DINAMIKA TANAH DAN KEGEMPAAN

KODE MK: 08013152053

TOPIK KE – 1

THEORY OF VIBRATION

Disusun oleh:

Yusep Muslih Purwana

Prodi Teknik Sipil FT UNS

Lab. Mekanika Tanah FT UNS, Jl Ir Sutami 36 a Surakarta

VIBRASI

Vibrasi adalah gerakan osilasi yang berulang pada interval tertentu akibat adanya beban dinamik, diantaranya:

- Earthquake load
- Wind
- Moving load
- Getaran mesin
- Ledakan
- Beban impak

Efek negative getaran:

1. Kelelahan struktur
2. Keausan, dll

Kegunaan getaran:

Conveyor, vibrator, stamper,

Vibratory hammer, vibrofloating, dll

Periodic dan Non Periodic Motion

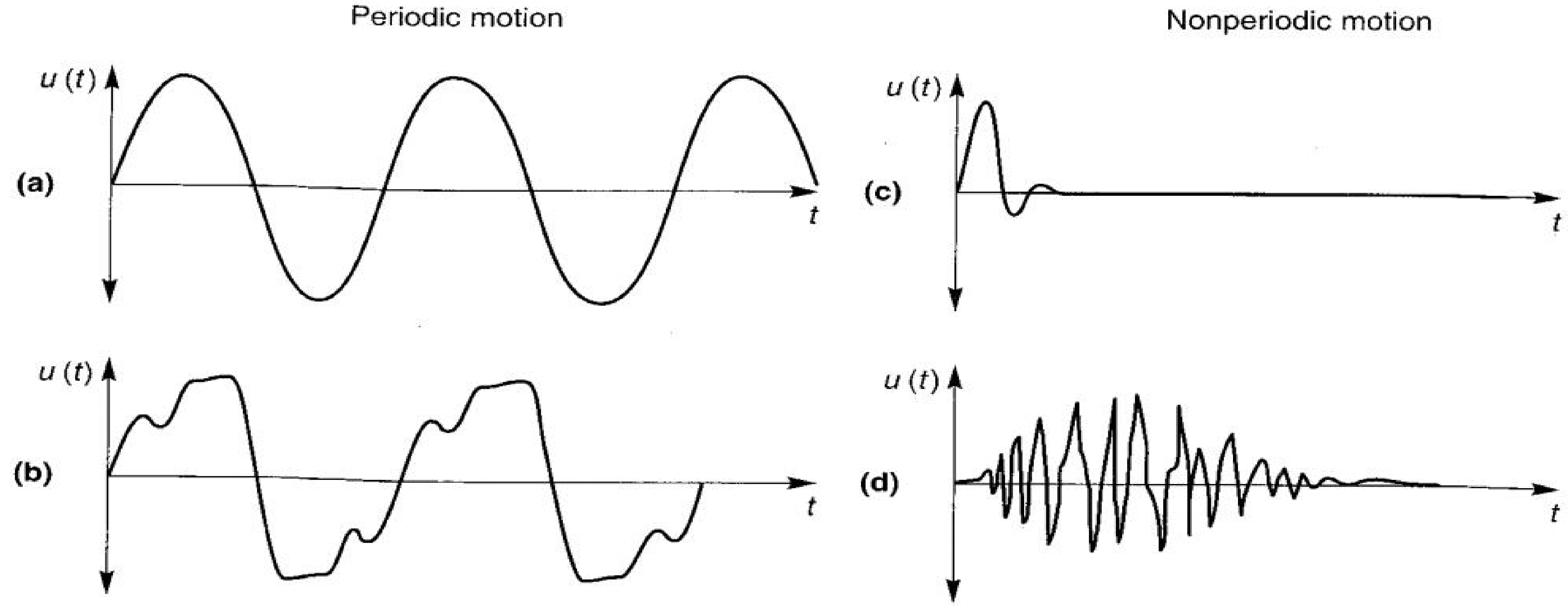
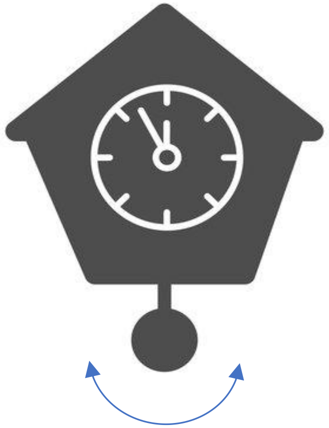
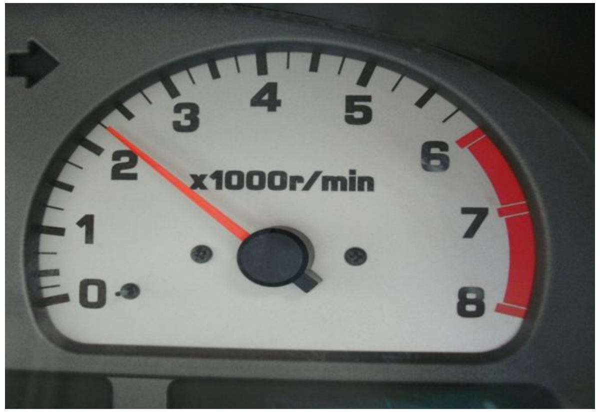
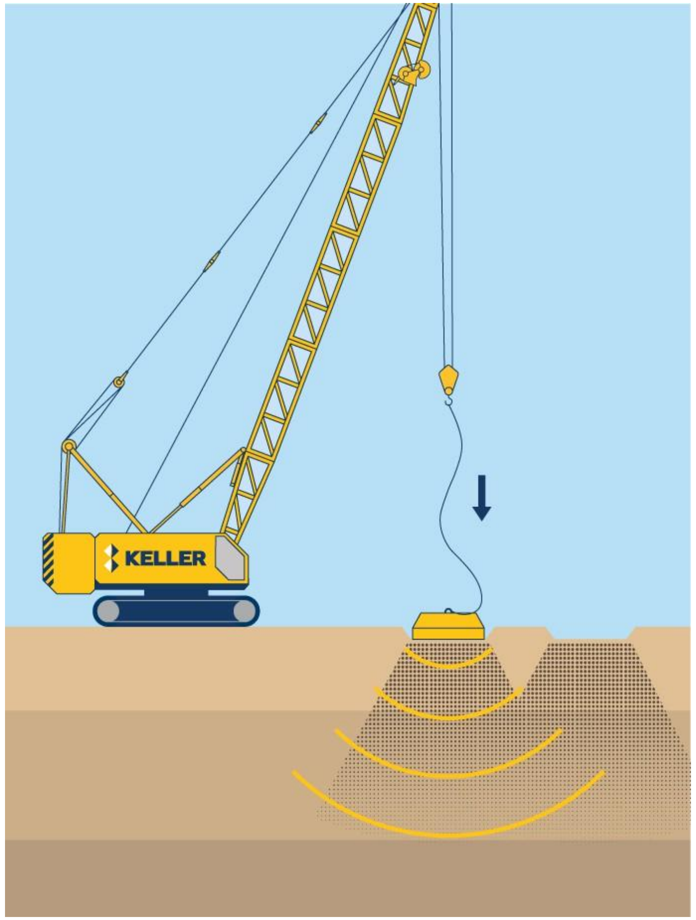


Figure A.1 Periodic and nonperiodic motion: (a) simple harmonic motion; (b) general periodic motion; (c) transient motion (response to impact loading); (d) transient motion (earthquake ground motion).

Periodic dan Non Periodic Motion Examples







Transient Motion as Periodic Motion

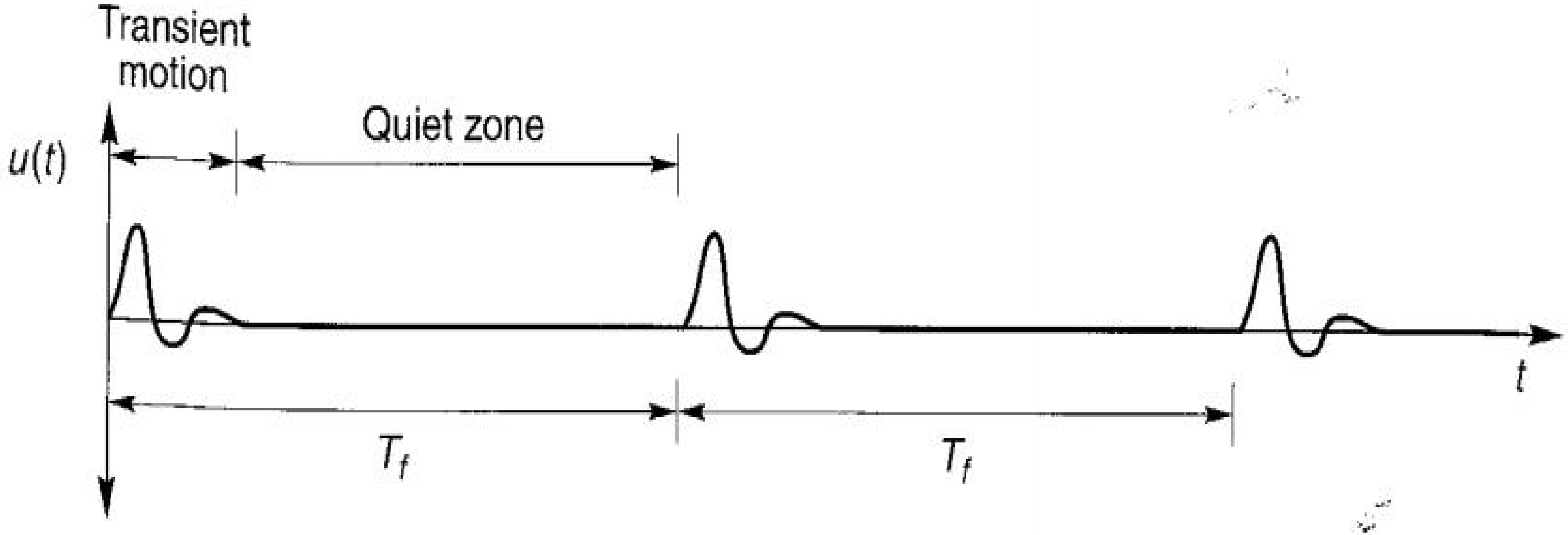


Figure A.2 Representation of a transient motion as a periodic motion using an artificial quiet zone. The motion repeats itself indefinitely at period T_f.

