Capacitance,
Current and Resistance

1

Capacitance

2

The capacitance C of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between the conductors:

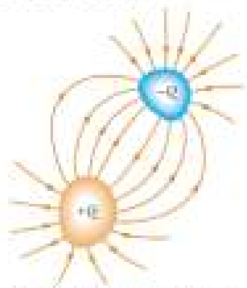


Figure 26.1 A capacitor consists of two conductors. When the capacitor is charged, the conductors carry charges of equal magnitude and opposite sign.

$$C = \frac{Q}{\Delta V}$$

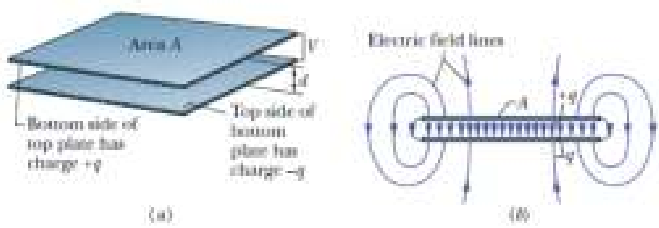
capacitance is always a positive quantity

The SI unit of capacitance is the farad (F), which was named in honor of Michael Faraday

$$1\ \text{F} = 1\ \text{C/V}$$

The farad is a very large unit of capacitance. In practice, typical devices have capacitances ranging from microfarads (10^{-6} F) to picofarads (10^{-12} F).

a *parallel-plate capacitor*



a *parallel-plate capacitor,* consisting of two parallel conducting plates of area *A* separated by a distance *d*. The symbol we use to represent a capacitor (⊣⊢) is based on the structure of a parallel-plate capacitor but is used for capacitors of all geometries.

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

$$\Delta V = Ed = \frac{Qd}{\epsilon_0 A}$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{Qd/\epsilon_0 A}$$

$$C = \frac{\epsilon_0 A}{d} \qquad \text{(parallel-plate capacitor).}$$

Example 1. Parallel-Plate Capacitor (Example 26.1 Raymond)

A parallel-plate capacitor with air between the plates has an area A = 2.00 x 10^{-4} m^2 and a plate separation d = 1.00 mm. Find its capacitance.

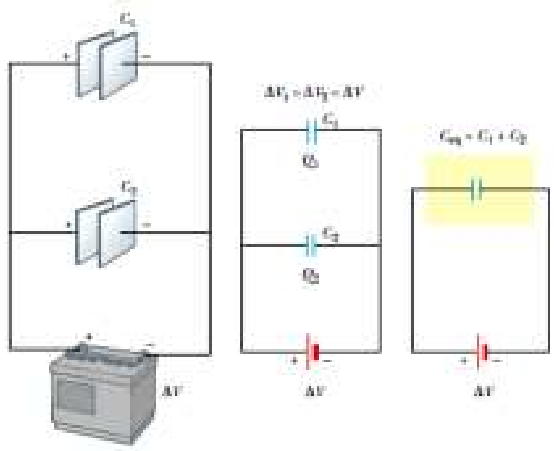
$$C = \frac{\epsilon_0 A}{d} = \frac{(8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2})(2.00 \times 10^{-4}\ \mathrm{m^2})}{1.00 \times 10^{-3}\ \mathrm{m}}$$

$$= 1.77 \times 10^{-12}\ \mathrm{F} = 1.77\ \mathrm{pF}$$

Combinations of Capacitors

Two or more capacitors often are combined in electric circuits. We can calculate the equivalent capacitance of certain combinations using methods described in this section. Throughout this section, we assume that the capacitors to be combined are initially uncharged.

Parallel Combination



the individual potential differences across capacitors connected in parallel are the same and are equal to the potential difference applied across the combination.

$$Q = Q_1 + Q_2$$

$$Q = C_{eq}\,\Delta V$$

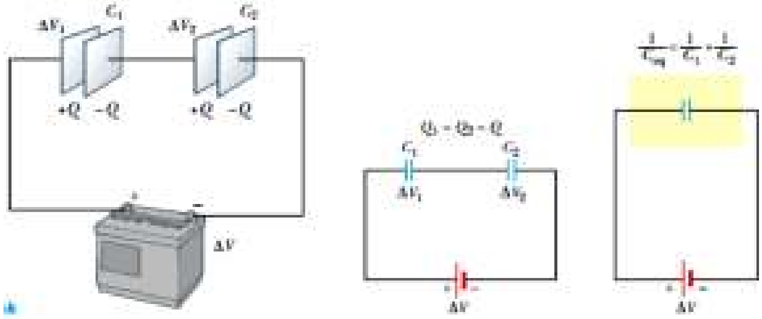
$$Q_1 = C_1\,\Delta V \qquad Q_2 = C_2\,\Delta V$$

