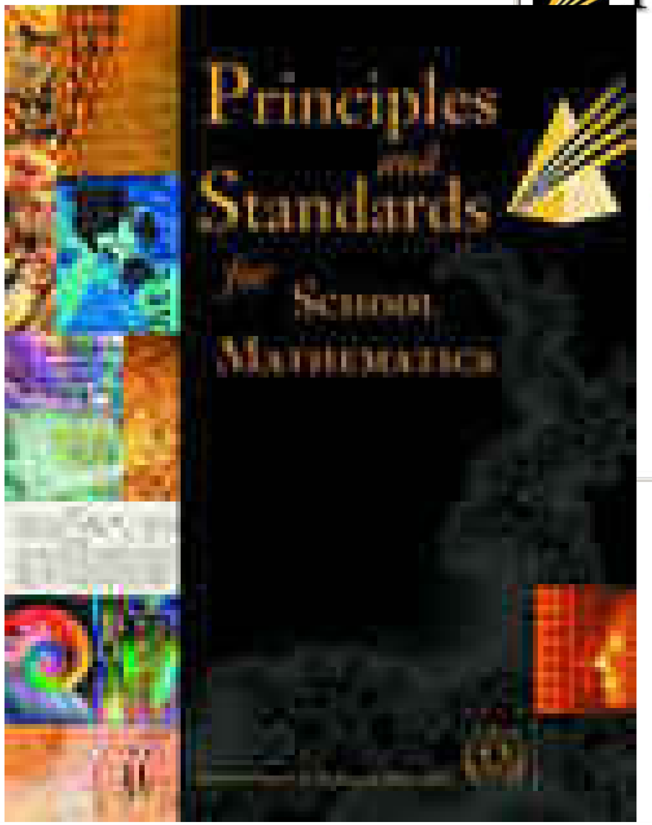
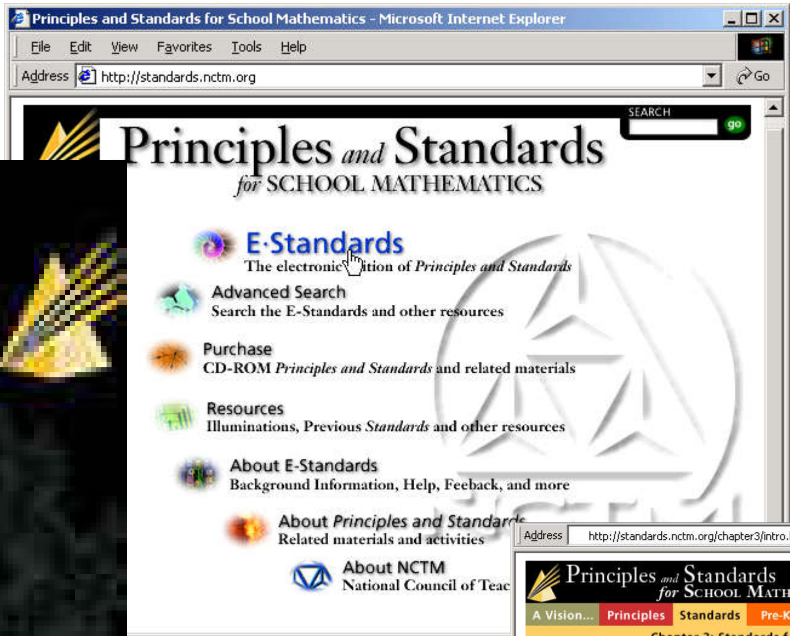
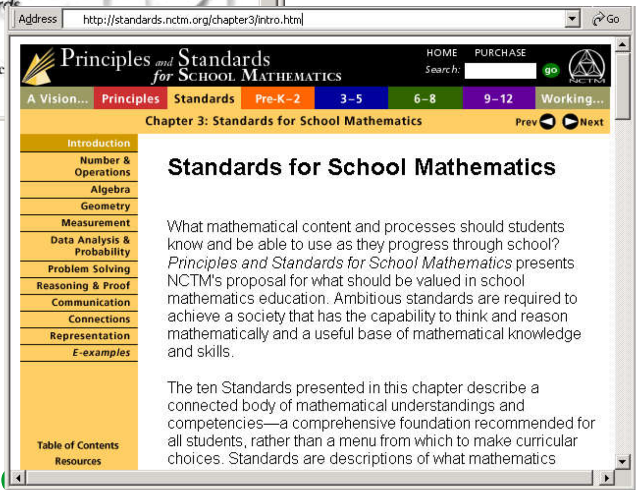
The NCTM *Standards*: A Vision of Mathematics Teaching and Learning

