# MOTION ALONG A STRAIGHT LINE

# 2

? A bungee jumper speeds up during the first part of his fall, then slows to a halt as the bungee cord stretches and becomes taut. Is it accurate to say that the jumper is *accelerating* as he slows during the final part of his fall?

What distance must an airliner travel down a runway before reaching takeoff speed? When you throw a baseball straight up in the air, how high does it go? When a glass slips from your hand, how much time do you have to catch it before it hits the floor? These are the kinds of questions you will learn to answer in this chapter. We are beginning our study of physics with *mechanics,* the study of the relationships among force, matter, and motion. In this chapter and the next we will study *kinematics,* the part of mechanics that enables us to describe motion. Later we will study *dynamics,* which relates motion to its causes.

In this chapter we concentrate on the simplest kind of motion: a body moving along a straight line. To describe this motion, we introduce the physical quantities *velocity* and *acceleration.* In physics these quantities have definitions that are more precise and slightly different from the ones used in everyday language. Both velocity and acceleration are *vectors:* As you learned in Chapter 1, this means that they have both magnitude and direction. Our concern in this chapter is with motion along a straight line only, so we won't need the full mathematics of vectors just yet. But using vectors will be essential in Chapter 3 when we consider motion in two or three dimensions.

We'll develop simple equations to describe straight-line motion in the important special case when the acceleration is constant. An example is the motion of a freely falling body. We'll also consider situations in which the acceleration varies during the motion; in this case, it's necessary to use integration to describe the motion. (If you haven't studied integration yet, Section 2.6 is optional.)

## LEARNING GOALS

*By studying this chapter, you will learn:*

- How to describe straight-line motion in terms of average velocity, instantaneous velocity, average acceleration, and instantaneous acceleration.
- How to interpret graphs of position versus time, velocity versus time, and acceleration versus time for straight-line motion.
- How to solve problems involving straight-line motion with constant acceleration, including free-fall problems.
- How to analyze straight-line motion when the acceleration is not constant.

## 2.1 Displacement, Time, and Average Velocity

Suppose a drag racer drives her AA-fuel dragster along a straight track (Fig. 2.1). To study the dragster's motion, we need a coordinate system. We choose the $x$-axis to lie along the dragster's straight-line path, with the origin $O$ at the starting line. We also choose a point on the dragster, such as its front end, and represent the entire dragster by that point. Hence we treat the dragster as a **particle.**

A useful way to describe the motion of the particle that represents the dragster is in terms of the change in the particle's coordinate $x$ over a time interval. Suppose that 1.0 s after the start the front of the dragster is at point $P_1$, 19 m from the origin, and 4.0 s after the start it is at point $P_2$, 277 m from the origin. The *displacement* of the particle is a vector that points from $P_1$ to $P_2$ (see Section 1.7). Figure 2.1 shows that this vector points along the $x$-axis. The $x$-component of the displacement is the change in the value of $x$, $(277\text{ m} - 19\text{ m}) = 258\text{ m}$, that took place during the time interval of $(4.0\text{ s} - 1.0\text{ s}) = 3.0\text{ s}$. We define the dragster's **average velocity** during this time interval as a *vector* quantity whose $x$-component is the change in $x$ divided by the time interval: $(258\text{ m})/(3.0\text{ s}) = 86\text{ m/s}$.

In general, the average velocity depends on the particular time interval chosen. For a 3.0-s time interval *before* the start of the race, the average velocity would be zero because the dragster would be at rest at the starting line and would have zero displacement.

