0
Articulated vehicle	0.5-0.9	6.0-10.0

### Grading resistance

When a vehicle goes up or down a slope, its weight produces a component of force that is always directed downwards, **Figure 5**. This force component opposes the forward motion, i.e. the grade climbing. When the vehicle goes down the grade, this force component aids the vehicle motion. The grading resistance can be expressed as

$$F_g = Mg \sin(\alpha)$$

where

$$M = \text{mass of vehicle [kg]} \quad (9)$$

$$g = \text{acceleration constant [m/s}^2]$$

$$\alpha = \text{road angle [radians]}$$

In order to simplify the calculation, the road angle  $\alpha$ , is usually replaced by the grade value, when the road angle is small. The grade value is defined as (**Figure 5**)

$$i = \frac{H}{L} = \tan(\alpha) \approx \sin(\alpha) \quad (10)$$



In some literature, the tire rolling resistance and the grading resistance taken together and is called **road resistance**. The road resistance is expressed as

$$F_{rd} = F_f + F_g = Mg(f_r \cos(\alpha) + \sin(\alpha))$$

where

$M$  = mass of vehicle [kg]

$g$  = acceleration constant [ $m/s^2$ ]

$f_r$  = rolling resistance coefficient

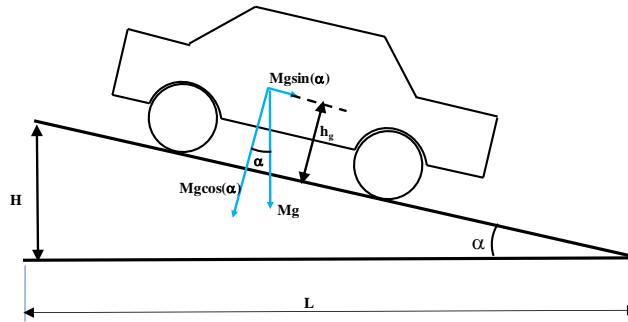


Figure 5: Vehicle going up a grade [1]

### Acceleration resistance

In addition to the driving resistance occurring in steady state motion, inertial forces also occur during acceleration and braking. The total mass of the vehicle and the inertial mass of those rotating parts of the drive accelerated or braked are the factors influencing the resistance to acceleration:

$$F_a = \left( M + \frac{\sum J_{rot}}{r_{dyn}^2} \right) \frac{dV}{dt}$$

where

$M$  = mass of vehicle [kg]

$J_{rot}$  = inertia of rotational components [ $kg \times m^2$ ]

$V$  = speed of the vehicle [km/h]

$r_{dyn}$  = dynamic radius of the tyre [m]

The rotational component is a function of the gear ratio. The moment of inertia of the rotating drive elements of engine, clutch, gearbox, drive shaft, etc., including all the road wheels are reduced to the driving axle. The acceleration resistance can be expressed as

$$F_a = \lambda M \frac{dV}{dt}$$

where

$\lambda$  = rotational inertia constant

$M$  = mass of the vehicle [kg]

$V$  = speed of the vehicle [m/s]

### Total driving resistance

The traction force ( $\mathbf{F}_t$ ) required at the drive wheels is made up of the driving resistance forces and is defined as

$$F_{resistance} = F_r + F_w + F_g + F_a \quad (14)$$

Substituting the values of all the forces in equation 14, gives

$$F_{resistance} = Mg f_r \cos(\alpha) + \frac{1}{2} \rho A_f C_D V^2 + Mg \sin(\alpha) + \lambda M \frac{dV}{dt} \quad (15)$$

The equation 15 may be used to calculate the power required ( $\mathbf{P}_{req}$ ):

$$P_{req} = F_{resistance} V \quad (16)$$

### Dynamic equation

In the longitudinal direction, the major external forces acting on a two axle vehicle (**Figure 1**) include:

- the rolling resistance of the front and rear tires ( $\mathbf{F}_{rf}$  and  $\mathbf{F}_{rr}$ ), which are represented by rolling resistance moment,  $\mathbf{T}_{rf}$  and  $\mathbf{T}_{rr}$
- the aerodynamic drag ( $\mathbf{F}_w$ )
- grade climbing resistance ( $\mathbf{F}_g$ )
- acceleration resistance ( $\mathbf{F}_a$ )

The dynamic equation of vehicle motion along the longitudinal direction is given by

$$M \frac{dV}{dt} = (F_{tf} + F_{tr}) - (F_{rf} + F_{rr} + F_w + F_g + F_a) \quad (17)$$

The first term on the right side is the total tractive effort and the second term is the total tractive resistance. To determine the maximum tractive effort, that the tire ground contact can support, the normal loads on the front and rear axles have to be determined. By summing the moments of all the forces about point  $\mathbf{R}$  (centre of the tire-ground area), the normal load on the front axle  $\mathbf{W}_f$  can be determined as

$$W_f = \frac{MgL_b \cos(\alpha) - \left( T_{rf} + T_{rr} + F_w h_w + Mgh_g \sin(\alpha) + Mh_g \frac{dV}{dt} \right)}{L} \quad (18)$$

Similarly, the normal load acting on the rear axle can be expressed as

$$W_r = \frac{MgL_a \cos(\alpha) - \left( T_{rf} + T_{rr} + F_w h_w + Mgh_g \sin(\alpha) + Mh_g \frac{dV}{dt} \right)}{L} \quad (19)$$