Principles and Standards for School Mathematics

NCTM – Hard Copy, CD-ROM, Web (with e-examples and illuminations)







standards.nctm.org

www.illuminations.nctm.org

Navigations Series

Who Was Involved in Development of the *Principles and Standards?*

- Teachers
- School administrators
- Mathematics supervisors
- University mathematicians
- Mathematics educators
- Researchers

Why *Principles and Standards for School Mathematics?*

- **The world is changing – call for change from NCTM, MAA, AMS, NRC, NSB, AERA, CEEB, ETS**
  - bombarded with "mathematical" information
  - workplace challenges to learn new skills
  - lives are being reshaped by changing technologies
- **Our students are different**
  - they are comfortable with and use technology
  - they have competing demands on their time
  - they are used to visual stimulation & access to information
- **School mathematics is not working well enough for enough students**

Time Line

- 1951 Max Beberman – UICSM Project
- 1957 SPUTNIK
- 1960 "New Math"
- 1963 FIMS – First International Mathematics Study
- 1969 NAEP – National Assessment of Ed Progress
- 1973 *Why Johnny Can't Read* – Morris Kline
  "Back to Basics"
- 1975 NACOME
- 1980 *Agenda For Action* – NCTM
  "Pragmatic – Problem Solving"
- 1982 SIMS – Second International Mathematics Study
- 1983 *A Nation at Risk*
- 1987 J. R. Flanders analysis of textbooks – prior to algebra

- 1989 *Curriculum Standards* – NCTM
  Everybody Counts – Lynn Steen & NRC
- 1991 *Teaching Standards* – NCTM
- 1992 NSF-Funded Integrated Standards-Based Curriculums
- 1995 *Assessment Standards* – NCTM
- 1996 TIMSS – Third International Mathematics and Science Study
- 1996 MAP 2000 – Mathematics Field Test
- 1998 National High-Stakes Test Debate – California
- 2000 *Principles & Standards for School Math – NCTM*
  MAP & MSIP – *Show-Me Standards, Curriculum Frameworks, Grade-Level Expectations*

Integrated Standards-Based Curriculums

The K-12 Mathematics Curriculum Center -- www2.edc.org.mcc.images.cumsum6.pdf

Elementary (The ARC Center – www.comap.com/arc)

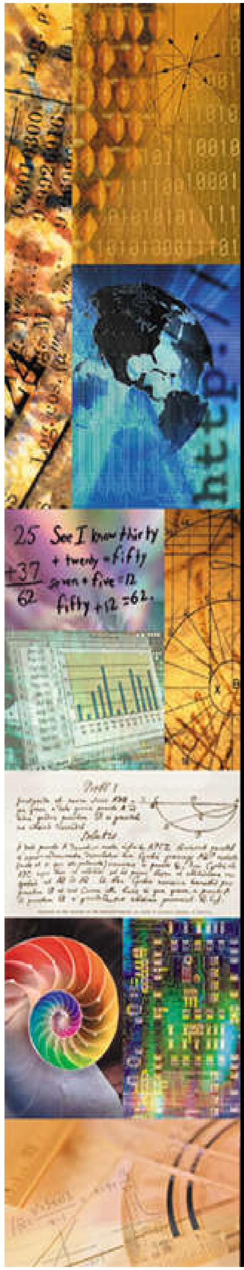
- Everyday Mathematics (K-6)
- Investigations in Number, Data, and Space (K-5)
- Math Trailblazers (TIMS) (K-5)

Middle School (The Show-Me Center – www.showmecenter.missouri.edu)

- Connected Mathematics (6-8)
- Mathematics in Context (5-8)
- MathScape: Seeing and Thinking Mathematically (6-8)
- MATHThematics (STEM) (6-8)

High School (Compass – www.ithaca.edu/compass)

- Core-Plus Mathematics Project (9-12)
- Interactive Mathematics Program (9-12)
- MATH Connections: A Secondary Mathematics Core Curriculum (9-11)
- Mathematics: Modeling Our World (ARISE) (9-12)
- SIMMS Integrated Mathematics: A Modeling Approach Using Technology (9-12)