Simple Harmonic Motion

- Simple harmonic motion ditandai dengan sinusoidal motion pada frekuensi konstan
- Parameter penting: Amplitude, frekuensi, dan fase
- Notasi pada gambar:

A = amplitude,

T= periode, = $2\pi/\omega$ atau 1/f

ω = circular frequency (rad/time)

f = frekuensi = 1/T = ω/π

ϕ = sudut fase

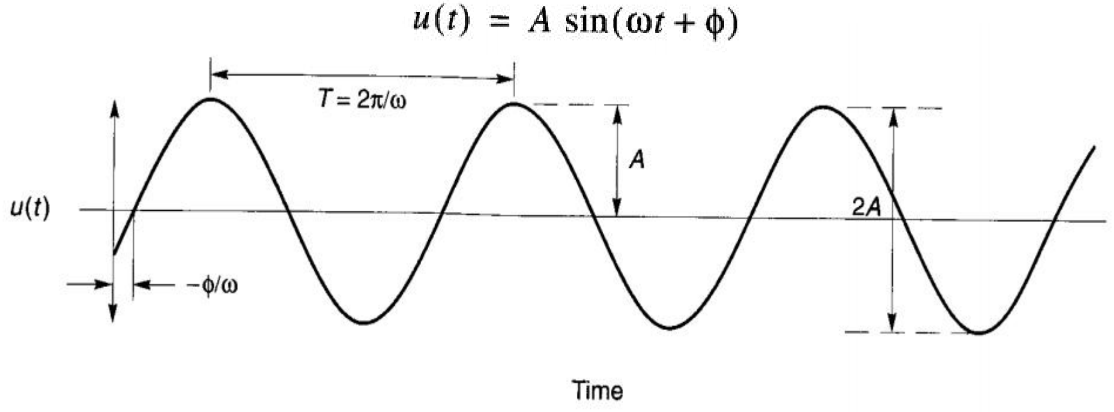
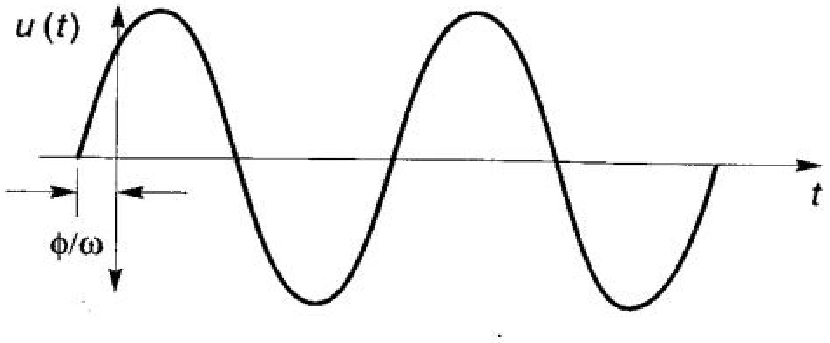


Figure A.3 Time history of simple harmonic displacement.

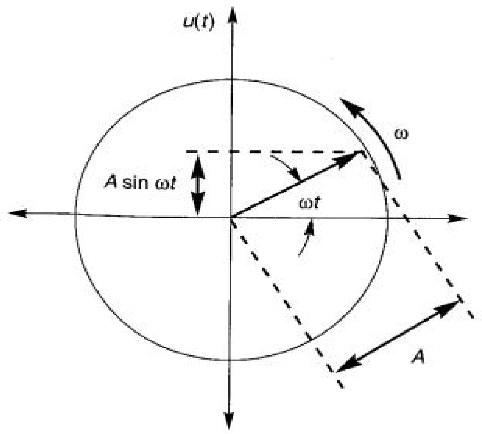


TRIGONOMETRIC NOTATION

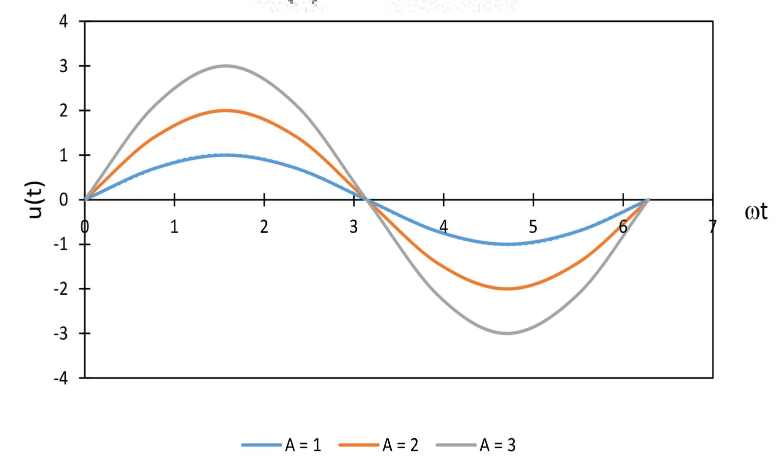
FOR

SIMPLE HARMONIC MOTION

Simple Harmonic Motion



$u(t) = A \sin\omega t$



- Displacement u(t), dpt diformulasikan:

  $u(t) = A \sin \omega t$

  dimana A = amplitodo max, ω = kecepatan sudut (rotasi/waktu),

  A max → $\omega t = \pi/2$, A = 0 → $\omega t = \pi$

- Waktu yg diperlukan utk 1 putaran (=T):

  $T = 2\pi/\omega$

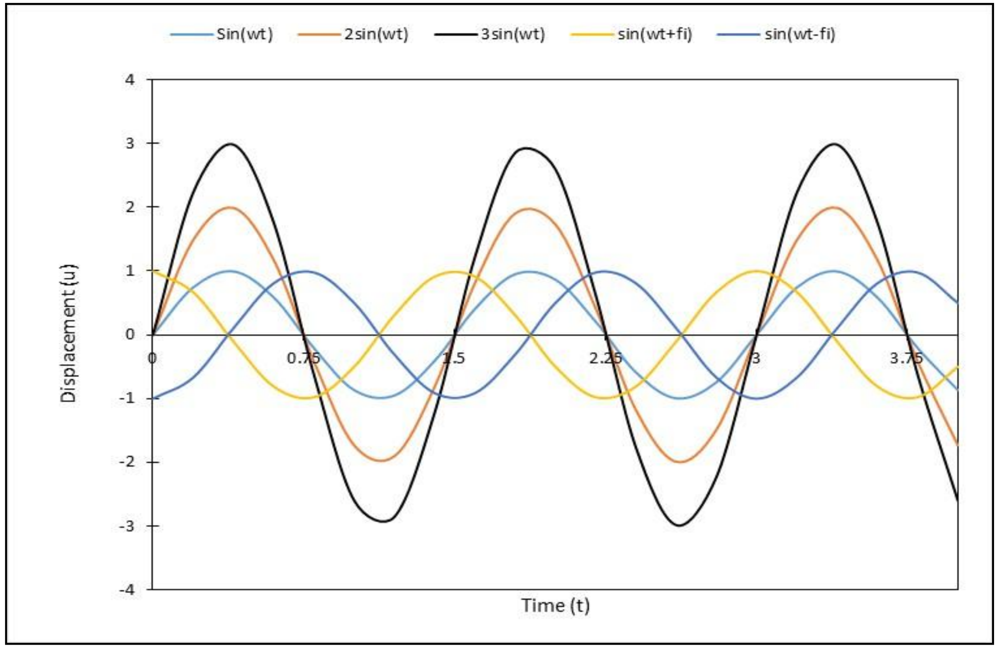
- Jumlah putaran per satuan waktu (=f)

  $f = 1/T = \omega/2\pi$

Simple Harmonic Motion

Contoh:

Suatu sistem berputar dengan 2 x setiap 3 detik. Gambarkan time history dari diplacement arah sumbu vertikal selama 4 detik untuk amplitudo 1, 2 dan 3. Persamaan yang digunakan adalah spt pada kolom 2 sd 6



t	u(t) =				
	sin (ωt)	2sin(ωt)	3sin(ωt)	sin (ωt+ϕ)	sin (wt-ϕ)
0	0	0	0	1	-1
0.2	0.74	1.49	2.23	0.67	-0.67
0.4	0.99	1.99	2.98	-0.10	0.10
0.6	0.59	1.18	1.76	-0.81	0.81
0.8	-0.21	-0.42	-0.62	-0.98	0.98
1	-0.87	-1.73	-2.60	-0.50	0.50
1.2	-0.95	-1.90	-2.85	0.31	-0.31
1.4	-0.41	-0.81	-1.22	0.91	-0.91
1.6	0.41	0.81	1.22	0.91	-0.91
1.8	0.95	1.90	2.85	0.31	-0.31
2	0.87	1.73	2.60	-0.50	0.50
2.2	0.21	0.42	0.62	-0.98	0.98
2.4	-0.59	-1.18	-1.76	-0.81	0.81
2.6	-0.99	-1.99	-2.98	-0.10	0.10
2.8	-0.74	-1.49	-2.23	0.67	-0.67
3	0.00	0.00	0.00	1.00	-1.00
3.2	0.74	1.49	2.23	0.67	-0.67
3.4	0.99	1.99	2.98	-0.10	0.10
3.6	0.59	1.18	1.76	-0.81	0.81
3.8	-0.21	-0.42	-0.62	-0.98	0.98
4	-0.87	-1.73	-2.60	-0.50	0.50

Simple Harmonic Motion (Cont)

Simple harmonic motion dapat juga digambarkan jumlah fungsi sinus dan cosinus. Misal a =1.5, b = 1