Let's generalize the concept of average velocity. At time $t_1$ the dragster is at point $P_1$, with coordinate $x_1$, and at time $t_2$ it is at point $P_2$, with coordinate $x_2$. The displacement of the dragster during the time interval from $t_1$ to $t_2$ is the vector from $P_1$ to $P_2$. The $x$-component of the displacement, denoted $\Delta x$, is the change in the coordinate $x$:

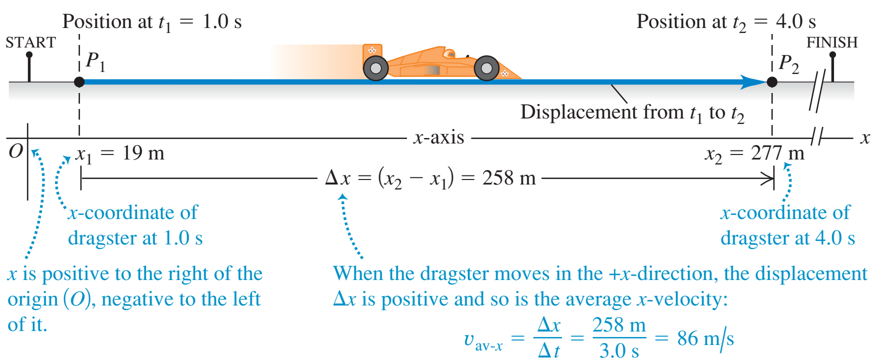
$$\Delta x = x_2 - x_1 \tag{2.1}$$

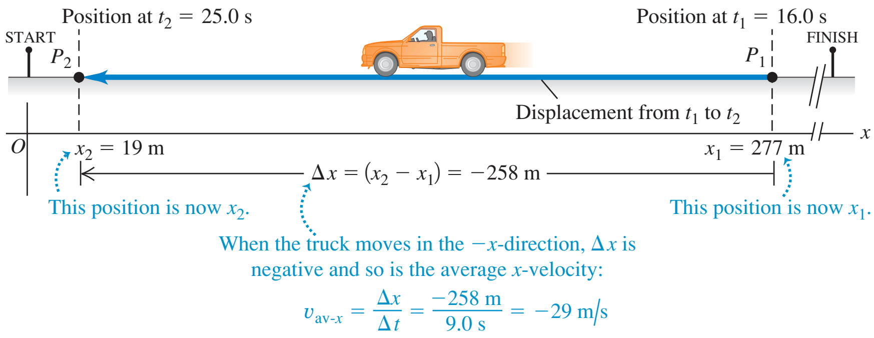
The dragster moves along the $x$-axis only, so the $y$- and $z$-components of the displacement are equal to zero.

**CAUTION** **The meaning of $\Delta x$** Note that $\Delta x$ is *not* the product of $\Delta$ and $x$; it is a single symbol that means "the change in the quantity $x$." We always use the Greek capital letter $\Delta$ (delta) to represent a *change* in a quantity, equal to the *final* value of the quantity minus the *initial* value—never the reverse. Likewise, the time interval from $t_1$ to $t_2$ is $\Delta t$, the change in the quantity $t$: $\Delta t = t_2 - t_1$ (final time minus initial time).

The $x$-component of average velocity, or **average $x$-velocity,** is the $x$-component of displacement, $\Delta x$, divided by the time interval $\Delta t$ during which

**2.1** Positions of a dragster at two times during its run.






**2.2** Positions of an official's truck at two times during its motion. The points $P_1$ and $P_2$ now indicate the positions of the truck, and so are the reverse of Fig. 2.1.

the displacement occurs. We use the symbol $v_{av\text{-}x}$ for average $x$-velocity (the subscript "av" signifies average value and the subscript $x$ indicates that this is the $x$-component):

$$v_{av\text{-}x} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} \quad \text{(average } x\text{-velocity, straight-line motion)} \tag{2.2}$$

As an example, for the dragster $x_1 = 19$ m, $x_2 = 277$ m, $t_1 = 1.0$ s, and $t_2 = 4.0$ s, so Eq. (2.2) gives

$$v_{av\text{-}x} = \frac{277\text{ m} - 19\text{ m}}{4.0\text{ s} - 1.0\text{ s}} = \frac{258\text{ m}}{3.0\text{ s}} = 86\text{ m/s}$$

The average $x$-velocity of the dragster is positive. This means that during the time interval, the coordinate $x$ increased and the dragster moved in the positive $x$-direction (to the right in Fig. 2.1).