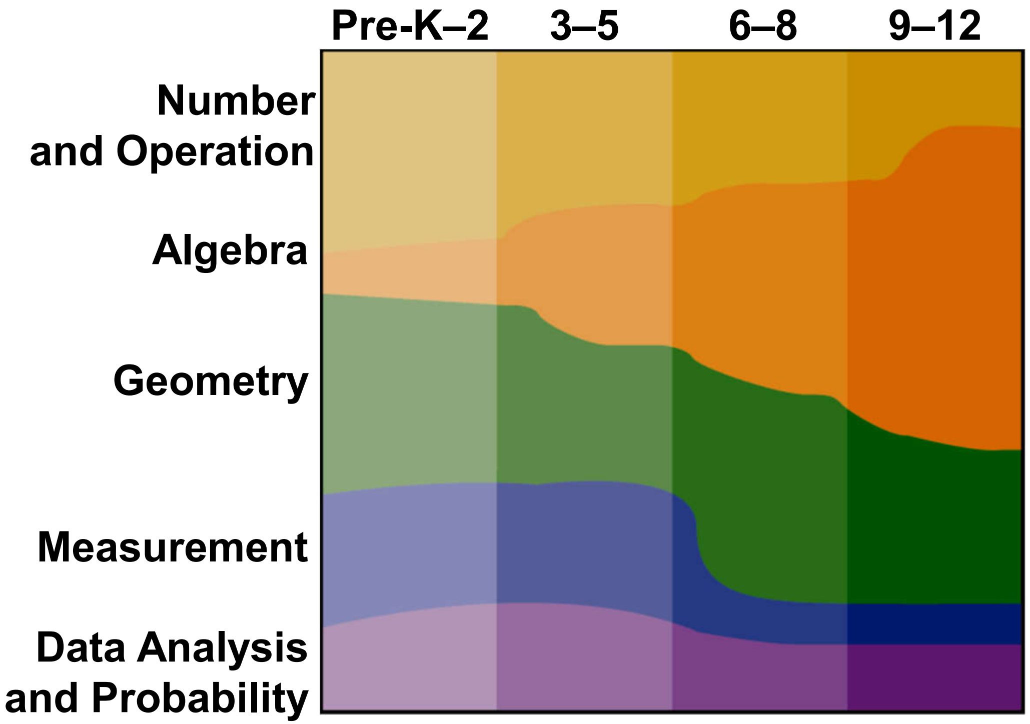
Standards

- Number and Operations
- Algebra
- Geometry
- Measurement
- Data Analysis and Probability

- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representation

Content Standards
Across the Grades

Number



BEST FOR You
ORGANICS COMPANY

Algebra Standard

Instructional programs from prekindergarten through grade 12 should enable all students to—

- Understand patterns, relations, and functions
- Represent and analyze mathematical situations and structures using algebraic symbols
- Use mathematical models to represent and understand quantitative relationships
- Analyze change in various contexts

Algebra Standard

One Expectation Across the Grades

- Understand patterns, relations and functions

Pre-K-2	▪ Recognize, describe, and extend patterns such as sequences of sounds and shapes or simple numeric patterns and translate from one representation to another.
3-5	▪ Describe, extend, and make generalizations about geometric and numeric patterns.
6-8	▪ Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible symbolic rules.
9-12	▪ Generalize patterns using explicitly defined and recursively defined functions.

Cost of Balloons

Number of Balloons	Cost of Balloons (in cents)
1	20
2	40
3	60
4	80
5	?
6	?
7	?

Looking
for Patterns

Grade-3 Level
Expectation in Missouri
(left)

Grade-6 Level
Expectation in Missouri
(below)