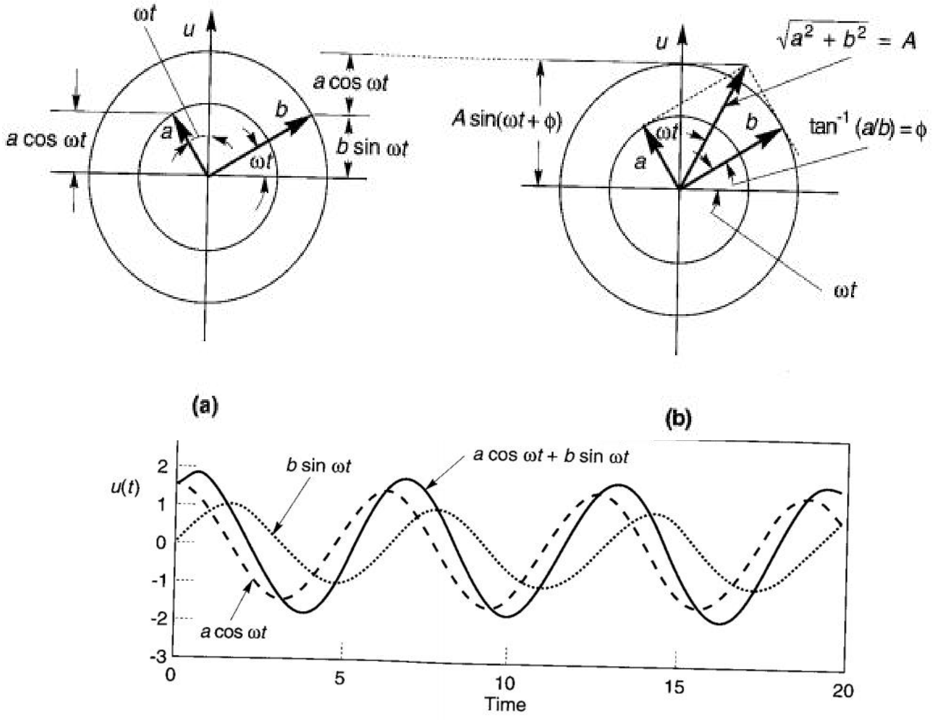
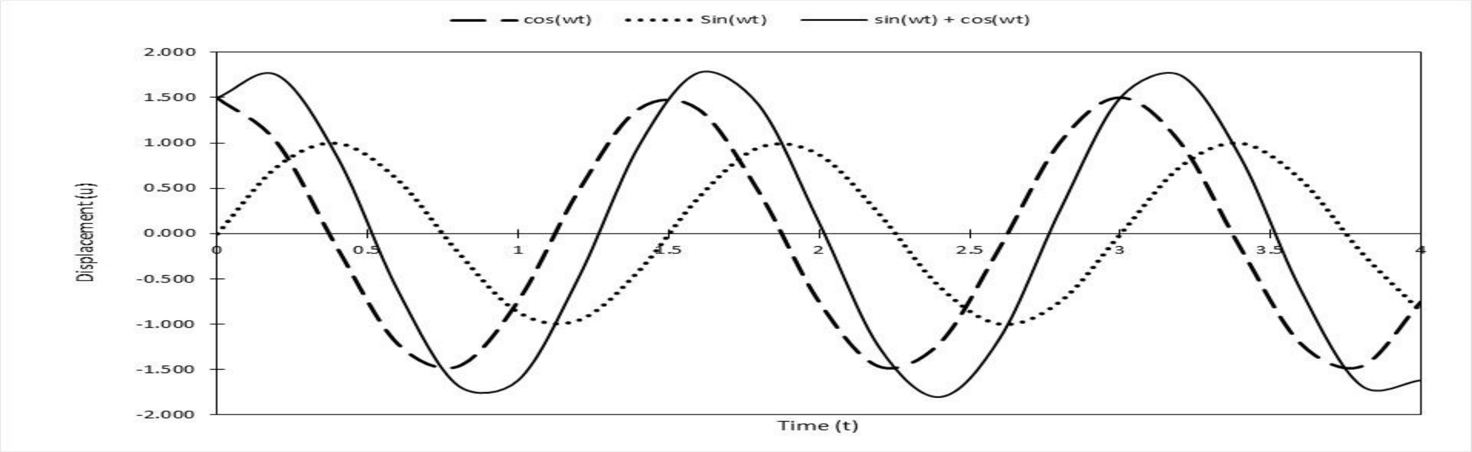
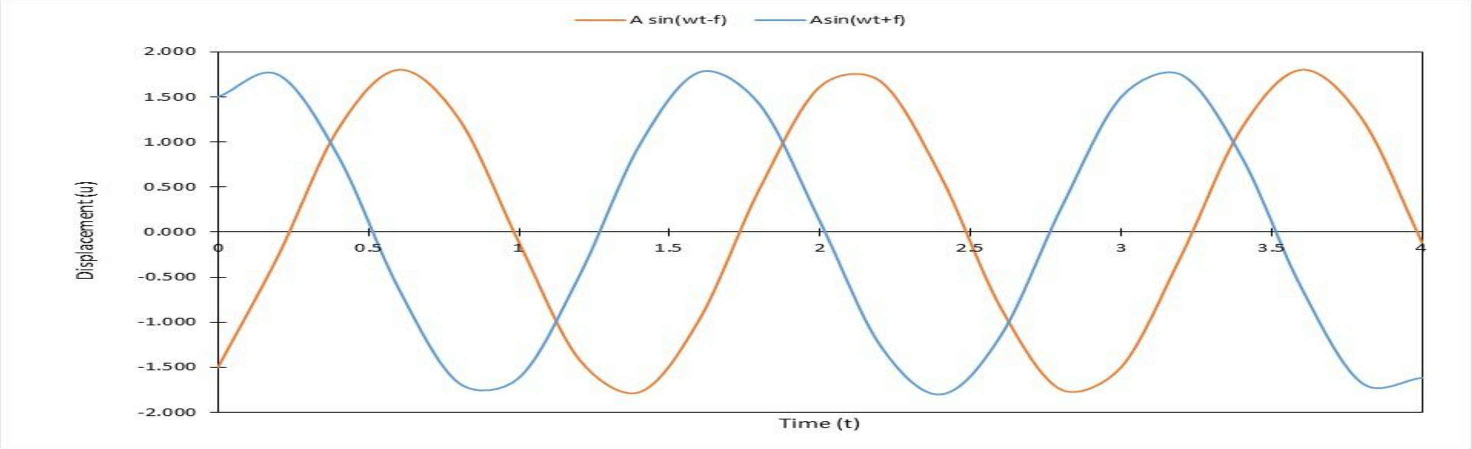


Figure A.6 Summation of sine and cosine functions of the same frequency produces a sinusoid of the same frequency. Amplitude and phase of the sinusoid depends on the amplitudes of the sine and cosine functions.

Misal	a =	1.5	A = $(a^2 + b^2)$^0.5 =	1.803	
	b =	1	ϕ = $\tan^{-1}(a/b)$ =	0.983	
t	u(t) =				
	a cos(wt)	b sin (wt)	a cos(wt)+ b sin (wt)	Asin (wt-f)	Asin (wt+f)
0	1.500	0	1.500	-1.500	1.500
0.2	1.004	0.743	1.747	-0.261	1.747
0.4	-0.157	0.995	0.838	1.151	0.838
0.6	-1.214	0.588	-0.626	1.801	-0.626
0.8	-1.467	-0.208	-1.675	1.259	-1.675
1	-0.750	-0.866	-1.616	-0.116	-1.616
1.2	0.464	-0.951	-0.488	-1.415	-0.488
1.4	1.370	-0.407	0.964	-1.777	0.964
1.6	1.370	0.407	1.777	-0.964	1.777
1.8	0.464	0.951	1.415	0.488	1.415
2	-0.750	0.866	0.116	1.616	0.116
2.2	-1.467	0.208	-1.259	1.675	-1.259
2.4	-1.214	-0.588	-1.801	0.626	-1.801
2.6	-0.157	-0.995	-1.151	-0.838	-1.151
2.8	1.004	-0.743	0.261	-1.747	0.261
3	1.500	0.000	1.500	-1.500	1.500
3.2	1.004	0.743	1.747	-0.261	1.747
3.4	-0.157	0.995	0.838	1.151	0.838
3.6	-1.214	0.588	-0.626	1.801	-0.626
3.8	-1.467	-0.208	-1.675	1.259	-1.675
4	-0.750	-0.866	-1.616	-0.116	-1.616

Simple Harmonic Motion (Cont)





SIMPLE HARMONIC MOTION MEASURES

Pengukuran Motion

- Gerak (motion) tidak hanya digambarkan dengan displacement, namun sering juga digambarkan dengan kecepatan dan percepatan.

$$u(t) = A\sin(\omega t + \phi) \quad \text{displacement}$$

$$\dot{u}(t) = \frac{du}{dt} = \omega A\cos(\omega t + \phi) \quad \text{velocity}$$

$$\ddot{u}(t) = \frac{d^2u}{dt^2} = -\omega^2 A\sin(\omega t + \phi) = -\omega^2 u \quad \text{acceleration}$$

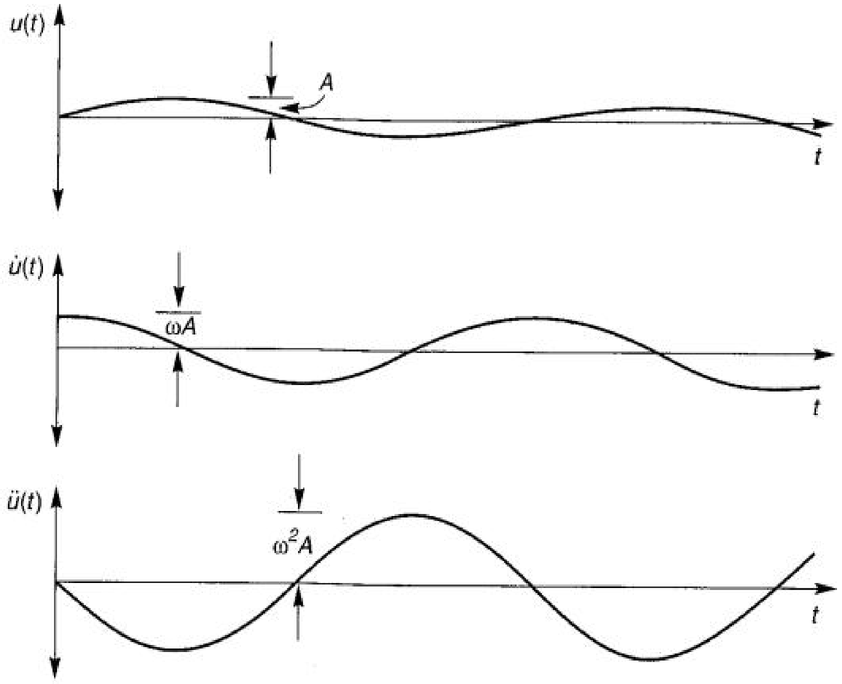
A: Amplitudo perpindahan

ωA: Amplitudo kecepatan

ω^2A: Amplitodo percepatan

- Kecepatan adalah turunan dari perpindahan, dan percepatan adalah turunan dari kecepatan.

Simple Harmonic Motion



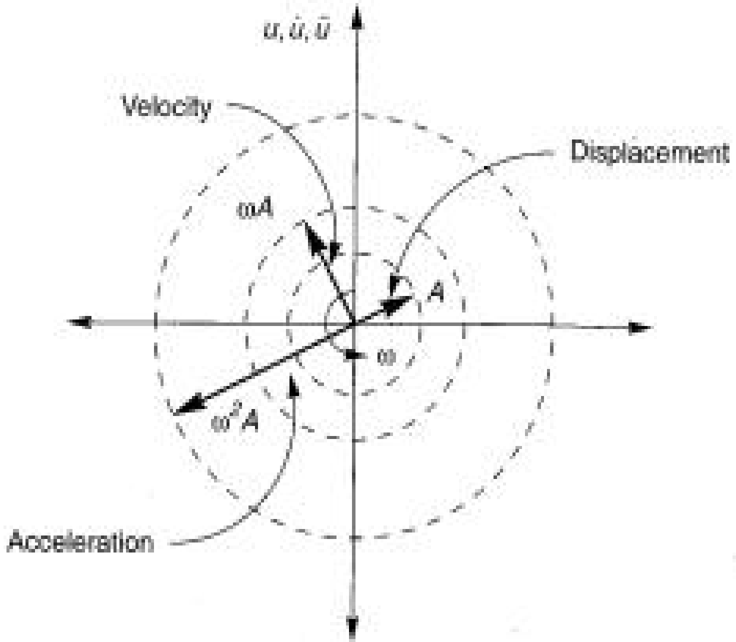
$$u(t) = A\sin(\omega t + \phi) \qquad \text{displacement}$$

$$\dot{u}(t) = \frac{du}{dt} = \omega A\cos(\omega t + \phi) \qquad \text{velocity}$$

$$\ddot{u}(t) = \frac{d^2u}{dt^2} = -\omega^2 A\sin(\omega t + \phi) = -\omega^2 u \qquad \text{acceleration}$$

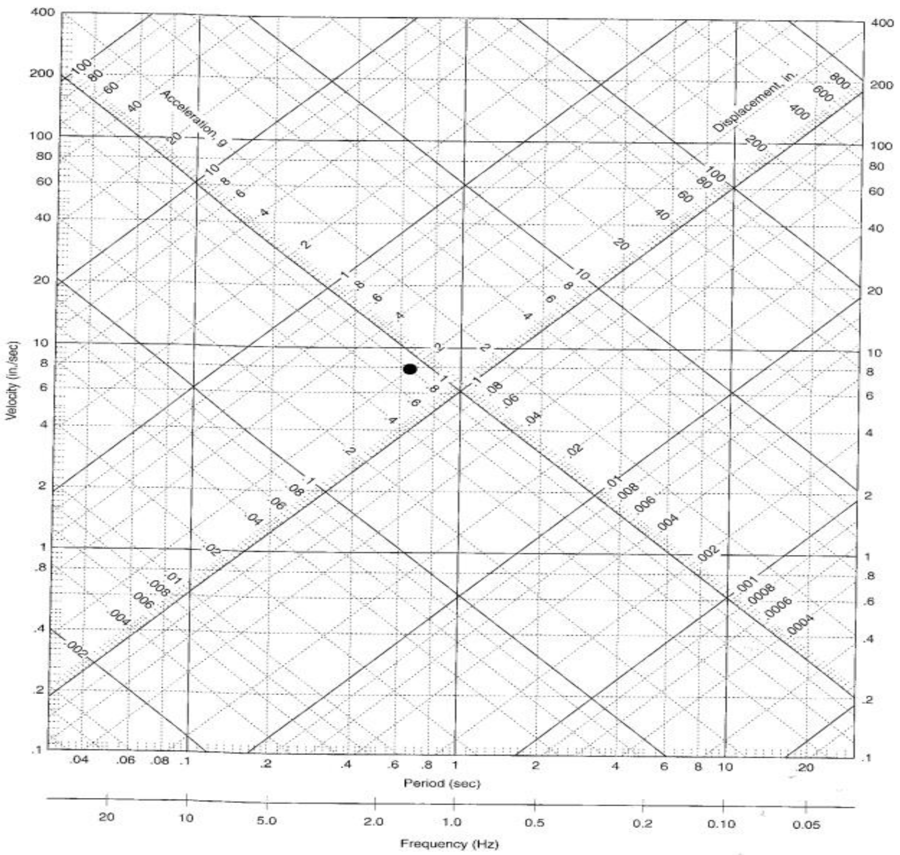
- Kecepatan mendahului displacement sebesar $\pi/2$
- Percepatan mendahului kecepatan sebesar $\pi/2$ →percepatan mendahului displacement sebesar π