In case of passenger cars, the height of the centre of application of aerodynamic resistance ( $\mathbf{h}_w$ ) is assumed to be near the height of centre of gravity of the vehicle ( $\mathbf{h}_g$ ). The **equation 18** and **19** can be simplified as

$$W_f = \frac{L_b}{L} Mg \cos(\alpha) - \frac{h_g}{L} \left( F_w + F_g + Mgf_r \frac{r_{dyn}}{h_g} \cos(\alpha) + M \frac{dV}{dt} \right) \quad (20)$$

and

$$W_r = \frac{L_a}{L} Mg \cos(\alpha) - \frac{h_g}{L} \left( F_w + F_g + Mgf_r \frac{r_{dyn}}{h_g} \cos(\alpha) + M \frac{dV}{dt} \right) \quad (21)$$

Using **equation 5, 17, 20** and **21** can be rewritten as

$$W_r = \frac{L_a}{L} Mg \cos(\alpha) - \frac{h_g}{L} \left( F_t - F_r \left( 1 - \frac{r_{dyn}}{h_g} \right) \right) \quad (22)$$

$$W_f = \frac{L_b}{L} Mg \cos(\alpha) + \frac{h_g}{L} \left( F_t - F_r \left( 1 - \frac{r_{dyn}}{h_g} \right) \right) \quad (23)$$

The first term on the right hand side of **equation 22** and **equation 23** is the static load on the front and the rear axles when the vehicle is at rest on level ground. The second term is the dynamic component of the normal load.

The maximum tractive effort ( $\mathbf{F}_{tmax}$ ) that the tire-ground contact can support is described by the product of the normal load and the coefficient of road adhesion ( $\mathbf{m}$ ). In **Table 3**, the values of coefficient of adhesion are given for different speeds of the vehicle and different road conditions. For the front wheel drive vehicle,  $\mathbf{F}_{tmax}$  is given by

$$F_{tmax} = \mu W_f = \mu \left[ \frac{L_b}{L} Mg \cos(\alpha) - \frac{h_g}{L} \left( F_{tmax} - F_r \left( 1 - \frac{r_{dyn}}{h_g} \right) \right) \right] \quad (24)$$

$$F_{tmax} = \frac{\mu Mg \cos(\alpha) \left[ L_b + f_r (h_g - r_{dyn}) \right] / L}{1 + \mu h_g / L} \quad (25)$$

For the rear wheel drive vehicle,  $F_{tmax}$  is given by

$$F_{tmax} = \mu W_r = \mu \left[ \frac{L_b}{L} Mg \cos(\alpha) + \frac{h_g}{L} \left( F_{tmax} - F_r \left( 1 - \frac{r_{dyn}}{h_g} \right) \right) \right] \quad (26)$$

$$F_{tmax} = \frac{\mu Mg \cos(\alpha) \left[ L_a - f_r (h_g - r_{dyn}) \right] / L}{1 - \mu h_g / L} \quad (27)$$

**Table 3: Coefficient of road adhesion**

Road speed [km/h]	Coefficient of road adhesion for dry roads	Coefficient of road adhesion for wet roads
50	0.85	0.65
90	0.8	0.6
130	0.75	0.55

**Adhesion, Dynamic wheel radius and slip**

When the tractive effort of a vehicle exceeds the maximum tractive effort limit imposed by the adhesive capability between the tyre and ground, the driven wheels will spin on the ground. The adhesive capability between the tyre and the ground is the main limitation of the vehicle performance especially when the vehicle is driven on wet, icy, snow covered or soft soil roads.

The maximum tractive effort on the driven wheels, transferred from the power plant through the transmission should not exceed the maximum values given by **equation 25** and **equation 27**. Otherwise, the driven wheels will spin on the ground, leading to vehicle instability. The slip between the tyres and the surface can be described as:

$$drive\ slip\ S_T = \frac{\omega_R r_{dyn} - V}{\omega_R r_{dyn}} \quad (28)$$

where

$\omega_R$  = angular speed of the tyre [rad / s]

The dynamic wheel radius ( $r_{dyn}$ ) is calculated from the distance travelled per revolution of the wheel, rolling without slip. The dynamic wheel radius is calculated from a distance travelled at 60km/h. The increasing tyre slip at higher speeds roughly offsets the increase in  $r_{dyn}$ . The values of  $r_{dyn}$  for different tyre sizes are given in table 4.

Table 4: Dynamic wheel radius of common tyre sizes

Tyre Size	Rolling Circumference [m]	$R_{dyn}$ [m]	Tyre Size	Rolling Circumference [m]	$R_{dyn}$ [m]
<b>Passenger cars</b>			<b>Passenger cars</b>		
135 R 13	1.67	0.266	205/65 R15	1.975	0.314
145 R 13	1.725	0.275	195/60 R15	1.875	0.298
155 R 13	1.765	0.281	205/60 R 15	1.91	0.304
145/70 R 13	1.64	0.261	<b>Light commercial vehicles</b>		
155/70 R13	1.68	0.267	185 R 14	1.985	0.316
165/70 R 13	1.73	0.275	215 R 14	2.1	0.334
175/70 R 13	1.77	0.282	205 R 14	2.037	0.324
175 R 14	1.935	0.308	195/75 R 16	2.152	0.343
185 R 14	1.985	0.316	205/75 R 16	2.2	0.35
195/70 R 14	1.94	0.309	<b>Trucks and buses</b>		
185/65 R 14	1.82	0.29	12 R 22.5	3.302	0.526
185/60 R 14	1.765	0.281	315/80 R 22.5	3.295	0.524
195/60 R	1.8	0.286	295/80 R	3.215	0.512