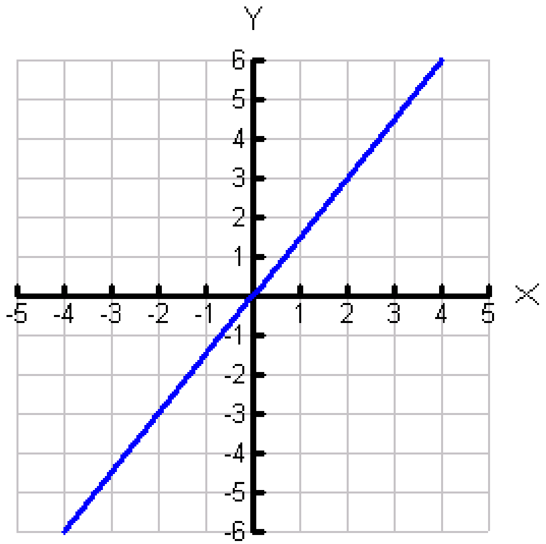
Cost of Balloons

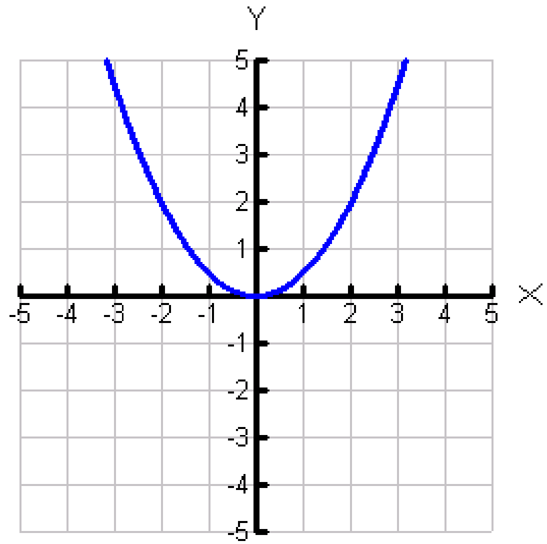
Number of Balloons	1	2	3	4	5		n
Cost of Balloons (in cents)	20	40	60	80	?		?

Write an equation which names the relationship between the two variables x and y for each of the following two graphs.

Looking for Patterns

Grade-8 Level Expectation in Missouri





BEST FOR YOU
ORGANICS COMPANY

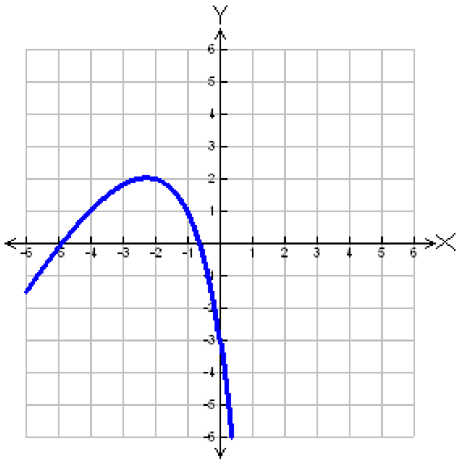
Different views and explanations of the rational function

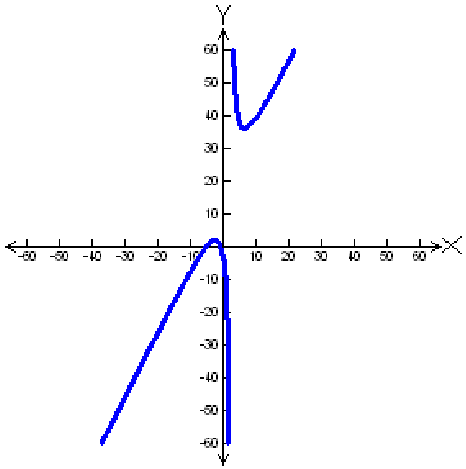
$$f(x) = \frac{2x^2 + 11x + 6}{x - 2}$$

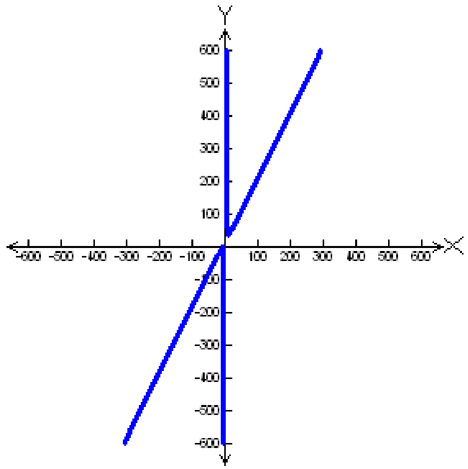
$$f(x) = 2x + 15 + \frac{36}{x - 2}$$

Looking for Patterns

Grade-10 Level Expectation in Missouri







Reasoning and Proof Standard

Instructional programs from prekindergarten through grade 12 should enable all students to—