If a particle moves in the *negative* $x$-direction during a time interval, its average velocity for that time interval is negative. For example, suppose an official's truck moves to the left along the track (Fig. 2.2). The truck is at $x_1 = 277$ m at $t_1 = 16.0$ s and is at $x_2 = 19$ m at $t_2 = 25.0$ s. Then $\Delta x = (19\text{ m} - 277\text{ m}) = -258$ m and $\Delta t = (25.0\text{ s} - 16.0\text{ s}) = 9.0$ s. The $x$-component of average velocity is $v_{av\text{-}x} = \Delta x/\Delta t = (-258\text{ m})/(9.0\text{ s}) = -29\text{ m/s}$. Table 2.1 lists some simple rules for deciding whether the $x$-velocity is positive or negative.

**CAUTION** **Choice of the positive $x$-direction** You might be tempted to conclude that positive average $x$-velocity must mean motion to the right, as in Fig. 2.1, and that negative average $x$-velocity must mean motion to the left, as in Fig. 2.2. But that's correct *only* if the positive $x$-direction is to the right, as we chose it to be in Figs. 2.1 and 2.2. Had we chosen the positive $x$-direction to be to the left, with the origin at the finish line, the dragster would have negative average $x$-velocity and the official's truck would have positive average $x$-velocity. In most problems the direction of the coordinate axis will be yours to choose. Once you've made your choice, you *must* take it into account when interpreting the signs of $v_{av\text{-}x}$ and other quantities that describe motion!

**Table 2.1 Rules for the Sign of $x$-Velocity**

| If the $x$-coordinate is: | . . . the $x$-velocity is: |
|---|---|
| Positive & increasing (getting more positive) | Positive: Particle is moving in $+x$-direction |
| Positive & decreasing (getting less positive) | Negative: Particle is moving in $-x$-direction |
| Negative & increasing (getting less negative) | Positive: Particle is moving in $+x$-direction |
| Negative & decreasing (getting more negative) | Negative: Particle is moving in $-x$-direction |

*Note:* These rules apply to both the average $x$-velocity $v_{av\text{-}x}$ and the instantaneous $x$-velocity $v_x$ (to be discussed in Section 2.2).

With straight-line motion we sometimes call $\Delta x$ simply the displacement and $v_{av\text{-}x}$ simply the average velocity. But be sure to remember that these are really the $x$-components of vector quantities that, in this special case, have *only* $x$-components. In Chapter 3, displacement, velocity, and acceleration vectors will have two or three nonzero components.

Figure 2.3 is a graph of the dragster's position as a function of time—that is, an ***x-t* graph.** The curve in the figure *does not* represent the dragster's path in space; as Fig. 2.1 shows, the path is a straight line. Rather, the graph is a pictorial way to represent how the dragster's position changes with time. The points $p_1$ and $p_2$ on the graph correspond to the points $P_1$ and $P_2$ along the dragster's path. Line $p_1p_2$ is the hypotenuse of a right triangle with vertical side $\Delta x = x_2 - x_1$