14			22.5		
195/70 R			215/75 R		
15	2	0.318	17.5	2.376	0.378
185/65			275/70 R		
R15	1.895	0.302	22.5	2.95	0.47
195/65			305/70 R		
R15	1.935	0.308	19.5	2.805	0.446

**References:**

[1] M. Ehsani, *Modern Electric, Hybrid Electric and Fuel Cell Vehicles: Fundamentals, Theory and Design*, CRC Press, 2005

**Suggested Reading:**

[1] I. Husain, *Electric and Hybrid Electric Vehicles*, CRC Press, 2003

[2] C. C. Chan and K. T. Chau, *Modern Electric Vehicle Technology*, Oxford Science Publication, 2001

[3] G. Lechner and H. Naunheimer, *Automotive Transmissions: Fundamentals, Selection, Design and Application*, Springer, 1999

## Lecture 4: Vehicle Power Plant and Transmission Characteristics

### Vehicle Power Plant and Transmission Characteristics

#### Introduction

The topics covered in this chapter are as follows:

- The drive train configuration
- Various types of vehicle power plants
- The need of gearbox in a vehicle
- The mathematical model of vehicle performance

#### Drive train Configuration

An automotive drive train is shown in **Figure 1**. It consists of:

- a power plant
- a clutch in a manual transmission or a torque converter in automatic transmission
- a gear box
- final drive
- differential shaft
- driven wheels

The torque and rotating speed from the output shaft of the power plant are transmitted to the driven wheels through the clutch or torque converter, gearbox, final drive, differential and drive shaft.

The clutch is used in manual transmission to couple or decouple the gearbox to the power plant. The torque converter in an automatic transmission is hydrodynamic device, functioning as the clutch in manual transmission with a continuously variable gear ratio.

The gearbox supplies a few gear ratios from its input shaft to its output shaft for the power plant torque-speed profile to match the requirements of the load. The final drive is usually a pair of gears that supply a further speed reduction and distribute the torque to each wheel through the differential.

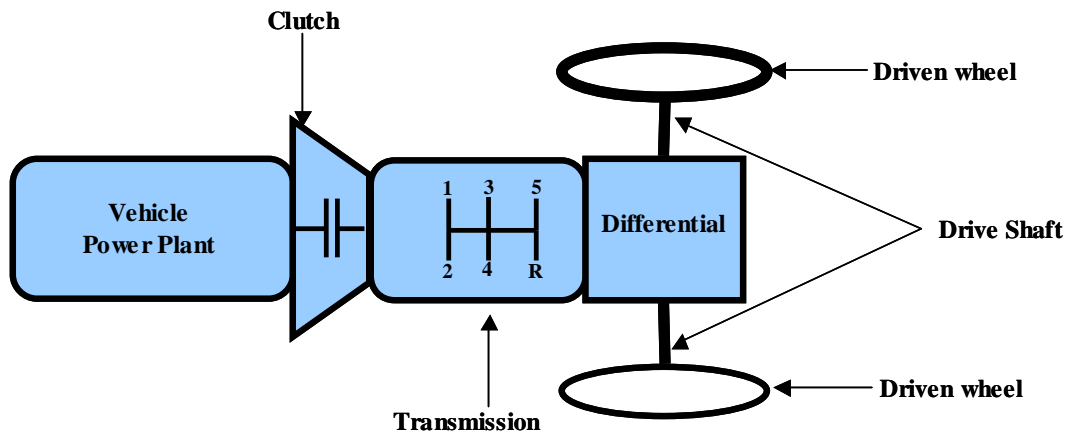


Figure 1: An automobile power train

### Vehicle power plant

There are two limiting factors to the maximum tractive effort of the vehicle:

- Maximum tractive effort that the tire-ground contact can support
- Tractive effort that the maximum torque of the power plant can produce with the given driveline gear ratios.

The smaller of these factors will determine the performance potential of the vehicle. Usually it is the second factor that limits the vehicles performance.

The classification of various types of power plants used in a vehicle is shown in Figure 2.