» Recognize reasoning and proof as fundamental aspects of mathematics

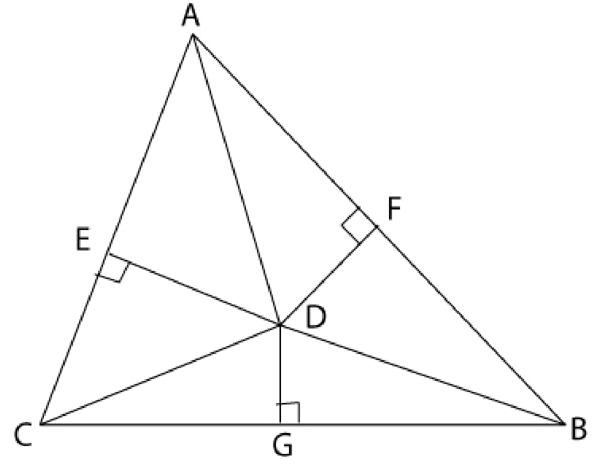
» Make and investigate mathematical conjectures

» Develop and evaluate mathematical arguments and proofs

» Select and use various types of reasoning and methods of proof

Δ

Find the Reasoning Flaw
to show that all triangles are equilateral



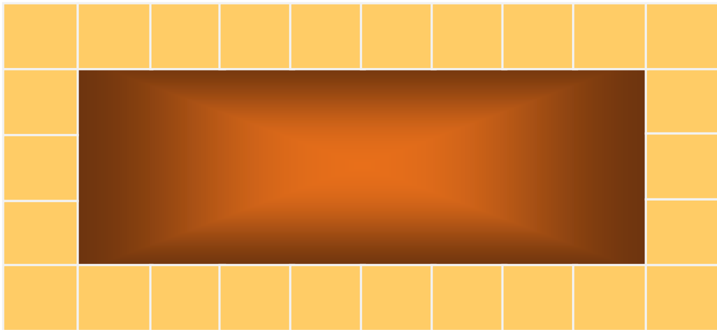
Consider any triangle ABC. Bisect angle A. Construct the perpendicular bisector to side BC. Let D be the intersection of the two lines. Construct lines perpendicular to sides AC and AB through point D. Draw segments DC and DB.

Then triangle AED is congruent to triangle AFD by SAA, and triangle BGD is congruent to triangle CGD by SAS. So triangle BFD is congruent to triangle CED by Hypotenuse-Leg. Therefore, AE + EC is congruent to AF + FB; that is AC=AB.

In a similar way, we can show that AB=BC. So triangle ABC is equilateral. Because triangle ABC is an arbitrary triangle, all triangles must be equilateral.

Resourceful Problem Solving

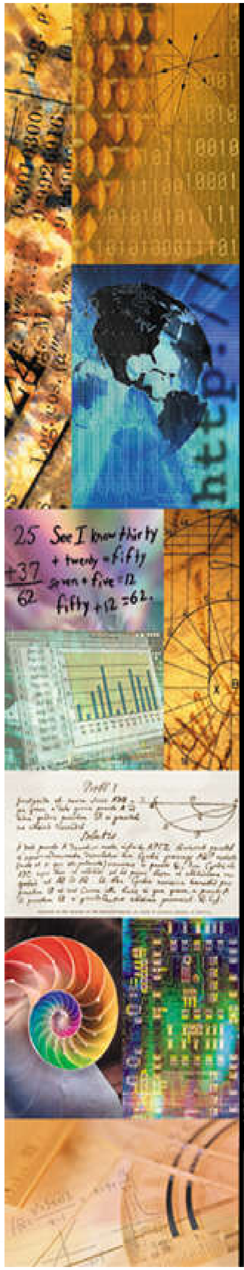
Explain in words, numbers, or tables visually and with symbols the number of tiles that will be needed for pools of various lengths and widths.



Student Responses

1) $T = 2(L + 2) + 2W$
2) $4 + 2L + 2W$
3) $(L + 2)(W + 2) - LW$

BEST FOR You
ORGANICS COMPANY



Principles

Describe particular features of high-quality mathematics programs

- Equity
- Curriculum
- Learning
- Teaching
- Assessment
- Technology

Statement of Principles

The Equity Principle

Excellence in mathematics education requires equity– high expectations and strong support for all students.

The Curriculum Principle

A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.

The Teaching Principle

Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.

Statement of Principles

The Learning Principle

Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.

The Assessment Principle

Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.