**2.3** The position of a dragster as a function of time.

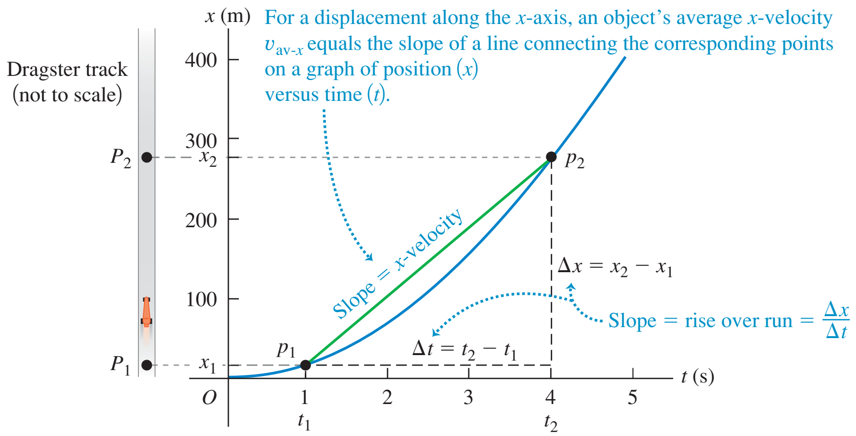



**Table 2.2 Typical Velocity Magnitudes**

| | |
|---|---|
| A snail's pace | $10^{-3}$ m/s |
| A brisk walk | 2 m/s |
| Fastest human | 11 m/s |
| Freeway speeds | 30 m/s |
| Fastest car | 341 m/s |
| Random motion of air molecules | 500 m/s |
| Fastest airplane | 1000 m/s |
| Orbiting communications satellite | 3000 m/s |
| Electron orbiting in a hydrogen atom | $2 \times 10^6$ m/s |
| Light traveling in a vacuum | $3 \times 10^8$ m/s |

and horizontal side $\Delta t = t_2 - t_1$. The average x-velocity $v_{av\text{-}x} = \Delta x/\Delta t$ of the dragster equals the *slope* of the line $p_1p_2$—that is, the ratio of the triangle's vertical side $\Delta x$ to its horizontal side $\Delta t$.

The average x-velocity depends only on the total displacement $\Delta x = x_2 - x_1$ that occurs during the time interval $\Delta t = t_2 - t_1$, not on the details of what happens during the time interval. At time $t_1$ a motorcycle might have raced past the dragster at point $P_1$ in Fig. 2.1, then blown its engine and slowed down to pass through point $P_2$ at the same time $t_2$ as the dragster. Both vehicles have the same displacement during the same time interval and so have the same average x-velocity.

If distance is given in meters and time in seconds, average velocity is measured in meters per second (m/s). Other common units of velocity are kilometers per hour (km/h), feet per second (ft/s), miles per hour (mi/h), and knots (1 knot = 1 nautical mile/h = 6080 ft/h). Table 2.2 lists some typical velocity magnitudes.

**Test Your Understanding of Section 2.1** Each of the following automobile trips takes one hour. The positive x-direction is to the east. (i) Automobile *A* travels 50 km due east. (ii) Automobile *B* travels 50 km due west. (iii) Automobile *C* travels 60 km due east, then turns around and travels 10 km due west. (iv) Automobile *D* travels 70 km due east. (v) Automobile *E* travels 20 km due west, then turns around and travels 20 km due east. (a) Rank the five trips in order of average x-velocity from most positive to most negative. (b) Which trips, if any, have the same average x-velocity? (c) For which trip, if any, is the average x-velocity equal to zero?

**2.4** The winner of a 50-m swimming race is the swimmer whose average velocity has the greatest magnitude—that is, the swimmer who traverses a displacement $\Delta x$ of 50 m in the shortest elapsed time $\Delta t$.

## 2.2 Instantaneous Velocity

Sometimes the average velocity is all you need to know about a particle's motion. For example, a race along a straight line is really a competition to see whose average velocity, $v_{av\text{-}x}$, has the greatest magnitude. The prize goes to the competitor who can travel the displacement $\Delta x$ from the start to the finish line in the shortest time interval, $\Delta t$ (Fig. 2.4).

But the average velocity of a particle during a time interval can't tell us how fast, or in what direction, the particle was moving at any given time during the interval. To do this we need to know the **instantaneous velocity,** or the velocity at a specific instant of time or specific point along the path.

**CAUTION** **How long is an instant?** Note that the word "instant" has a somewhat different definition in physics than in everyday language. You might use the phrase "It lasted just an instant" to refer to something that lasted for a very short time interval. But in physics an instant has no duration at all; it refers to a single value of time.