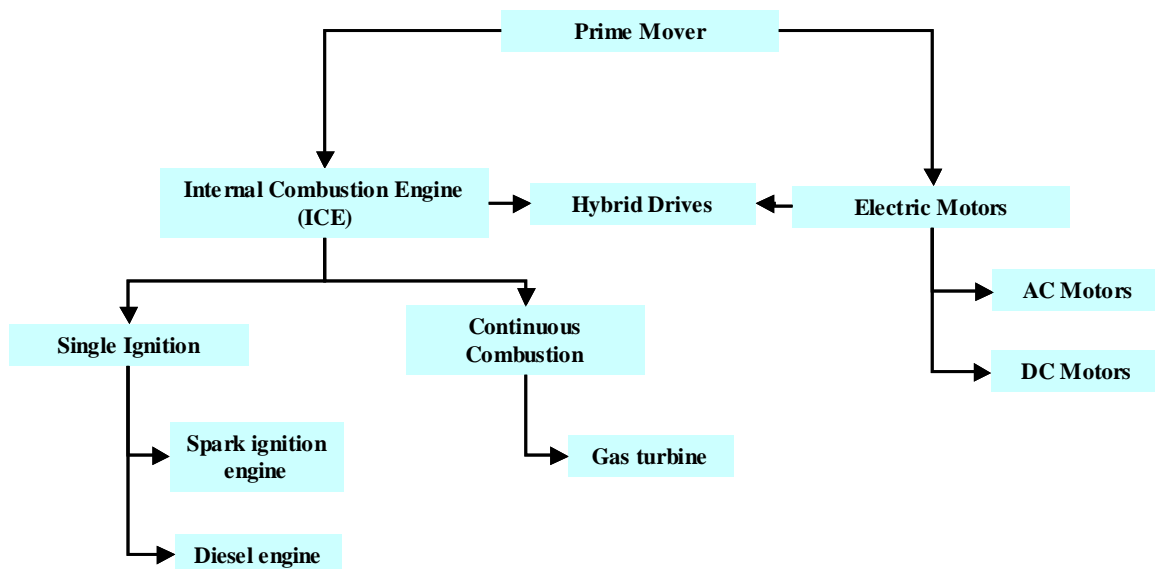


Figure 2: Classification of vehicle power plat



In selecting a suitable power plant, the following factors are considered:

- **Operating performance**
- **Economy**
- **Environment friendliness**

For vehicular applications, the ideal performance characteristic of a power plant is constant power output over the full speed range. Consequently, the torque varies hyperbolically with respect to speed as shown in **Figure 3**. This ideal performance characteristic of the power plant will ensure that the maximum power is available at any vehicle speed, thus resulting in optimal vehicle performance. In practice however, the torque is constrained to be constant at low speeds. This is done so as not to be over the maxima limited by the adhesion between the tyre-ground contact areas. The **internal combustion (IC)** engines are the most commonly used power plants for the land vehicles. In hybrid and electric vehicle technology, the **electric motor** is used.

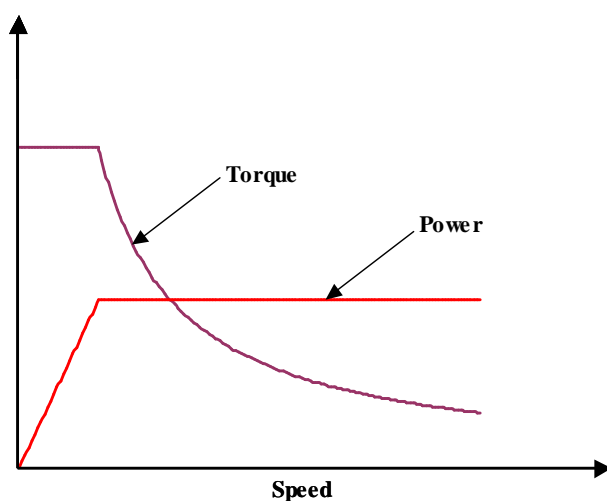


Figure 3: Ideal performance characteristics for a vehicle power plant

### Internal combustion engine

The internal combustion engines used in the vehicles are based on two principles:

- **spark ignition (petrol engines)** principle
- **Diesel** principle.

The key features of the ICs based spark ignition principle are:

- high power/weight ratio
- good performance
- low combustion noise.

The disadvantages of the ICs based spark ignition principle are:

- quality of fuel required
- higher fuel consumption.

The advantages of the diesel engines are:

- low fuel consumption
- low maintenance requirement due to absence of ignition system
- low fuel quality required

The disadvantages of the diesel engine are

- high level of particulate emission
- greater weight and higher price
- higher levels of noise

The two typical characteristic curves used to describe the engine characteristic are:

- torque vs. engine speed curve at full load (100% acceleration pedal position)
- power vs. engine speed curve at full load (100% acceleration pedal position)

These two characteristic curves are shown in **Figure 4**. In **Figure 4** the following nomenclature is used:

$P_{\max} = P_n$  = Maximum engine power = Nominal power

$P(T_{\max})$  = Engine power at maximum torque

$T_{\max}$  = Maximum engine torque

$T(P_{\max}) = T_n$  = Engine torque at maximum power = Nominal Torque

$n(P_{\max}) = n_n$  = Engine speed at maximum power = Nominal speed

$n(T_{\max})$  = Engine speed at maximum torque

Various indices are used to facilitate comparison between different types of engine. The two most important indices are:

- **torque increase (torque elasticity)** defined as

$$\tau = \frac{T_{\max}}{T_n} \tag{1}$$

where

$T_{\max}$  = maximum engine torque

$T_n$  = engine torque at maximum power, also known as nominal torque

- **engine speed ratio** defined as

$$v = \frac{n_n}{n(T_{\max})}$$

where

$n_n$  = engine speed at maximum power, also known as nominal speed

$n(T_{\max})$  = engine speed at maximum torque

(2)