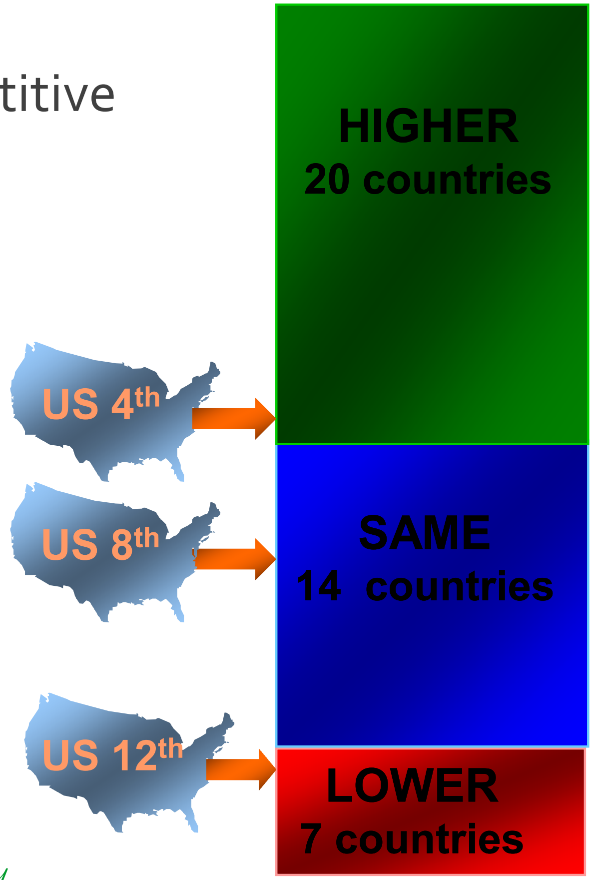
The Technology Principle

Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.

School Mathematics Is Not Working Well Enough for Enough Students

Internationally (TIMSS, 1994-1995), our students are not mathematically competitive

- 4th grade – average
- 8th grade – below average
- 12th grade – among lowest of 21
    - at 25th percentile, like FIMS & SIMS
    - particularly poor in Geometry
    - better in creative constructed responses questions



Source: US TIMSS Research Center, 1996–1998

What Explains the Poor US Showing?

» It is **NOT** that …

- other countries spent more time studying mathematics
- US teachers assign less homework than other countries
- US students spent too much time watching TV, visiting friends, etc.

» They documented that in the US …

- we teach more math topics than elsewhere at almost every grade
- math topics remain in curricula for more grades than elsewhere
- reform topics may be added but traditional content also remains
- our textbooks emphasize simple student performance – we do not stand out in our emphasis on problem solving
- our texts are fragmented in their organization
- our teachers use more activities and move frequently among them
- our teachers work hard, but unable (curricula & texts) to "work smart"

Source: US TIMSS Research Center, Dr Curtis McKnight, http://ustimss.msu.edu

Nationally, however, there has been a clear and consistent pattern of higher student achievement

- Student mathematics proficiency on the **NAEP** (National Assessment of Educational Progress) has significantly increased at grades 4, 8 and 12 between 1990 and 1996, representing approximately one grade level of growth at each grade level. Also 4^{th} and 8^{th} grade scores in 2003 are higher than ever[1]

- Average **SAT**-Math scores have increased from 500 in 1991 to 512 in 1998 to 519 in 2003.[2]

- Connecticut, Michigan, Texas and North Carolina have reported some of the greatest student mathematics gains between 1990 and 1996 – 4 states with strong and consistent investments in standards and assessments reflecting the NCTM Standards vision.[3]

Sources: [1]NAEP 2004; [2]SAT 2004; [3]NCTM, 2000

Students Can Do Basics, ...

347 + 453

90%

864 – 38

73%

… But Students Cannot Solve Problems

33%

Ms. Yost's class has read 174 books, and Mr. Smith's class has read 90 books.
How many more books do they need to read to reach the goal of reading 575 books?

Source: NAEP 1996

What Does Research Tell Us? [1]

» **What we can't expect from research …**

- Standards are not determined by research, but are instead statements of priorities and goals
- What is "best" cannot be proven by research.
- Research cannot imagine new ideas.

» **What we can expect from research …**

- Research can influence the nature of the standard.
- Research can document the current situation.
- Research can document the effectiveness of new ideas.
- Research can suggest explanations for success or failure.

[1] Hiebert, J. (1999) Relationships between research and the NCTM Standards, *Journal for Research in Mathematics Education*, *30*(1):3-19.

How Is Mathematics Learned?