To find the instantaneous velocity of the dragster in Fig. 2.1 at the point $P_1$, we move the second point $P_2$ closer and closer to the first point $P_1$ and compute the average velocity $v_{\text{av-}x} = \Delta x/\Delta t$ over the ever-shorter displacement and time interval. Both $\Delta x$ and $\Delta t$ become very small, but their ratio does not necessarily become small. In the language of calculus, the limit of $\Delta x/\Delta t$ as $\Delta t$ approaches zero is called the **derivative** of $x$ with respect to $t$ and is written $dx/dt$. *The instantaneous velocity is the limit of the average velocity as the time interval approaches zero; it equals the instantaneous rate of change of position with time.* We use the symbol $v_x$, with no "av" subscript, for the instantaneous velocity along the $x$-axis, or the **instantaneous *x*-velocity:**

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \quad \text{(instantaneous } x\text{-velocity, straight-line motion)} \qquad (2.3)$$

The time interval $\Delta t$ is always positive, so $v_x$ has the same algebraic sign as $\Delta x$. A positive value of $v_x$ means that $x$ is increasing and the motion is in the positive $x$-direction; a negative value of $v_x$ means that $x$ is decreasing and the motion is in the negative $x$-direction. A body can have positive $x$ and negative $v_x$, or the reverse; $x$ tells us where the body is, while $v_x$ tells us how it's moving (Fig. 2.5). The rules that we presented in Table 2.1 (Section 2.1) for the sign of average $x$-velocity $v_{\text{av-}x}$ also apply to the sign of instantaneous $x$-velocity $v_x$.

Instantaneous velocity, like average velocity, is a vector quantity; Eq. (2.3) defines its $x$-component. In straight-line motion, all other components of instantaneous velocity are zero. In this case we often call $v_x$ simply the instantaneous velocity. (In Chapter 3 we'll deal with the general case in which the instantaneous velocity can have nonzero $x$-, $y$-, and $z$-components.) When we use the term "velocity," we will always mean instantaneous rather than average velocity.

The terms "velocity" and "speed" are used interchangeably in everyday language, but they have distinct definitions in physics. We use the term **speed** to denote distance traveled divided by time, on either an average or an instantaneous basis. Instantaneous *speed,* for which we use the symbol $v$ with *no* subscripts, measures how fast a particle is moving; instantaneous *velocity* measures how fast *and* in what direction it's moving. Instantaneous speed is the magnitude of instantaneous velocity and so can never be negative. For example, a particle with instantaneous velocity $v_x = 25\ \text{m/s}$ and a second particle with $v_x = -25\ \text{m/s}$ are moving in opposite directions at the same instantaneous speed 25 m/s.

**2.5** Even when he's moving forward, this cyclist's instantaneous $x$-velocity can be negative—if he's traveling in the negative $x$-direction. In any problem, the choice of which direction is positive and which is negative is entirely up to you.

**CAUTION** **Average speed and average velocity** Average speed is *not* the magnitude of average velocity. When César Cielo set a world record in 2009 by swimming 100.0 m in 46.91 s, his average speed was $(100.0\ \text{m})/(46.91\ \text{s}) = 2.132\ \text{m/s}$. But because he swam two lengths in a 50-m pool, he started and ended at the same point and so had zero total displacement and zero average *velocity!* Both average speed and instantaneous speed are scalars, not vectors, because these quantities contain no information about direction.