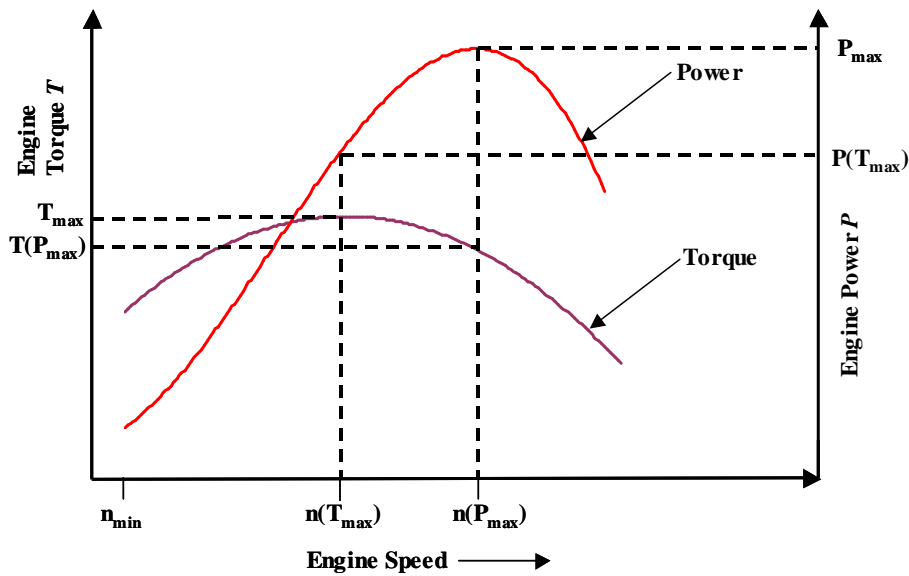


Figure 4: Characteristic curves of an internal combustion engine

The higher value of the product  $\tau v$  better engine power at low and medium engine speeds.

This in turn means less frequent gear changing.

## Electric Motor

The electric motors have are ideal for vehicle application because of the torque speed characteristics of the motors (**Figure 5**). Electric motors are capable of delivering a high starting torque. It is very important to select proper type of motor with a suitable rating. For example, it is not accurate to simply refer to a 10 h.p. motor or a 15 h.p. motor, because horsepower varies with volts and amps, and peak horsepower is much higher than the continuous rating.

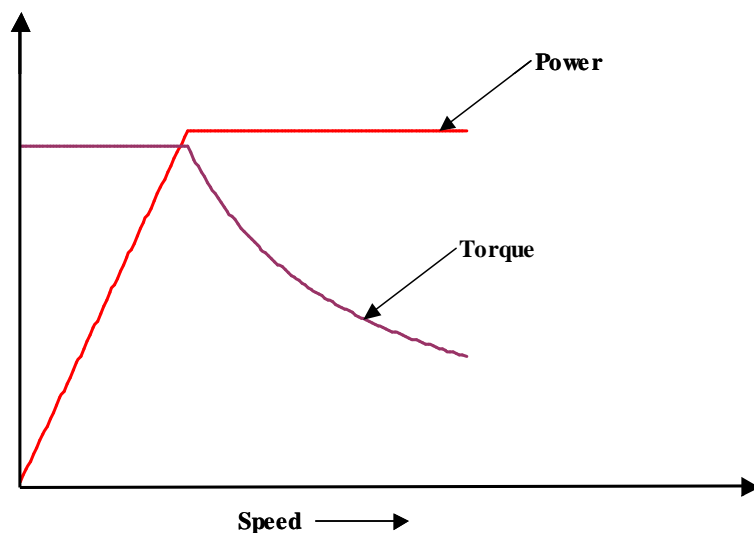


Figure 5: Torque vs. speed and power vs. speed characteristics of electric motor

It is also confusing to compare electric motors to IC engines, since electric motors are designed for a continuous rating under load and IC engines are rated at their peak horsepower under loaded condition. The commonly used motors in EVs are:

- AC motors
- Permanent magnet (PM) motors
- Series wound DC motors
- Shunt wound DC motors

The DC series motors were used in a number of prototype Electric Vehicle (EVs) and prior to that mainly due to the ease of control. However, the size and maintenance requirements of DC motors are making their use obsolete. The recent EVs and Hybrid Electric Vehicles (HEVs) use AC, PM and Switched Reluctance motors. A classification of motors used in EVs is shown in **Figure 6**.