- **Mental Discipline** (19th Century) – Mind is a muscle

- **Behaviorism** (Early 1900's) – Condition observable human behavior
  - J. B. Watson – sequence of stimulus and response actions (Pavlov's dog)
  - E. L. Thorndike (1922) – connectionism (cat escaped puzzle box)
  - Max Wertheimer (1922) – gestalt theory - group stimuli (parallelogram area)
  - B. F. Skinner (1950) – behavior modification - positive/negative reinforcement

- **Incidental Learning** (Dewey, 1920's) – "hands on", integrated curriculum

- **Developmental Learning** (Piaget , 1960's) – develop in 4 stages (yrs), sensorimotor (0-2), preoperational (3-7), concrete operational (8-11), formal operational (12-15)

- **Constructivism** (Introduced 1970's) – learner actively constructs concepts
  - Jerome Bruner (1966) – learning in 3 phases, enactive (touch), iconic (pictorial), symbolic (abstract)
  - L. Vygotsky (1978) – social constructivism, learn first socially then individually
  - E. von Glasersfeld (1987) – researched specifically in mathematics education

- Other Current Learning Theories
  - **Theory of Multiple Intelligences** (H. Gardner, 1993) – 8 intelligences
  - **Brain-Compatible Learning** (E. Jensen, 1999) – connected synapse (time)

Learning Theories' Impact on Teaching

Behaviorism	Constructivism
Directed Instruction – Teacher Centered	Facilitated Instruction – Learner Centered
Students viewed as blank slates onto which information is etched by teacher	Students viewed as thinkers with emerging theories about the world
Students work primarily alone	Students work primarily in groups
Strict adherence to fixed curriculum and correct answers is highly valued	Pursuit of student questions and students' conceptions are highly valued
Curricular activities rely heavily on textbooks and workbooks	Curricular activities rely heavily on primary sources of data and manipulative materials
Assessment of student learning is separate from teaching and occurs almost entirely through testing	Assessment of student learning is interwoven with teaching and includes observation of students at work & exhibits

Current State of Classroom Teaching

- **A consistent predictable method of teaching math in the U.S. for nearly a century, even in the face of pressures to change.**

  (Dixon et al., 1998; Fey, 1979; Stigler & Hiebert, 1997; Stodolsky, 1988, Weiss, 1978)

  Often cited account…

  "First, answers were given for the previous day's assignment. A brief explanation, sometimes none at all, was given of the new material, and problems were assigned for the next day. The remainder of the class was devoted to students working independently on the homework while the teacher moved about the room answering questions. The most noticeable thing about math classes was the repetition of this routine." (Welch, 1978, p.6)

- **"Teachers are essentially teaching the same way they were taught in school."** (Conference Bd of Mathematical Sciences, 1975, p.77)

- **Most characteristic of traditional math teaching is the emphasis on teaching procedures, especially computation procedures.**

- **From the TIMSS 8th-grade video study in US classrooms …**
  - for 78% of the topics covered, procedures and ideas were only demonstrated or stated, not explained or developed.
  - 96% of the time students were doing seatwork, they were practicing procedures they had been shown how to do.

    (Stigler & Hiebert, Phi Delta Kappan, 1997)

- "One of the most reliable findings from research on teaching and learning: Students learn what they have an opportunity to learn. Achievement data indicate that is what they are learning: simple calculation procedures, terms and definitions." (Hiebert, JRME, 1999, p.12)

- "It is curious that the current debate about the future of mathematics education in this country often is treated as a comparison between the traditional "proven" approaches and the new "experimental" approaches." (Schoenfeld, J. Math Behavior, 1994)

What About Technology?

In her 1997 meta-analysis of all U.S. research studies involving technology[1], and her later 2001 meta-analysis of research involving CAS environments specifically[2], M. Kathleen Heid reports findings from multiple researchers about using technology for mathematics instruction. She reflects on issues about the nature of technology use, learning issues, curriculum issues, and teacher preparation issues. Some of her discussion and findings are listed below:

- The use of calculators does not lead to an atrophy of basic skills.
- Symbolic manipulation skills may be learned more quickly in areas such as introductory algebra and calculus after students have developed conceptual understanding through the use of cognitive technologies.
- Concepts-before-skills approach using CAS in algebra and calculus courses, and inductive-before-deductive investigatory approach in geometry have been tested.
- Graphics-oriented technology may level the playing field for males and females.
- CAS students were more flexible with problem solving approaches and more able to perceive a problem structure.
- CAS students were able to move and make connections between representations.
- 5 of 7 researching experts (on a panel) reported better understanding by CAS students

[1] Heid, M. K. (1997). The technological revolution and the reform of school mathematics, *American Journal of Education*, *106*(1)5-61.