$$C_{eq}\,\Delta V = C_1\,\Delta V + C_2\,\Delta V$$

$$C_{eq} = C_1 + C_2 \qquad \text{(parallel combination)}$$

$$C_{eq} = C_1 + C_2 + C_3 + \cdots \qquad \text{(parallel combination)}$$

Series Combination



$$\Delta V = \Delta V_1 + \Delta V_2$$

$$\Delta V = \frac{Q}{C_{eq}}$$

$$\Delta V_1 = \frac{Q}{C_1} \qquad \Delta V_2 = \frac{Q}{C_2}$$

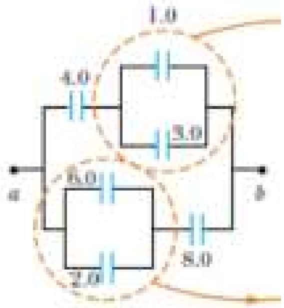
$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

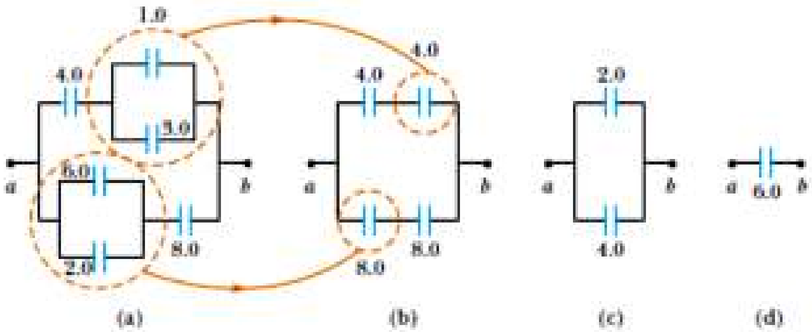
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{(series combination)}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \quad \text{(series combination)}$$

Example 2. Equivalent Capacitance (Example 26.4 Raymond)

Find the equivalent capacitance between *a* and *b* for the combination of capacitors shown in Figure. All capacitances are in microfarads.





Example 3.
(Sample Problem 25.03, Halliday)

Capacitor 1, with $C1$ = 3.55 μF, is charged to a potential difference $V0$ = 6.30 V, using a 6.30 V battery. The battery is then removed, and the capacitor is connected as in Fig. to an uncharged capacitor 2, with $C2$ 8.95 μF. When switch S is closed, charge flows between the capacitors. Find the charge on each capacitor when equilibrium is reached.

$$q_0 = C_1V_0 = (3.55 \times 10^{-6}\ \text{F})(6.30\ \text{V})$$
$$= 22.365 \times 10^{-6}\ \text{C}.$$

When switch S in Fig. 25-11 is closed and capacitor 1 begins to charge capacitor 2, the electric potential and charge on capacitor 1 decrease and those on capacitor 2 increase until

$$V_1 = V_2 \quad \text{(equilibrium).}$$

From Eq. 25-1, we can rewrite this as

$$\frac{q_1}{C_1} = \frac{q_2}{C_2} \quad \text{(equilibrium).}$$

Because the total charge cannot magically change, the total after the transfer must be

$$q_1 + q_2 = q_0 \quad \text{(charge conservation);}$$

thus

$$q_2 = q_0 - q_1.$$

We can now rewrite the second equilibrium equation as

$$\frac{q_1}{C_1} = \frac{q_0 - q_1}{C_2}.$$

Solving this for q_1 and substituting given data, we find

$$q_1 = 6.35\ \mu\text{C}. \qquad \text{(Answer)}$$

The rest of the initial charge ($q_0 = 22.365\ \mu$C) must be on capacitor 2:

$$q_2 = 16.0\ \mu\text{C}. \qquad \text{(Answer)}$$

Current and Resistance

Electric Current

The current is the rate at which charge flows through this surface.

The SI unit of current is the ampere (A):

$$I = \frac{dQ}{dt}$$

$$1\,\text{A} = \frac{1\,\text{C}}{1\,\text{s}}$$

Resistance

$$R = \frac{\Delta V}{I}$$

resistance has SI units of volts per ampere. One volt per ampere is defined to be one ohm (Ω)

$$1\ \Omega = \frac{1\ \text{V}}{1\ \text{A}}$$

Most electric circuits use circuit elements called **resistors** to control the current level in the various parts of the circuit.