Simple Harmonic Motion



- Kecepatan mendahului displacement sebesar p/2
- Percepatan mendahului kecepatan sebesar p/2
  →percepatan mendahului displacement sebesar p
- Frekuensi, displacement, velocity, dan acceleration saling terkait satu dengan lain. Apabila frekuensi dan amplitude diketahui memungkinkan perhitungan lainnya.
- Sifat ini memungkinkan penggambaran ketiganya dalam satu grafik yang disebut: TRIPARTITE PLOT

Tripartite Plot



- Menggambarkan plot tiga parameter sekaligus: displacement, kecepatan, dan percepatan.
- Contoh:

    Titik node pada gambar menunjukkan harmonic motion pada periode 0.65 detik, displacement amplitude 0.8 in, velocity amplitude 8 in/detik, dan acceleration amplitude 0.2 g

Fourier Series

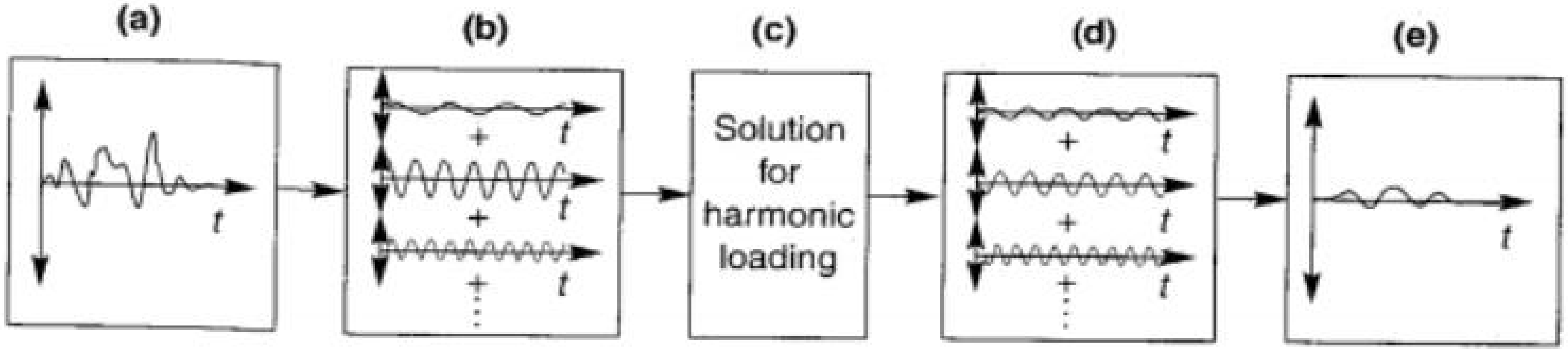


Figure A.12 Process by which Fourier series representation of complicated loading can allow relatively simple solutions for harmonic loading to be used to produce the total response: (a) time history of loading; (b) representation of time history of loading as sum of series of harmonic loads; (c) calculation of response for each harmonic load; (d) representation of response as sum of series of harmonic responses; (e) summation of harmonic responses to produce time history of response.

Degree of freedom

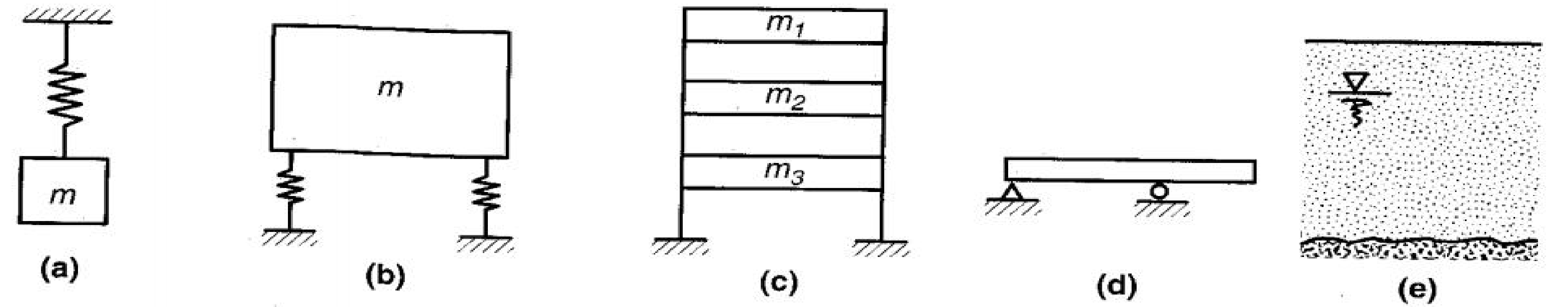
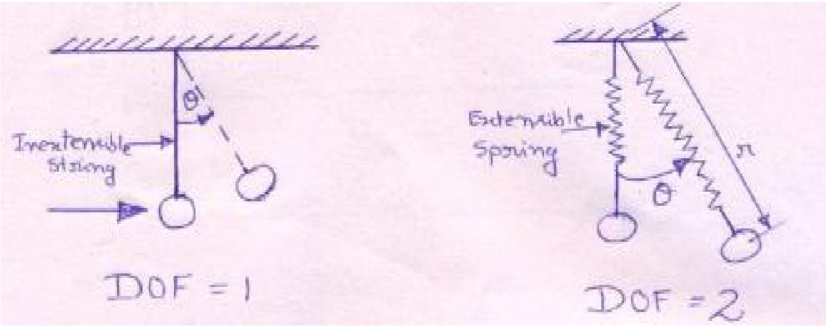


Figure B.1 Vibrating systems with various numbers of degrees of freedom: (a) one DOF, vertical translation; (b) two DOF, vertical translation and rocking; (c) three DOF, horizontal translation; (d) infinite DOF; (e) infinite DOF.



Review SDOF

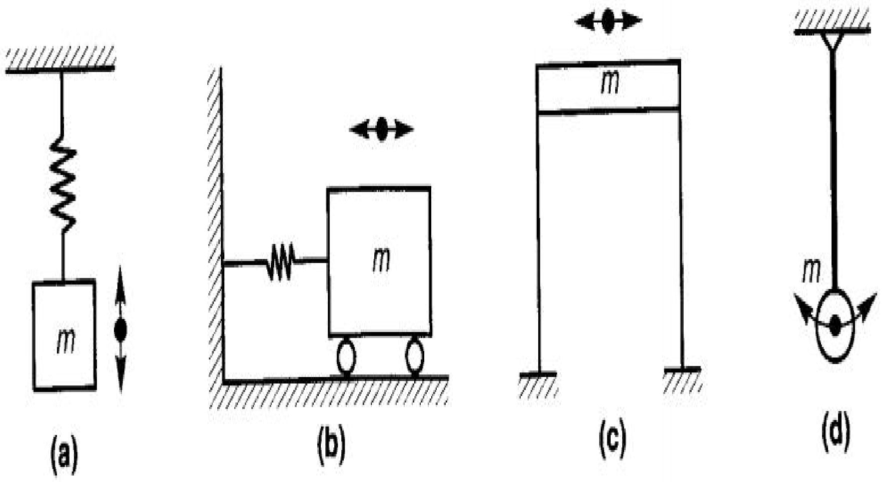


Figure B.2 Various SDOF systems. The degrees of freedom are (a) vertical translation, (b) and (c) horizontal translation, and (d) rotation.

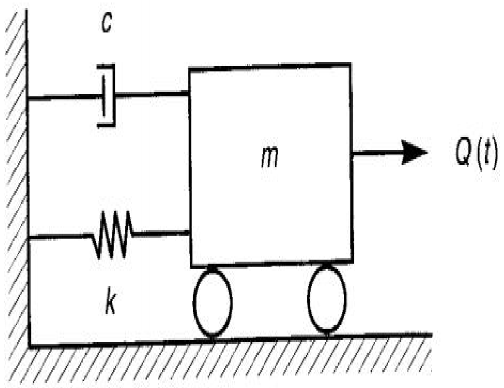
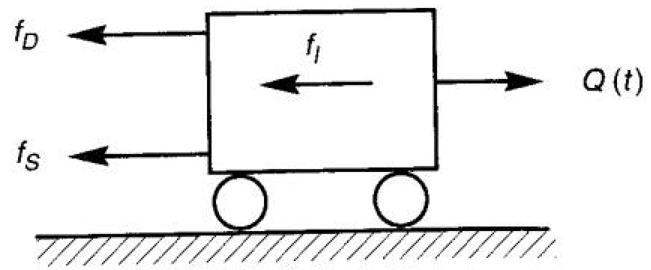


Figure B.3 Damped SDOF system subjected to external dynamic load, $Q(t)$.