2 Heid, M. K. (2001). Research on mathematics learning in CAS environments. Presented at the 11th annual ICTCM Conference, New Orleans.

Do Skills Lead to Understanding?

Can Drill Help Develop Increase Mathematical Reasoning?

Can Calculators & Computers Increase Mathematical Reasoning?

Steen, L.A. (1999). Twenty questions about mathematical reasoning. in L.V. Stiff & F.R. Curcio (Eds), Developing mathematical reasoning in grades K-12: 1999 Yearbook (pp. 270-285). Reston, VA : NCTM.

Do Skills Lead to Understanding?

"The public mantra for improving mathematics education focuses on skills, knowledge, and performance – *what* students "know and are able to do." To this public agenda, mathematics educators consistently add reasoning and understanding – *why* and *how* mathematics works as it does. Experienced teachers know that knowledge and performance are not reliable indicators of either reasoning or understanding.

Nonetheless, the public values (and hence demands) mathematics education not so much for its power to enhance reasoning as for the quantitative skills that are so necessary in today's world. It is not that adults devalue understanding but that they expect basic skills first (Wadsworth 1997). They believe in a natural order of learning – first skills, then higher order reasoning. But, do skills naturally lead to understanding? Or is it the reverse – that understanding helps secure skills?"

- **Lynn Arthur Steen, 1999**

Can Drill Help Develop Mathematical Reasoning?

"Critics of current educational practice indict "drill and kill" methods for two crimes against mathematics: disinterest and anxiety. Yet despite the earnest efforts to focus mathematics on reasoning, one out of every two students thinks that learning mathematics is mostly memorization (Kenney and Silver 1997).

Research shows rather convincingly that real competence comes only with extensive practice (Bjork and Druckman 1994). Nevertheless, practice is certainly not sufficient to ensure understanding. Both the evidence of research and the wisdom of experience suggest that students who can draw on both recalled and deduced mathematical facts make more progress than those who rely on one without the other (Askey and William 1995).“

- Lynn Arthur Steen, 1999

Can Calculators and Computers Increase Mathematical Reasoning?

"At home and at work, calculators and computers are "power tools" that remove human impediments to mathematical performance – they extend the power of the mind as well as substitute for it – by performing countless calculations without error or effort.

Calculators and computers are responsible for a "rebirth of experimental mathematics" (Mandelbrot 1994). They provide educators with wonderful tools for generating and validating patterns that can help children learn to reason mathematically and master basic skills.

Calculators and computers hold tremendous potential for mathematics. Depending on how they are used, they can either enhance mathematical reasoning or substitute for it, either develop mathematical reasoning or limit it."

- **Lynn Arthur Steen, 1999**

What is Mathematics?
Three Current Belief Groups

Thompson, Alba G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. Grouws (Ed.) Handbook of Research on Mathematics Teaching and Learning, Macmillan Publishing Co., New York, page 132.

The Problem Solving View

Mathematics is a continually expanding field of human creation and invention, in which patterns are generated and then distilled into knowledge. Thus mathematics is a process of enquiry and coming to know, adding to the sum of knowledge. Mathematics is not a finished product, for its results remain open to revision.

The Platonist View

Mathematics is a static but unified body of knowledge, a crystalline realm of interconnecting structures and truths, bound together by filaments of logic and meaning. Thus mathematics is a monolith, a static immutable product. Mathematics is discovered, not created.

The Instrumentalist View

Mathematics, like a bag of tools, is made up of an accumulation of facts, rules and skills to be used by the trained artisan skillfully in the pursuance of some external end. Thus mathematics is a set of unrelated but utilitarian rules and facts.

Four Views of Teaching

Thompson, Alba G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. Grouws (Ed.) Handbook of Research on Mathematics Teaching and Learning, Macmillan Publishing Co., New York, page 136.

Learner Focused

Mathematics teaching that focuses on the learner's personal construction of mathematical knowledge.

Content Focused with Emphasis on Concepts

Mathematics teaching that is driven by the content itself but emphasizes conceptual understanding.

Content Focused with Emphasis on Performance

Mathematics teaching that emphasizes student performance and mastery of mathematical rules and procedures.

Classroom Focused

Mathematics teaching based on knowledge about effective classrooms.