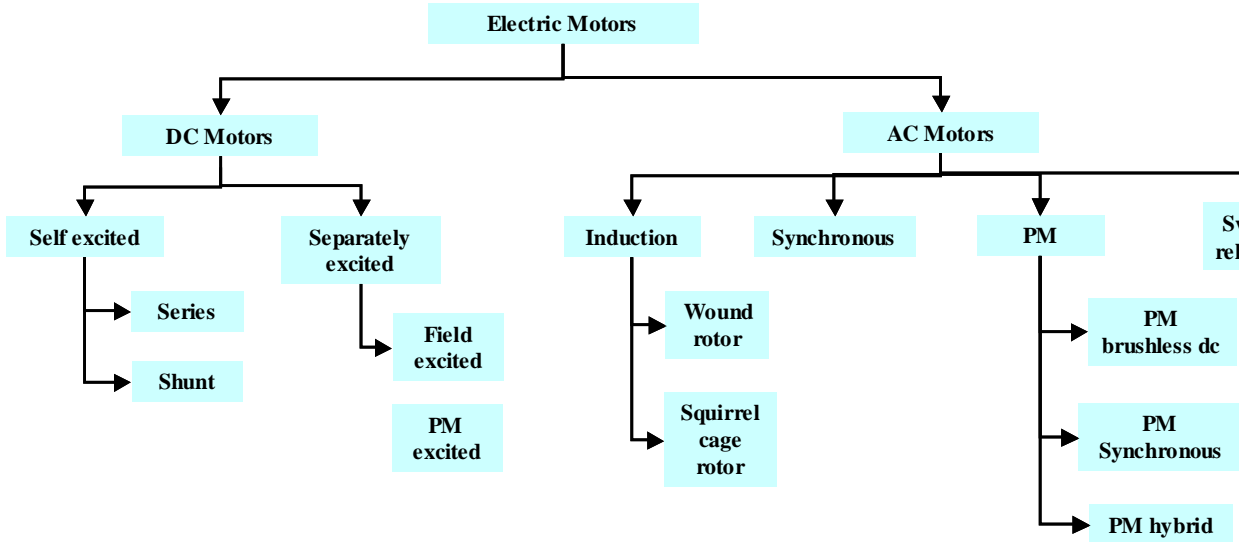


Figure 6: Classification of electric motors used in EVs

The AC Induction Motor (IM) technology is very mature and significant research and development activities have taken place in the area of induction motor drives. The control of IM is more complex than DC motors, but the availability of fast digital processors, computational complexity can easily be managed. The competitor to the induction motor is the permanent magnet (PM) motor. The permanent magnet motors have magnets on the rotor, while the stator construction is same as that of induction motor. The PM motors can be surface mounted type or the magnets can be inset within the rotor. The PM motors can also be classified as sinusoidal type or trapezoidal type depending on the flux density distribution in the air gap. Permanent magnet motors with sinusoidal air gap flux distribution are called Permanent Magnet synchronous Motors (PMSM) and the with trapezoidal air gap flux distribution are called Brushless DC (BLDC) motors.

### The need for gearbox

Internal combustion engines today drive most of the automobiles. These internal combustion engines work either on the principle of spark ignition or diesel principle. In addition to the many advantages of the internal combustion engine, such as high power to weight ratio and relatively compact energy storage, it has two fundamental disadvantages:

- i. *Unlike the electric motors, the internal combustion engine cannot produce torque at zero speed.*
- ii. *The internal combustion engine produces maximum power at a certain engine speed.*
- iii. *The efficiency of the engine, i.e. its fuel consumption, is very much dependent on the operating point in the engine’s performance map.*

With a maximum available engine power  $P_{max}$  and a road speed of  $v$ , the *ideal traction hyperbola*  $F_{ideal}$  and the *effective traction hyperbola*  $F_{effec}$  can be calculated as follows:

$$F_{ideal} = \frac{P_{max}}{v}$$

$$F_{ideal} = \frac{P_{max}}{v} \eta_{tot}$$

where

$\eta_{tot}$  = efficiency of the drivetrain

(1)

Hence, if the full load engine power  $P_{max}$  were available over the whole speed range, the traction hyperbolas shown in **Figure 7** would result. However, the  $P_{max}$  is not available for the entire speed range. The actual traction profile of the ICE ( $F_{engine}$ ) is shown in **Figure 7**. From **Figure 7** it is evident that the entire shaded area cannot be used.

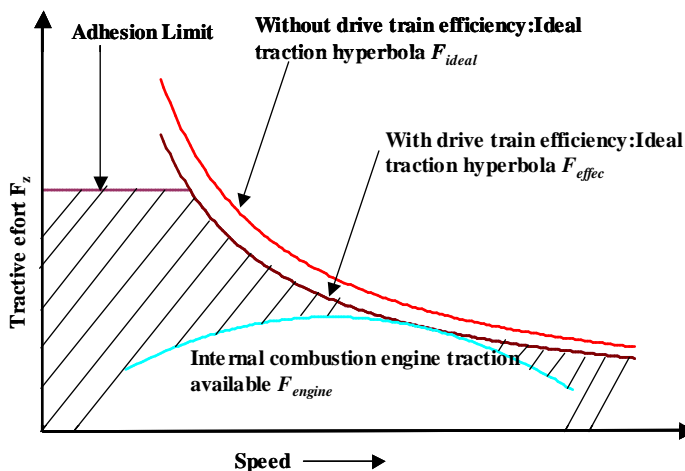


Figure 7: Traction force vs. speed map of an internal combustion engine without gearbox

In order to utilize the shaded area, shown in **Figure 7**, additional output converter is required. The output converter must convert the characteristics of the combustion engine in such a way that it approximates as closely as possible to the ideal *traction hyperbola* (**Figure 8**).

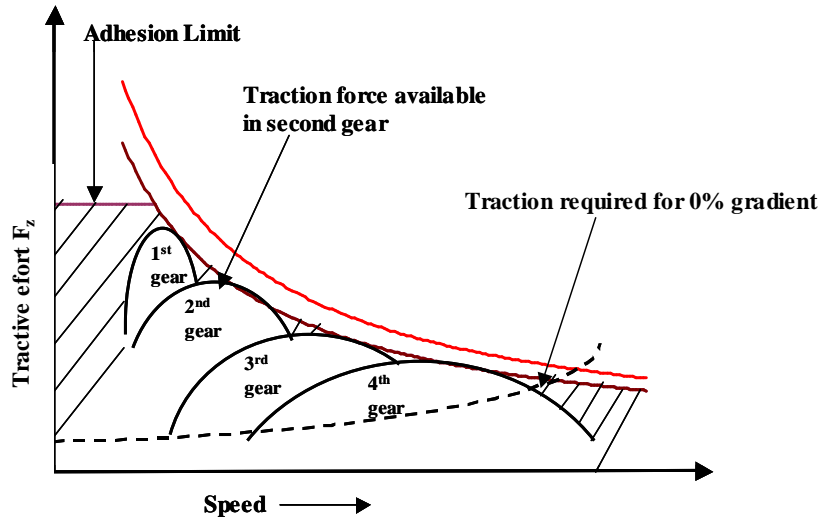


Figure 8: Traction force vs. speed map of an internal combustion engine with gearbox

The proportion of the shaded area, i.e. the proportion of impossible driving states, is significantly smaller when an output converter is used. Thus, the power potential of the engine is better utilized. The Figure 8 shows how increasing the number of gears gives a better approximation of the *effective traction hyperbola*.