## Example 2.1 Average and instantaneous velocities

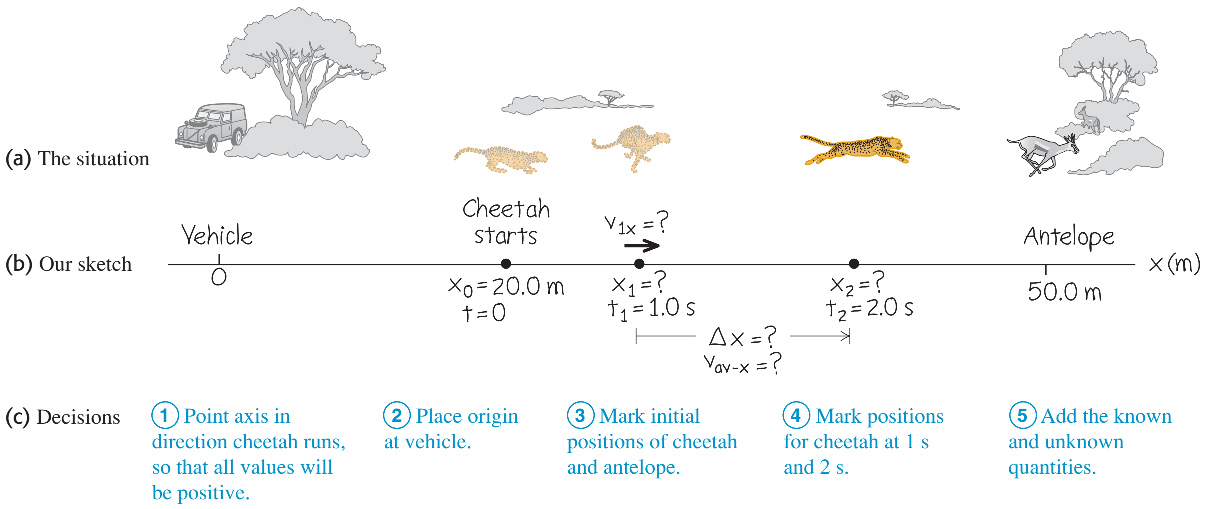
A cheetah is crouched 20 m to the east of an observer (Fig. 2.6a). At time $t = 0$ the cheetah begins to run due east toward an antelope that is 50 m to the east of the observer. During the first 2.0 s of the attack, the cheetah's coordinate $x$ varies with time according to the equation $x = 20\ \text{m} + (5.0\ \text{m/s}^2)t^2$. (a) Find the cheetah's displacement between $t_1 = 1.0\ \text{s}$ and $t_2 = 2.0\ \text{s}$. (b) Find its average velocity during that interval. (c) Find its instantaneous velocity at $t_1 = 1.0\ \text{s}$ by taking $\Delta t = 0.1\ \text{s}$, then 0.01 s, then 0.001 s. (d) Derive an expression for the cheetah's instantaneous velocity as a function of time, and use it to find $v_x$ at $t = 1.0\ \text{s}$ and $t = 2.0\ \text{s}$.

**SOLUTION**

**IDENTIFY and SET UP:** Figure 2.6b shows our sketch of the cheetah's motion. We use Eq. (2.1) for displacement, Eq. (2.2) for average velocity, and Eq. (2.3) for instantaneous velocity.

*Continued*

**2.6** A cheetah attacking an antelope from ambush. The animals are not drawn to the same scale as the axis.




**EXECUTE:** (a) At $t_1 = 1.0$ s and $t_2 = 2.0$ s the cheetah's positions $x_1$ and $x_2$ are

$$x_1 = 20 \text{ m} + (5.0 \text{ m/s}^2)(1.0 \text{ s})^2 = 25 \text{ m}$$
$$x_2 = 20 \text{ m} + (5.0 \text{ m/s}^2)(2.0 \text{ s})^2 = 40 \text{ m}$$

The displacement during this 1.0-s interval is

$$\Delta x = x_2 - x_1 = 40 \text{ m} - 25 \text{ m} = 15 \text{ m}$$

(b) The average $x$-velocity during this interval is

$$v_{av\text{-}x} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{40 \text{ m} - 25 \text{ m}}{2.0 \text{ s} - 1.0 \text{ s}} = \frac{15 \text{ m}}{1.0 \text{ s}} = 15 \text{ m/s}$$