Persamaan gerak SDOF

$$f_I(t) + f_D(t) + f_S(t) = Q(t)$$



$$m\ddot{u}(t) + c\dot{u}(t) + ku(t) = Q(t)$$

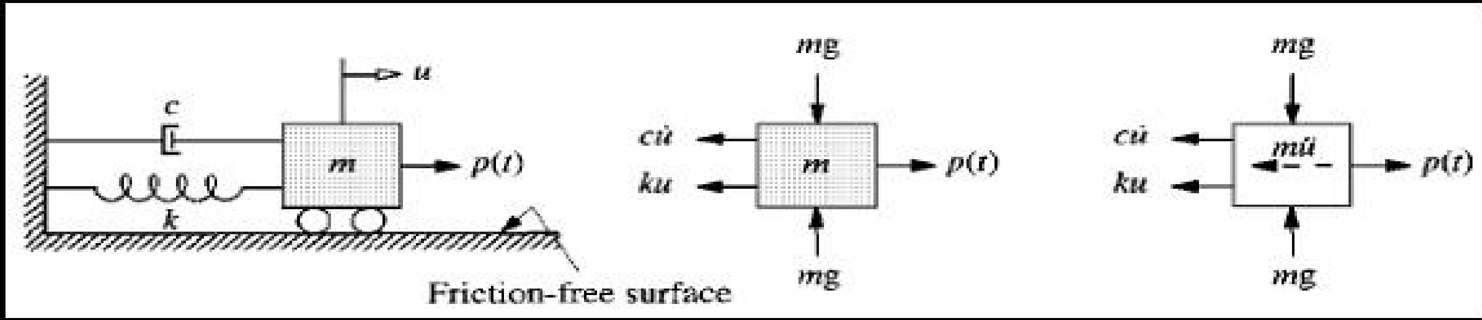
- Q(t) = external loading
- f_I = gaya inersia
- f_D = viscous damping force
- f_S = elastic spring force

$$f_I(t) = \frac{d}{dt}\left(m\frac{du(t)}{dt}\right) = m\frac{d^2u(t)}{dt^2} = m\ddot{u}(t)$$

$$f_D(t) = c\frac{du(t)}{dt} = c\dot{u}(t) \qquad f_S(t) = ku(t)$$

Simple Vibrating System (SDOF system)

Mass-Spring-Damper (MSD) System



- m ⟶ Kinetic Energy
- k ⟶ Potential Energy
- c ⟶ Dissipation

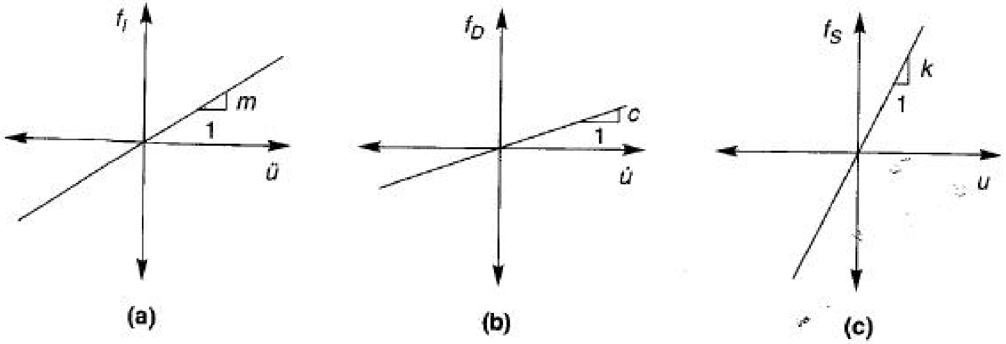
D'Allembart's principle

- For any object in motion, the externally applied forces, inertial force and forces of resistance form a system of forces in equilibrium.

Persamaan Gerak SDOF Akibat Beban Eksternal

$$m\ddot{u}(t) + c\dot{u}(t) + ku(t) = Q(t)$$

- Linear system:



- Non linear system:

  → Numerical solution

Persamaan Gerak SDOF akibat Vibrasi Support (Damped base shaking)

$$m\ddot{u}_t + c\dot{u} + ku = 0$$

or substituting $\ddot{u}_t(t) = \ddot{u}_b(t) + \ddot{u}(t)$ and rearranging,

$$m\ddot{u} = c\dot{u} + ku = -m\ddot{u}_b$$

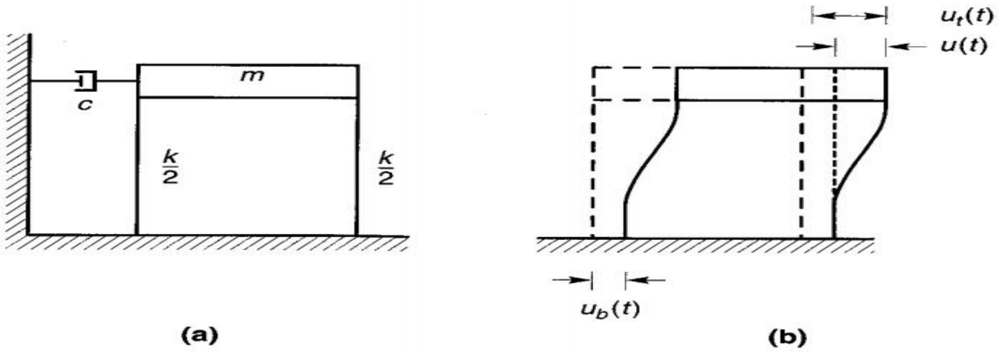


Figure B.6 Damped SDOF system subjected to base shaking.

Artinya: Respon suatu system akibat shaking adalah ekivalen dengan masa yang diberi beban eksternal

$$Q(t) = -m\ddot{u}_b(t)$$

Respon System akibat Shaking adalah Ekivalen dengan Masa dengan Beban Eksternal

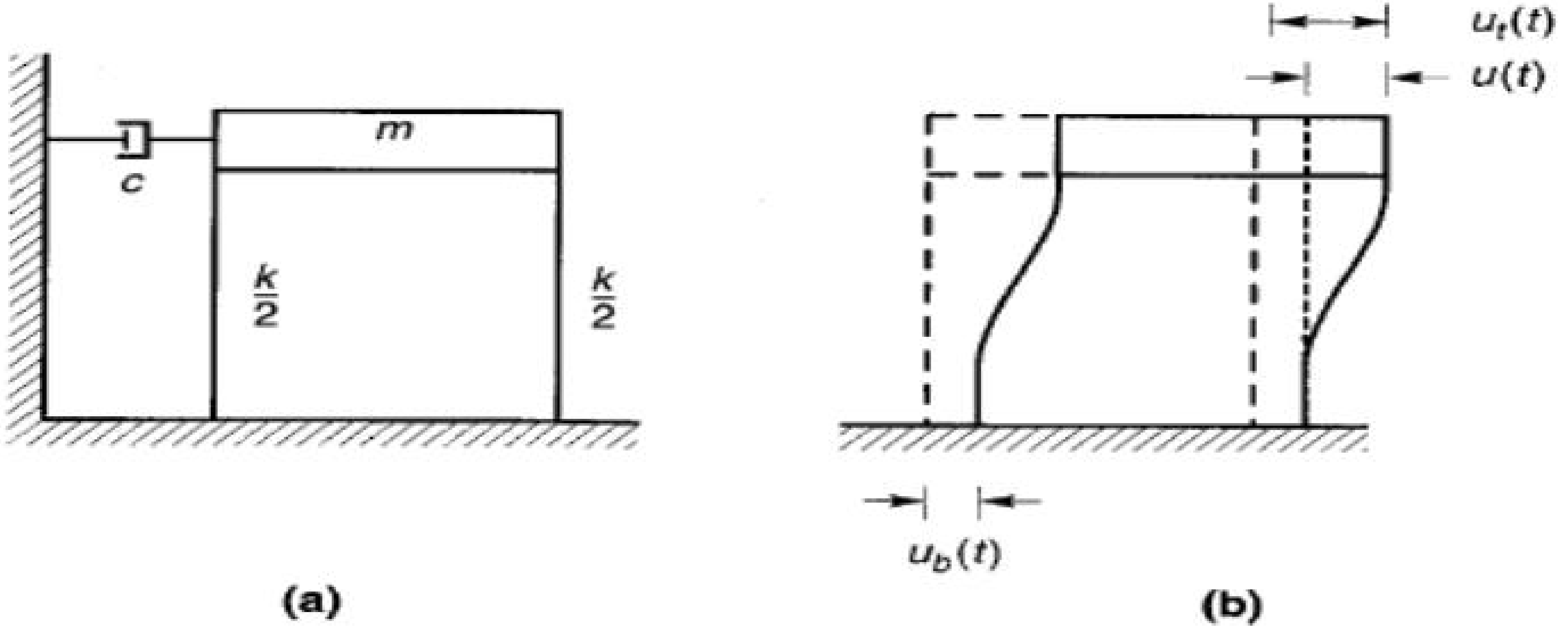


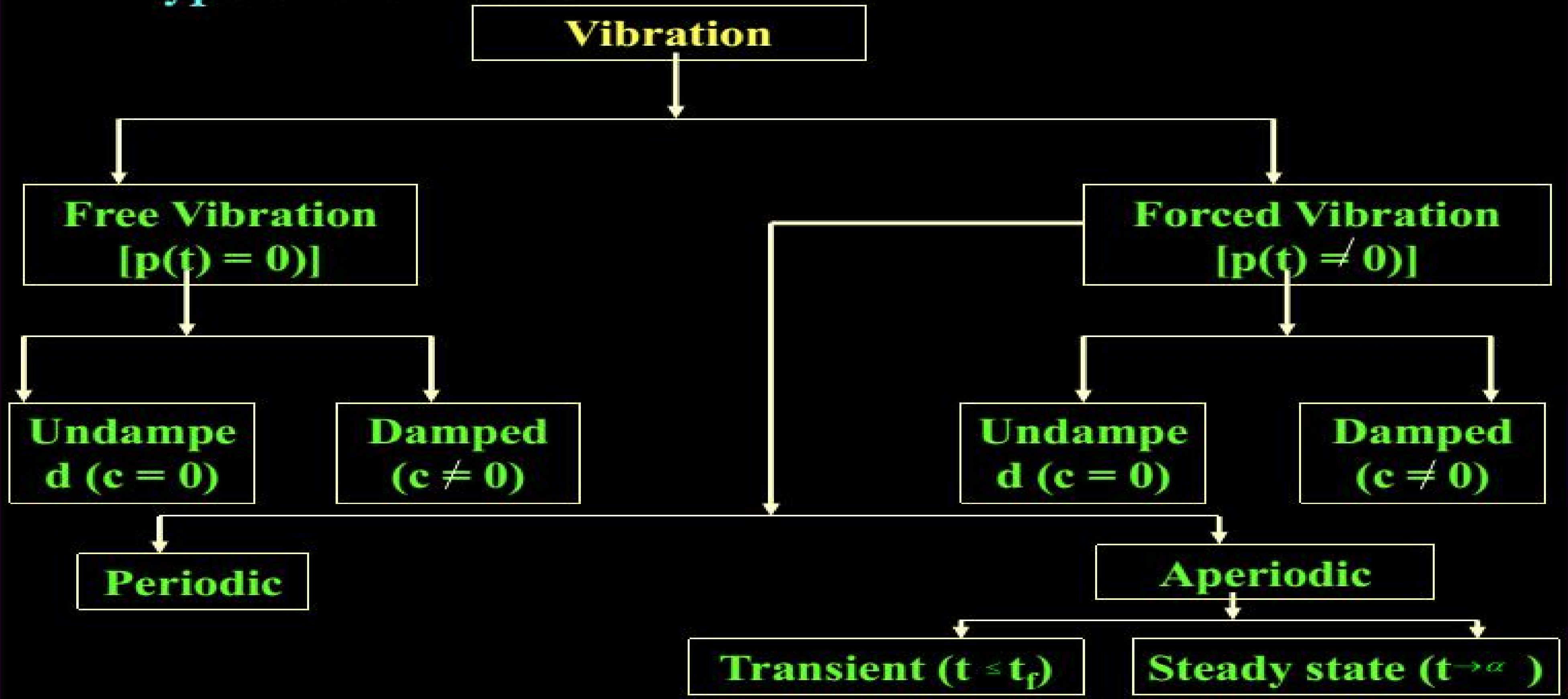
Figure B.6 Damped SDOF system subjected to base shaking.