### Drive train tractive effort and vehicle speed

After having dealt with the configuration of the drivetrain, this section deals with the *tractive effort*. The torque transmitted from the power plant to the driven wheels ( $T_w$ ) is given by:

$$T_w = i_g i_o \eta_t T_p$$

where

$$i_g = \text{gear ratio of the transmission} \tag{1}$$

$$i_o = \text{gear ratio of the final drive}$$

$$\eta_t = \text{efficiency of the driveline from the power plant to the driven wheels}$$

$$T_p = \text{torque output from the power plant [Nm]}$$

The tractive effort on the driven wheels (**Figure 9**) is expressed as

$$F_t = \frac{T_w}{r_{dyn}}$$

where

$$r_{dyn} = \text{dynamic radius of the tyre [m]} \tag{2}$$

Substituting value of  $T_w$  from **equation 1** into **equation 2** gives

$$F_t = \frac{T_p i_g i_o \eta_t}{r_{dyn}} \quad (3)$$

The total mechanical efficiency of the transmission between the engine output shaft and driven wheels is the product of the efficiencies of all the components of the drive train.

The rotating speed of the driven wheel is given by

$$N_w = \frac{N_p}{i_g i_o} \text{ [rpm]} \quad (4)$$

where

$N_p$  = rotational speed of the transmission [rpm]

The rotational speed  $N_p$  of the transmission is equal to the engine speed in the vehicle with a manual transmission and the turbine speed of a torque converter in the vehicle with an automatic transmission. The *translation speed of the wheel (vehicle speed)* is expressed as

$$V = \frac{\pi N_w r_{dyn}}{30} \text{ [m/s]} \quad (5)$$

By substituting the value of  $N_w$  from **equation 4** into **equation 5**, the *vehicle speed* can be expressed as

$$V = \frac{\pi N_p r_{dyn}}{30 i_g i_o} \text{ [m/s]} \quad (6)$$

### Vehicle performance

The performance of a vehicle is determined by the following factors:

- maximum cruising speed
- gradeability
- acceleration



**Maximum Cruising Speed**

The maximum speed of a vehicle is defined as the constant cruising speed that the vehicle can achieve with full power plant load on a flat road. The maximum speed of a vehicle is determined by the equilibrium between the tractive effort of the vehicle and the resistance and maximum speed of the power plant and gear ratios of the transmission. This equilibrium is:

$$\frac{T_p i_g i_0 \eta_t}{r_{dyn}} = M g f_r \cos(\alpha) + \frac{1}{2} \rho_a C_D A_f V^2$$

where

(30)

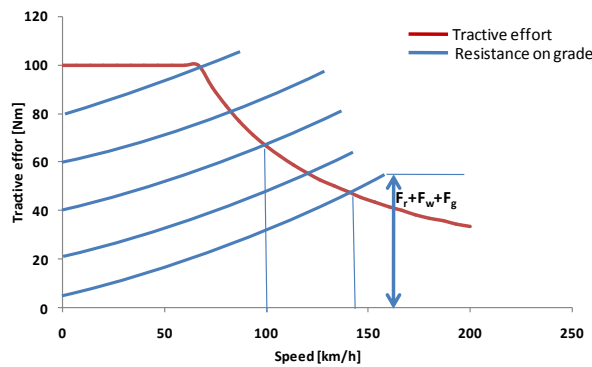
$i_g$  = gear ratio of the transmission

$i_0$  = gear ratio of the final drive

$\eta_t$  = efficiency of the driveline from the power plant to the driven wheels

$T_p$  = torque output of the power plant [Nm]

**equation 30** shows that the vehicle reaches its maximum speed when the tractive effort, represented by the left hand side term, equals the resistance, represented by the right hand side. The intersection of the tractive effort curve and the resistance curve is the maximum speed of the vehicle, **Figure 9**.



**Figure 9:** Tractive effort of an electric motor powered vehicle with a single speed transmission and its resistance

For some vehicles, no intersection exists between the tractive effort curve and the resistance curve, because of a large power plant. In such a case the maximum speed of the vehicle is determined by the maximum speed of the power plant. This maximum speed is given by

$$V_{\max} = \frac{\pi n_{p\max} r_{dyn}}{30 i_0 i_{g\min}}$$

where

$$i_{g\min} = \text{minimum gear ratio of the transmission} \quad (31)$$

$$i_0 = \text{gear ratio of the final drive}$$

$$n_{p\max} = \text{maximum speed of the power plant (motor or engine) [rpm]}$$

$$T_p = \text{torque output of the power plant [Nm]}$$

$$r_{dyn} = \text{dynamic radius of the tyre [m]}$$

### Gradeability

Gradeability is defined as the grade angle that the vehicle can negotiate at a certain constant speed. For heavy commercial vehicles the gradeability is usually defined as the maximum grade angle that the vehicle can overcome in the whole speed range.