Resistance

- for many materials (including most metals), the ratio of the current density to the electric field is a constant σ that is independent of the electric field producing the current.
- The resistance as the ratio of the potential difference across a conductor to the current in the conductor: $R \equiv \frac{\Delta V}{I}$
- The resistance has SI units of volts per ampere. One volt per ampere is defined to be one ohm (Ω). if a potential difference of 1 V across a conductor causes a current of 1 A, the resistance of the conductor is 1 Ω.
- The inverse of conductivity is resistivity[3] ρ: $\rho = \frac{l}{\sigma}$, where ρ has the units ohm-meters (Ω.m). Because $R = \rho \frac{l}{\sigma A}$ we can express the resistance of a uniform block of material along the length l as $R = \rho \frac{l}{A}$

Resistivities and Temperature Coefficients of Resistivity for Various Materials

Material	Resistivity[a] (Ω · m)	Temperature Coefficient[b] α [(°C)$^{-1}$]
Silver	1.59×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminum	2.82×10^{-8}	3.9×10^{-3}
Tungsten	5.6×10^{-8}	4.5×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Platinum	11×10^{-8}	3.92×10^{-3}
Lead	22×10^{-8}	3.9×10^{-3}
Nichrome[c]	1.50×10^{-6}	0.4×10^{-3}
Carbon	3.5×10^{-5}	-0.5×10^{-3}
Germanium	0.46	-48×10^{-3}
Silicon	640	-75×10^{-3}
Glass	10^{10} to 10^{14}	
Hard rubber	$\sim 10^{13}$	
Sulfur	10^{15}	
Quartz (fused)	75×10^{16}	

Example 4. The Resistance of a Conductor
(Example 27.2 Raymond)

Calculate the resistance of an aluminum cylinder that has a length of 10.0 cm and a cross-sectional area of 2.00 x 10^{-4} m^2. Repeat the calculation for a cylinder of the same dimensions and made of glass having a resistivity of 3.0 x 10^{10} Ωm.

Solution

$$R = \rho \frac{\ell}{A} = (2.82 \times 10^{-8}\ \Omega\cdot\text{m})\left(\frac{0.100\ \text{m}}{2.00 \times 10^{-4}\ \text{m}^2}\right)$$

$$= 1.41 \times 10^{-5}\ \Omega$$

$$R = \rho \frac{\ell}{A} = (3.0 \times 10^{10}\ \Omega\cdot\text{m})\left(\frac{0.100\ \text{m}}{2.00 \times 10^{-4}\ \text{m}^2}\right)$$

$$= 1.5 \times 10^{13}\ \Omega$$

Example 5 The Resistance of Nichrome Wire
(Example 27.3 Raymond)

a. Calculate the resistance per unit length of Nichrome wire, which has a radius of 0.321 mm.

b. If a potential difference of 10 V is maintained across a 1.0-m length of the Nichrome wire, what is the current in the wire?

The resistivity of Nichrome is 1.5 x 10^{-6} Ωm

$$A = \pi r^2 = \pi(0.321 \times 10^{-3}\ \text{m})^2 = 3.24 \times 10^{-7}\ \text{m}^2$$

$$\frac{R}{\ell} = \frac{\rho}{A} = \frac{1.5 \times 10^{-6}\ \Omega\cdot\text{m}}{3.24 \times 10^{-7}\ \text{m}^2} = 4.6\ \Omega/\text{m}$$

$$I = \frac{\Delta V}{R} = \frac{10\ \text{V}}{4.6\ \Omega} = 2.2\ \text{A}$$

Electrical Power

- The rate at which the system loses potential energy as the charge passes through the resistor is equal to the rate at which the system gains internal energy in the resistor. Thus, the power $\mathcal{P}$, representing the rate at which energy is delivered to the resistor, is :

$$\mathcal{P} = I\,\Delta V$$

- $\Delta V = I.R$ for a resistor, we can express the power delivered to the resistor in the alternative forms

$$\mathcal{P} = I^2 R = \frac{(\Delta V)^2}{R}$$

Example 6. Power in an Electric Heater
(Example 27.7 Raymond)

An electric heater is constructed by applying a potential difference of 120 V to a Nichrome wire that has a total resistance of 8.00 Ω. Find the current carried by the wire and the power rating of the heater.

Solution Because $\Delta V = IR$, we have

$$I = \frac{\Delta V}{R} = \frac{120\ \text{V}}{8.00\ \Omega} = 15.0\ \text{A}$$

We can find the power rating using the expression $\mathcal{P} = I^2 R$:

$$\mathcal{P} = I^2 R = (15.0\ \text{A})^2(8.00\ \Omega) = 1.80 \times 10^3\ \text{W}$$

$$\mathcal{P} = 1.80\ \text{kW}$$