Respon Linear SDOF System

- Evaluasi respon dinamik system SDOF linear dilakukan dengan menggunakan persamaan diferensial.
- Forced Vibration terjadi tatkala system dibebani eksternal loading (Qt) baik periodic maupun non periodic.
- Free vibration terjadi akibat adanya shaking tanpa adanya external loading. Ini terjadi misalnya akibat pemberian initial displacement atau akhir dari transient forced vibration.
- Beberapa kondisi SDOF adalah sbb:

1. **Undamped free vibrations:** $c = 0, \quad Q(t) = 0$
2. **Damped free vibrations:** $c > 0, \quad Q(t) = 0$
3. **Undamped forced vibrations:** $c = 0, \quad Q(t) \neq 0$
4. **Damped forced vibrations:** $c > 0, \quad Q(t) \neq 0$

Type of vibrations



$$m\ddot{u} + c\dot{u} + ku = p(t)$$

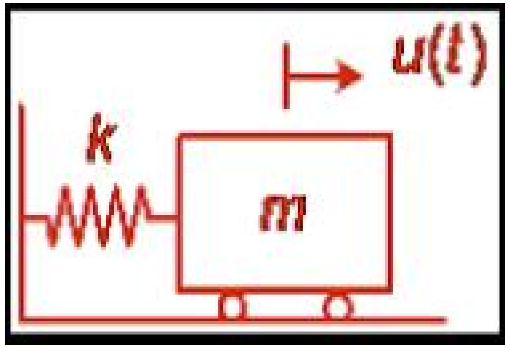
By: Yusep Muslih Purwana
Prodi Teknik Sipil FT UNS

Undamped Free Vibration

- Undamped, c = 0

→ $m\ddot{u} + ku = 0$ $\ddot{u} + \frac{k}{m}u = 0$

- Solution: $u = C_1 \sin\sqrt{\frac{k}{m}}t + C_2 \cos\sqrt{\frac{k}{m}}t$



dengan C_1 dan C_2 konstan, ditentukan dengan *intital condition*

- Undamped natural frequency dari system: $\omega_0 = \sqrt{\frac{k}{m}}$

- Natural frequency dan natural periode adalah:

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$T_0 = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{m}{k}}$$

Undamped Free Vibration (Cont)

Dengan substitusi diperoleh:

$$u = C_1 \sin \omega_0 t + C_2 \cos \omega_0 t$$

Pada kondisi awal t = 0, maka

$$u_0 = C_1 \sin(0) + C_2 \cos(0) = C_2$$

$$\dot{u}_0 = \omega_0 C_1 \cos(0) - \omega_0 C_2 \sin(0) = \omega_0 C_1$$

$C_1 = \dot{u}_0/\omega_0$ and $C_2 = u_0$.

$$u = \frac{\dot{u}_0}{\omega_0} \sin \omega_0 t + u_0 \cos \omega_0 t$$

$$u = A \sin(\omega_0 t + \phi) \qquad A = \sqrt{u_0^2 + \left(\frac{\dot{u}_0}{\omega_0}\right)^2}$$

$$\phi = \tan^{-1} \frac{u_0 \omega_0}{\dot{u}_0}$$

Menggambarkan vibrasi harmonic pada frekuensi natural undamped

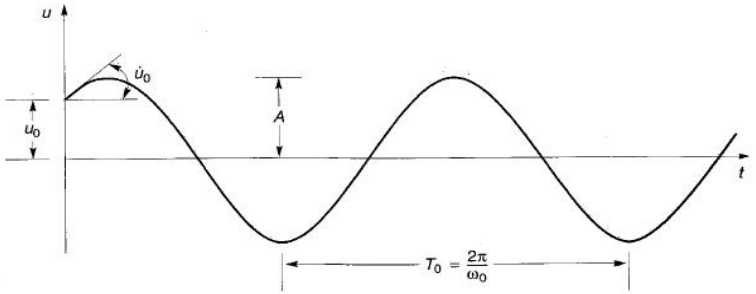


Figure B.7 Time history of displacement for undamped free vibration with initial displacement u_0 and initial velocity $\dot{u}_0$.

Undamped Free Vibration (Cont)

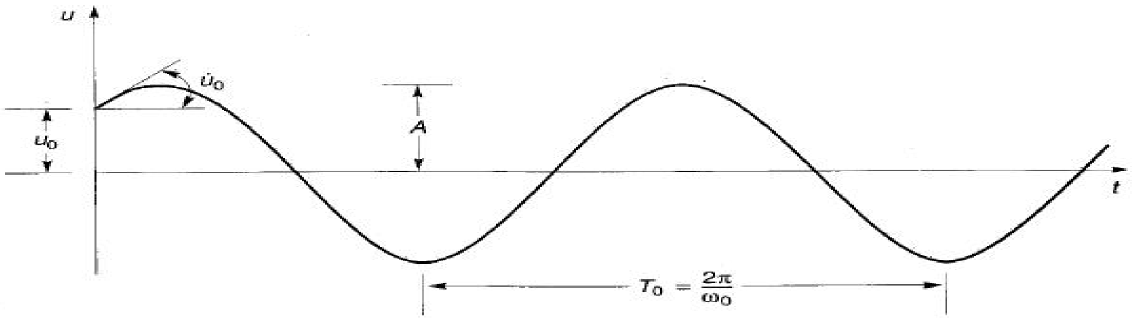


Figure B.7 Time history of displacement for undamped free vibration with initial displacement u_0 and initial velocity $\dot{u}_0$.

$$u = A\sin(\omega_0 t + \phi)$$

$$A = \sqrt{u_0^2 + \left(\frac{\dot{u}_0}{\omega_0}\right)^2}$$

$$\phi = \tan^{-1}\frac{u_0\omega_0}{\dot{u}_0}$$

Penyelesain ini menunjukkan bahwa:

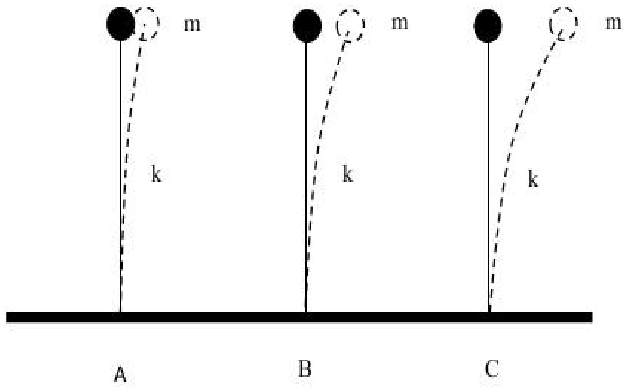
1. Respon system tergantung dari initial displacement dan velocity
2. Amplitode constant sepanjang waktu
3. Tidak ada kehilangan energy, sehingga akan bergerak terus menerus
4. Sistem ini hanya ideal, pada kenyataannya tidak ada di lapangan
5. Akan tetapi utk system dengan damping yang rendah, respon pada perode pendek masih mungkin terjadi

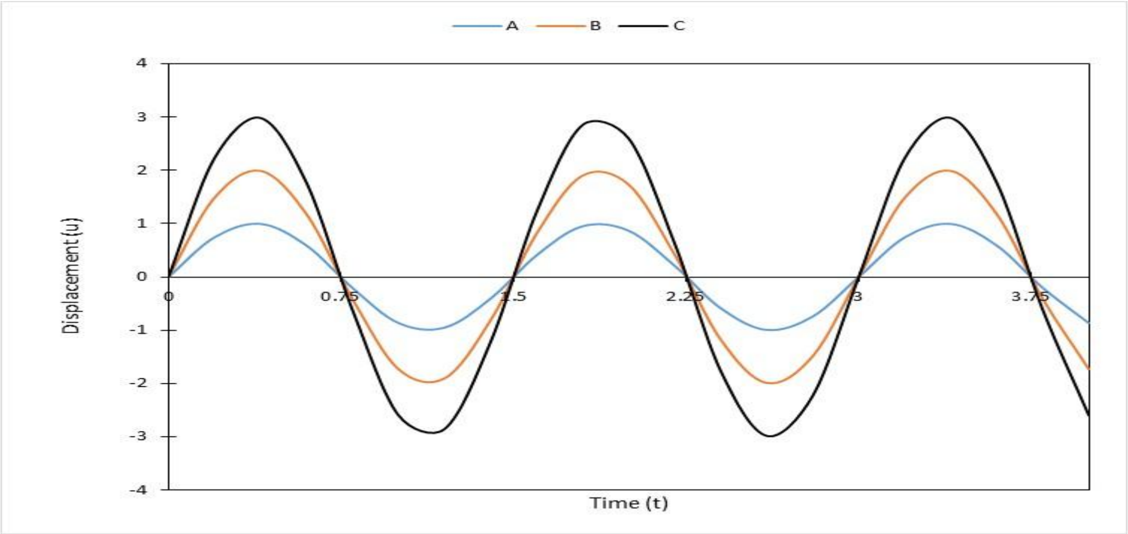
CATATAN: Natural Frequency

- **Natural frequency ω_o** adalah frekuensi yang dimiliki oleh suatu undamped system yang dibiarkan bergetar tanpa diberikan eksitasi (undamped free vibration system). Sistem akan berosilasi dengan frekuensi tertentu.
- Frekuensi ini nilainya bergantung pada kekakuan (k) dan massa (m), diformulasikan:

$$\omega_o = \sqrt{\frac{k}{m}} \qquad \text{(rad/sec)}$$

$$f_o = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \text{(Hertz)}$$





Example (diambil dari Kramer)

Example B.1

The SDOF structure shown in Figure EB.1a consists of a 10-kip weight supported by a massless column. Application of a 5-kip static horizontal force to the weight produces a horizontal deflection of 0.04 in. Compute (a) the natural circular frequency, (b) the natural period of vibration, and (c) the time history of response if the horizontal force was suddenly removed.

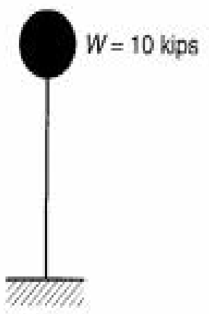


Figure EB.1a

Solution (a) The problem statement indicates that the stiffness of the column is

$$k = \frac{5 \text{ kips}}{0.04 \text{ in.}} = 125 \text{ kips/in.}$$

The natural circular frequency is given by

$$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{kg}{W}} = \sqrt{\frac{(125 \text{ kips/in.})(12 \text{ in./ft})(32.2 \text{ ft/sec}^2)}{10 \text{ kips}}} = 69.5 \text{ rad/sec}$$

(b) The natural period would be

$$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi \text{ rad}}{69.5 \text{ rad/sec}} = 0.09 \text{ sec}$$

(c) The horizontal force produced a static deflection of 0.04 in. Consequently, the initial conditions for free vibration would be

$$u_0 = 0.04 \text{ in.} \qquad \dot{u}_0 = 0$$

Then

$$u(t) = \frac{\dot{u}_0}{\omega_0} \sin \omega_0 t + u_0 \cos \omega_0 t = (0.04 \text{ in.}) \cos (69.5t)$$

The response is plotted in Figure EB.1b.