When the vehicle is driving on a road with relatively small grade and constant speed, the tractive effort and resistance equilibrium can be expressed as

$$\frac{T_p i_0 i_g \eta_t}{r_{dyn}} = M g f_r + \frac{1}{2} \rho_a C_D A_f V^2 + M g i \quad (32)$$

Hence,

$$i = \frac{T_p i_0 i_g \eta_t / r_{dyn} - M g f_r - 1/2 \rho_a C_D A_f V^2}{M g} = d - f_r \quad (33)$$

where

$$d = \frac{T_p i_0 i_g \eta_t / r_{dyn} - 1/2 \rho_a C_D A_f V^2}{M g} \quad (34)$$

The factor **d** is called the performance factor. When the vehicle drives on a road with a large grade, the gradeability of the vehicle can be calculated as

$$\sin(\alpha) = \frac{d - f_r^2 \sqrt{1 - d^2 + f_r^2}}{1 + f_r^2} \quad (35)$$

### Acceleration Performance

The acceleration of a vehicle is defined by its acceleration time and distance covered from zero speed to a certain high speed on a level ground. The acceleration of the vehicle can be expressed as

$$a = \frac{dV}{dt} = \frac{F_t - F_f - F_w}{M\delta} = \frac{T_p i_0 i_g \eta_t / r_{dyn} - Mgf_r - 1/2\rho_a C_D A_f V^2}{M\delta} = \frac{g}{\delta} (d - f_r) \quad (36)$$

where  $\delta$  is the rotational inertia factor taking into account the equivalent mass increase due to the angular moments of the rotating components. This mass factor can be written as

$$\delta = 1 + \frac{I_w}{Mr_{dyn}^2} + \frac{i_0^2 i_g^2 I_p}{Mr^2} \quad (37)$$

$I_w$  = total angular inertial moment of the wheels

$I_p$  = total angular inertial moment of the rotating components associated with the power plant

To determine the value of  $\delta$ , it is necessary to determine the values of the mass moments of inertia of all the rotating parts. In case the mass moments of inertia are not available then, the rotational factor ( $\delta$ ) can be approximated as:

$$\begin{aligned} \delta &= 1 + \delta_1 + \delta_2 i_g^2 i_0^2 \\ \delta_1 &\approx 0.04 \\ \delta_2 &\approx 0.0025 \end{aligned} \quad (38)$$

The acceleration rate along with vehicle speed for a petrol engine powered vehicle with a four gear transmission and an electric motor powered vehicle with a single gear transmission are shown in **Figure 10** and **Figure 11** respectively.

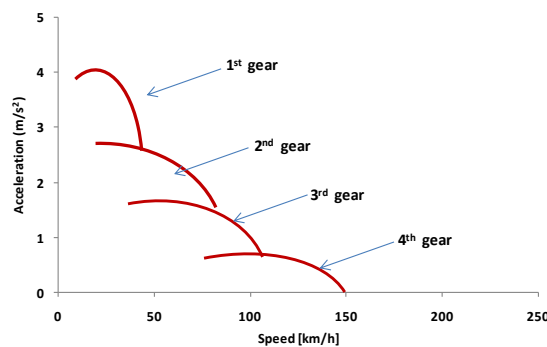


Figure 10: Acceleration rate of a petrol engine powered vehicle with four gears

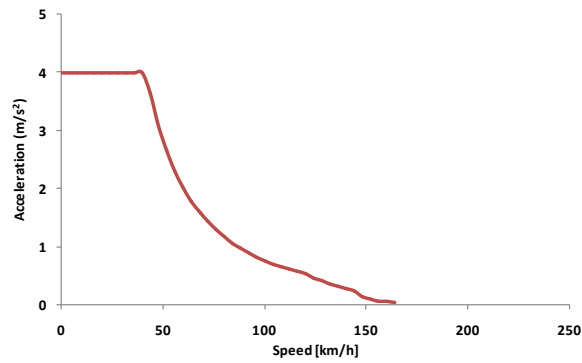


Figure 11: Acceleration rate of an electric machine powered vehicle with a single gear

From **equation 36**, the acceleration time  $t_a$  and distance  $S_a$  from a lower speed  $V_1$  to a higher speed  $V_2$  can be expressed as

$$t_a = \int_{V_1}^{V_2} \frac{M \delta}{T_p i_g i_0 \eta_t / r_{dyn} - M g f_r - 1/2 \rho_a C_D A_f V^2} dV \quad (39)$$

and

$$S_a = \int_{V_1}^{V_2} \frac{M \delta V}{T_p i_g i_0 \eta_t / r_{dyn} - M g f_r - 1/2 \rho_a C_D A_f V^2} dV \quad (40)$$

The power plant torque  $T_p$  in **equation 39** and **equation 40** is a function of speed of the power plant. The speed of the power plant is in turn a function of the vehicle speed and gear ratio of the transmission. Hence, analytical solution of **equation 39** and **equation 40** is not possible. Numerical methods are usually used to solve these equations.

### Suggested Reading

[1] I. Husain, *Electric and Hybrid Electric Vehicles*, CRC Press, 2003

[2] G. Lechner and H. Naunheimer, *Automotive Transmissions: Fundamentals, Selection, Design and*

*Application*, Springer, 1999