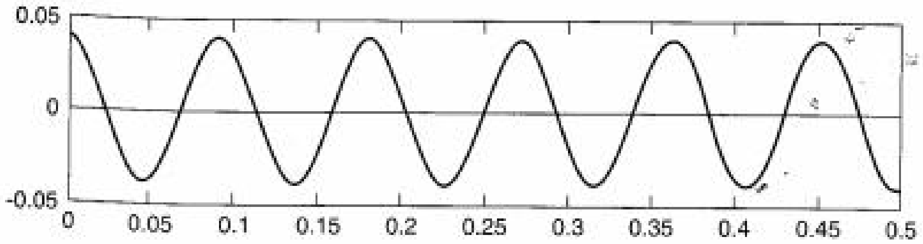


Figure EB.1b

Damped Free Vibration

Persamaan utk system ini:

$$m\ddot{u} + c\dot{u} + ku = 0$$

$$\ddot{u} + 2\frac{c}{2\sqrt{km}}\omega_0 u + \omega_0^2 u = 0$$

Nilai $2\sqrt{km}$ disebut crital damping coefisien c_c. Damping ratio ξ didefinisikan sebagi rasio damping coefcient dengan critical damping coefficient.

$$\xi = \frac{c}{c_c} = \frac{c}{2\sqrt{km}} = \frac{c}{2m\omega_0} = \frac{c\omega_0}{2k}$$

Sehingga persamaan menjadi:

$$\ddot{u} + 2\xi\omega_0\dot{u} + \omega_0^2 u = 0$$

- Pada system real, energy akan hilang akibat adanya friksi, panas, tahanan udara, atau lainnya.
- Sehingga respon pada damped free vibration dari SDOF semakin lama semakin kecil dan akhirnya hilang
- Dari persamaan terakhir, terlihat bahwa solusi persamaan tergantung pada damping ratio ξ.
- Untuk ξ < 100% ($c < c_c$) → underdumped, untuk ξ = 100% ($c = c_c$) → critically dumped untuk ξ > 100% ($c > c_c$) → over dumped
- Pada struktur umumnya berlaku kondisi underdumped

Under, Critical, and Overdamped

$$m\ddot{z} + c\dot{z} + kz = 0$$

Let $z = Ae^{rt}$ be a solution to Eq. (2.44), where A is a constant. Substitution of this into Eq. (2.44) yields

$$mAr^2e^{rt} + cAre^{rt} + kAe^{rt} = 0$$

or

$$r^2 + \left(\frac{c}{m}\right)r + \frac{k}{m} = 0 \qquad (2.45)$$

The solutions to Eq. (2.45) can be given as

$$r = -\frac{c}{2m} \pm \sqrt{\frac{c^2}{4m^2} - \frac{k}{m}} \qquad (2.46)$$

If $c/2m < \sqrt{k/m}$, the roots of Eq. (2.45) are complex :

$$r = -\frac{c}{2m} \pm i\sqrt{\frac{k}{m} - \frac{c^2}{4m^2}}$$

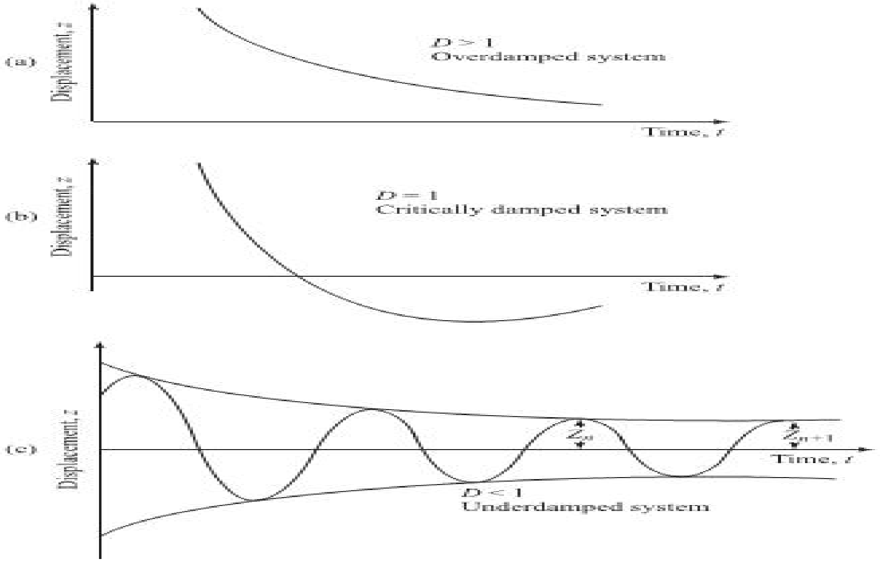
$$D = \frac{c}{c_c} = \frac{c}{2\sqrt{km}}$$

This is referred to as a case of *underdamping*.

1. If $c/2m > \sqrt{k/m}$, both roots of Eq. (2.45) are real and negative. This is referred to as an *overdamped* case.
2. If $c/2m = \sqrt{k/m}$, $r = -c/2m$. This is called the *critical damping* case. Thus, for this case,

$$c = c_c = 2\sqrt{km} \qquad (2.47a)$$

$$z = e^{-D\omega_n t}\left[z_0 \cosh\left(\omega_n\sqrt{D^2-1}t\right) + \frac{v_0 + D\omega_n z_0}{\omega_n\sqrt{D^2-1}}\sinh\left(\sqrt{D^2-1}t\right)\right]$$



Damped Free Vibration (Cont)

- Kebanyakan struktur berada dalam kondisi underdamped, gambar menunjukkan bahwa amplitude dispalecement semakin kecil.
- Rasio amplitude antara dua puncak:

$$\frac{u_n}{u_n + 1} = \exp\left(2\pi\xi\frac{\omega_0}{\omega_d}\right)$$

- Misal, penurunan logarithmic $\delta = \ln(u_n/u_n+1)$. Maka:

$$\delta = 2\pi\xi\frac{\omega_0}{\omega_d} = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

- Sehingga:

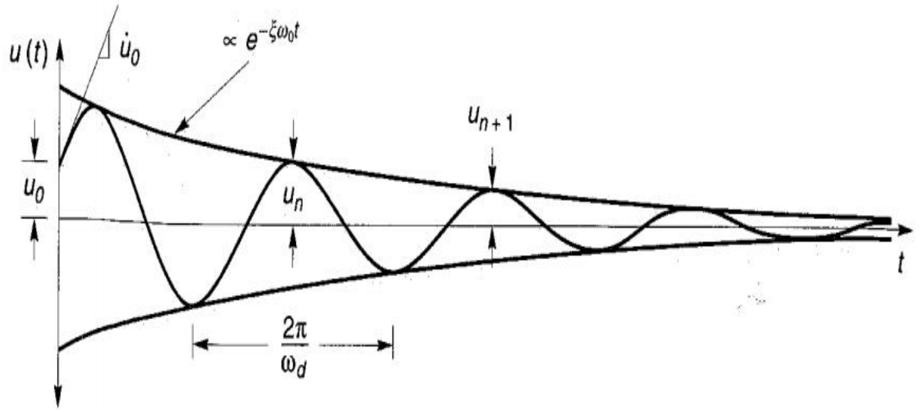
$$\xi = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}$$

Solusi untuk persamaan gerak kondisi damped free vibration adalah:

$$u = e^{-\xi\omega_0 t}\left(\frac{\dot{u}_0 + \xi\omega_0 u_0}{\omega_d}\sin\omega_d t + u_0\cos\omega_d t\right)$$

Dimana $\omega_d = \omega_0\sqrt{1-\xi^2}$

Adalah damped natural frequency. Apabila digambarkan diperoleh grafik sbb:



Damped Free Vibration (Cont)

Contoh:

Bandul seperti contoh diatas dengan berat, simpangan awal, serta kekakuan yang sama. Namun kali ini dianggap pada system berlaku damping ratio sebesar 5%. Tentukan:

1. Damped natural frequency wd
2. Gambar time history sampai 0.5 detik

Jawab:

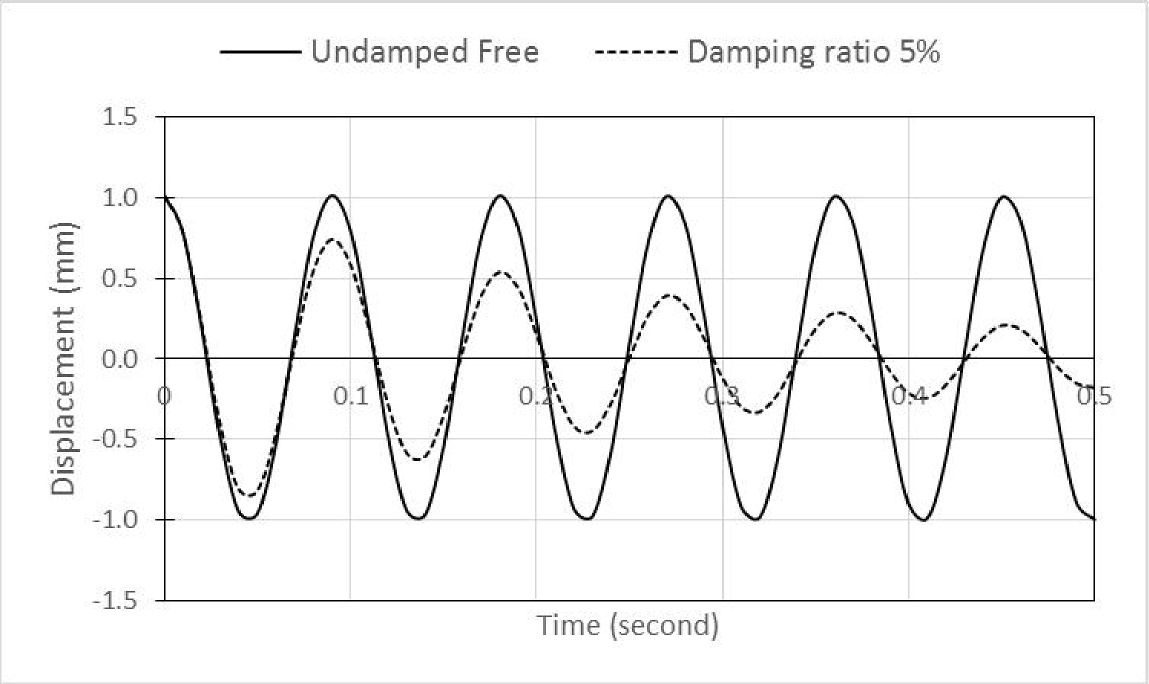
$$\omega_d = \omega_0\sqrt{1-\xi^2}$$

= $69.48x(1\text{-}0.05^2)$^$^{0.5}$

= 69.4 rad/dt

Time history di sebelah kanan

$$u = e^{-\xi\omega_0 t}\left(\frac{\dot{u}_0 + \xi\omega_0 u_0}{\omega_d}\sin\omega_d t + u_0\cos\omega_d t\right)$$



Undamped Forced Vibration

- Respon system dengan kondisi undamped forced vibration adalah:

$$m\ddot{u} + ku = Q_0 \sin \overline{\omega} t$$

- Solusi persamaan ini adalah:

$$u(t) = u_c(t) + u_p(t) = C_1 \sin \omega_0 t + C_2 \cos \omega_0 t + \frac{Q_0/\mathrm{k}}{1-\beta^2} \sin \overline{\omega} t$$

- Setelah melalui berbagai langkah substitusi, solusi akhir dari persamaan ini adalah

$$u = \left[\frac{\dot{u}_0}{\omega_0} - \frac{Q_0 \beta}{k(1-\beta^2)}\right] \sin \omega_0 t + u_0 \cos \omega_0 t + \frac{Q_0/k}{1-\beta^2} \sin \overline{\omega} t$$

- Untuk initial condition kondisi equilibrium, dimana displacement dan kecepatan awal adalah 0, maka:

$$u = \frac{Q_0}{k} \frac{1}{1-\beta^2} (\sin \overline{\omega} t - \beta \sin \omega_0 t)$$

- Menunjukkan bahwa respon mengandung 2 komponen:

1) akibat beban bekerja, dan
2) Efek free vibration akibat initial condition yang terjadi pada frekuensi natural dari sistem

Damped Forced Vibration

- Respon system kondisi ini adalah:

$$m\ddot{u} + c\dot{u} + ku = Q_0 \sin \bar{\omega} t$$

- Dengan substitusi menggunakan hubungan $\xi = c/2m\omega_0$ dan $w_0^2 = k/m$, diperoleh:

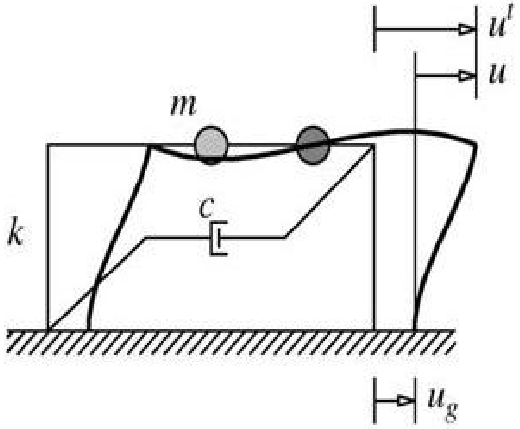
$$\ddot{u} + 2\xi\omega_0\dot{u} + \omega_0^2 u = \frac{Q_0}{m} \sin \bar{\omega} t$$

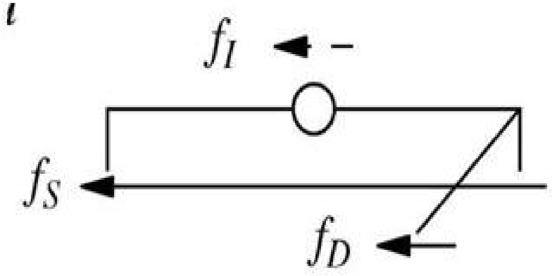
- Solusi umum untuk kondisi ini adalah:

$$u(t) = e^{-\xi\omega_0 t}(C_1 \sin \omega_d t + C_2 \cos \omega_d t) + \frac{Q_0}{k} \frac{1}{(1-\beta^2)^2 + (2\xi\beta)^2} [(1-\beta^2) \sin \bar{\omega} t - 2\xi\beta \cos \bar{\omega} t]$$

Earthquake Excitation

Single-Degree-of-Freedom System





Where,

$$\omega = \sqrt{\frac{k}{m}}$$

$$\zeta = \frac{c}{2m\omega}$$

$$f_I + f_D + f_S = 0$$

$$m\ddot{u}^t + c\dot{u} + ku \quad = 0$$

$$m(\ddot{u}_g + \ddot{u}\ ) + c\dot{u} + ku \ = 0$$

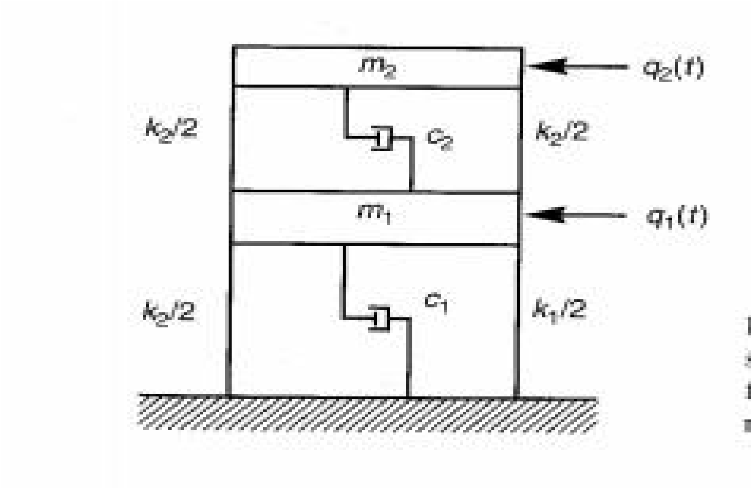
$$m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g(t)$$

$$\ddot{u} + 2\zeta\omega\dot{u} + \omega^2 u = -u_g\,(t)$$

- *the circular frequency*

- *damping*

Two Story Building (TDOF)



- Perpindahan massa 1 dan 2 dari posisi seimbang adalah u_1 dan u_2
- Untuk masing-masing massa, beban eksternal yang diberikan harus diimbangi oleh inersia, dumping, dan gaya elastis yang menahan, sehingga dpt ditulis:

$$f_{I1} + f_{D1} + f_{S1} = q_1(t)$$
$$f_{I2} + f_{D2} + f_{S2} = q_2(t)$$

- Dalam bentuk matriks, ditulis:

$$\mathbf{f}_I + \mathbf{f}_D + \mathbf{f}_S = \mathbf{q}(t)$$

- Gaya yang menahan motion pada setiap level dapat dituliskan dalam koefisien dimana parameter motion setiap level dikalikan
- Gaya elastic yg menahan motion level 1 adalah: $f_{S1} = k_{11}u_1 + k_{12}u_2$

Two Story Building (TDOF)

- Dalam bentuk matriks, dapat ekspresi diatas ditulis:

$$\begin{Bmatrix} f_{S1} \\ f_{S2} \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

  atau **f**s = **ku**

- Dimana koef kekakuan k_{ij} adalah gaya pada level *i* akibat perpindahan satuan level *j*
- Hal yang sam diberlakukan juga untuk matriks damping dan matriks massa, sehingga dapat ditulis:

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{c}\dot{\mathbf{u}} + \mathbf{k}\mathbf{u} = \mathbf{q}(t)$$

- Persamaan gerak untuk base shaking dapat ditulis menjadi:

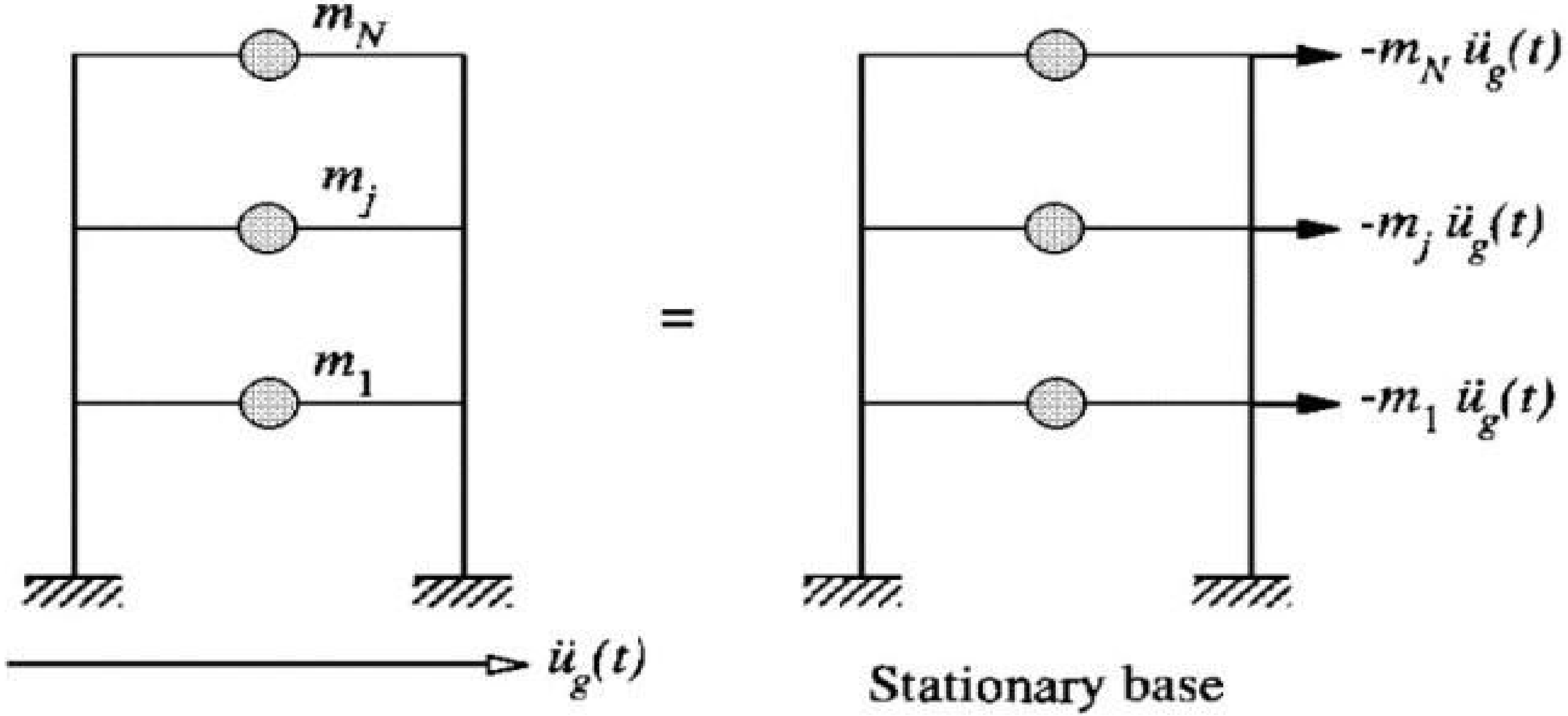
$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{c}\dot{\mathbf{u}} + \mathbf{k}\mathbf{u} = -\mathbf{m}\mathbf{1}\ddot{u}_b(t)$$

- Persamaan ini menunjukkan bahwa respon struktur bertingkat N akibat base motion sama dengan respon terhadap beban eksternal ekivalen dimana

$$q_i = -m_i\ddot{u}_b(t)\ (i = 1, N)$$

adalah beban yang diberikan terhadap lantai ke *i*

Multi Story Building (MDOF)



Undamped Free Vibration

- Untuk kondisi ini, persamaan gerak dapat ditulis sebagai:

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{k}\mathbf{u} = \mathbf{0}$$

- Dengan asumsi bahwa respon utk setiap massa adalah harmonic, maka dapat ditulis:

$$\mathbf{u}(t) = \mathbf{U}\sin(\omega t + \boldsymbol{\theta})$$

- Dimana **U** adalah vector yang mengandung amplitude perpindahan dan t adalah vector yang mengandung sudut fase setiap level struktur. Apabila persamaan ini diturunkan dua kali, diperoleh:

$$\ddot{\mathbf{u}}(t) = -\omega^2\mathbf{U}\sin(\omega t + \boldsymbol{\theta}) = -\omega^2\mathbf{u}(t)$$

- Substitusi kedua persamaan tsb menghasilkan persamaan:

$$-\mathbf{m}\omega^2\mathbf{U}\sin(\omega t + \boldsymbol{\theta}) + \mathbf{k}\mathbf{U}\sin(\omega t + \boldsymbol{\theta}) = \mathbf{0}$$ atau $$[\mathbf{k} - \omega^2\mathbf{m}]\mathbf{U} = \mathbf{0}$$

Undamped Free Vibration

- Solusi dari persamaan non trivial

$$-\mathbf{m}\omega^2\mathbf{U}\sin(\omega t+\theta)+\mathbf{k}\mathbf{U}\sin(\omega t+\theta) = 0 \qquad [\mathbf{k}-\omega^2\mathbf{m}]\mathbf{U} = 0$$

- Hanya dapat diperoleh hanya jika: $\det(\mathbf{k}-\omega^2\mathbf{m}) = |\mathbf{k}-\omega^2\mathbf{m}| = 0$
- Persamaan terakhir ini disebut sebagai persamaan frekuensi (atau persamaan karakteristik) dari sistim dimana utk sistim dengan N derajad kebebasan memberikan polynomial dengan derajad ke N dalam ω^2.
- Akar-akar ke N dari persamaan frekuensi $\{\omega_1^2, \omega_2^2, \omega_3^2, \ldots, \omega_N^2\}$ merupakan frekuensi dimana sistim tanpa damping bergetar tanpa ada gaya eksternal; disebut sebagai frekuensi natural sirkular

Terimakasih