(c) With $\Delta t = 0.1$ s the time interval is from $t_1 = 1.0$ s to a new $t_2 = 1.1$ s. At $t_2$ the position is

$$x_2 = 20 \text{ m} + (5.0 \text{ m/s}^2)(1.1 \text{ s})^2 = 26.05 \text{ m}$$

The average $x$-velocity during this 0.1-s interval is

$$v_{av\text{-}x} = \frac{26.05 \text{ m} - 25 \text{ m}}{1.1 \text{ s} - 1.0 \text{ s}} = 10.5 \text{ m/s}$$

Following this pattern, you can calculate the average $x$-velocities for 0.01-s and 0.001-s intervals: The results are 10.05 m/s and 10.005 m/s. As $\Delta t$ gets smaller, the average $x$-velocity gets closer to 10.0 m/s, so we conclude that the instantaneous $x$-velocity at $t = 1.0$ s is 10.0 m/s. (We suspended the rules for significant-figure counting in these calculations.)

(d) To find the instantaneous $x$-velocity as a function of time, we take the derivative of the expression for $x$ with respect to $t$. The derivative of a constant is zero, and for any $n$ the derivative of $t^n$ is $nt^{n-1}$, so the derivative of $t^2$ is $2t$. We therefore have

$$v_x = \frac{dx}{dt} = (5.0 \text{ m/s}^2)(2t) = (10 \text{ m/s}^2)t$$

At $t = 1.0$ s, this yields $v_x = 10$ m/s, as we found in part (c); at $t = 2.0$ s, $v_x = 20$ m/s.

**EVALUATE:** Our results show that the cheetah picked up speed from $t = 0$ (when it was at rest) to $t = 1.0$ s ($v_x = 10$ m/s) to $t = 2.0$ s ($v_x = 20$ m/s). This makes sense; the cheetah covered only 5 m during the interval $t = 0$ to $t = 1.0$ s, but it covered 15 m during the interval $t = 1.0$ s to $t = 2.0$ s.

MasteringPHYSICS

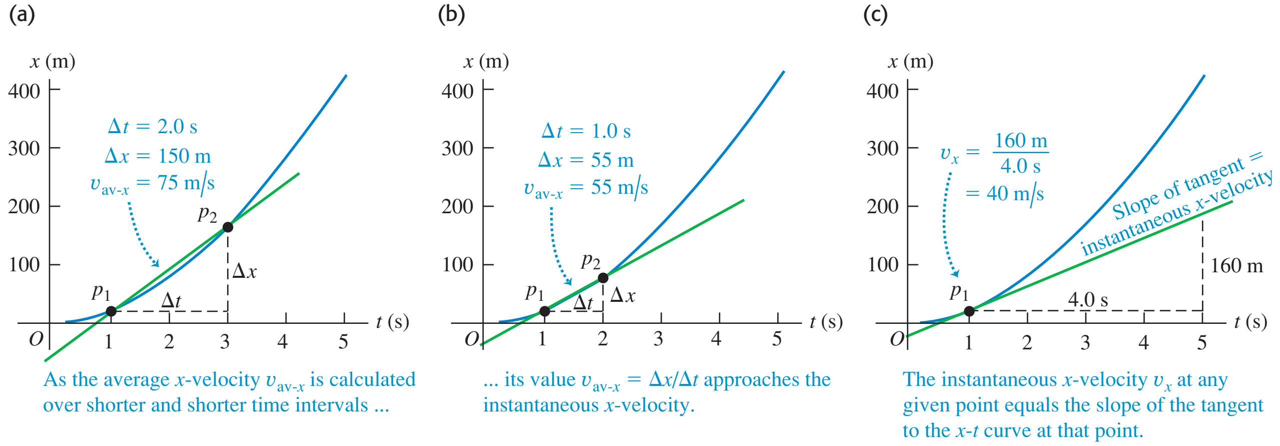
**ActivPhysics 1.1:** Analyzing Motion Using Diagrams

## Finding Velocity on an $x$-$t$ Graph

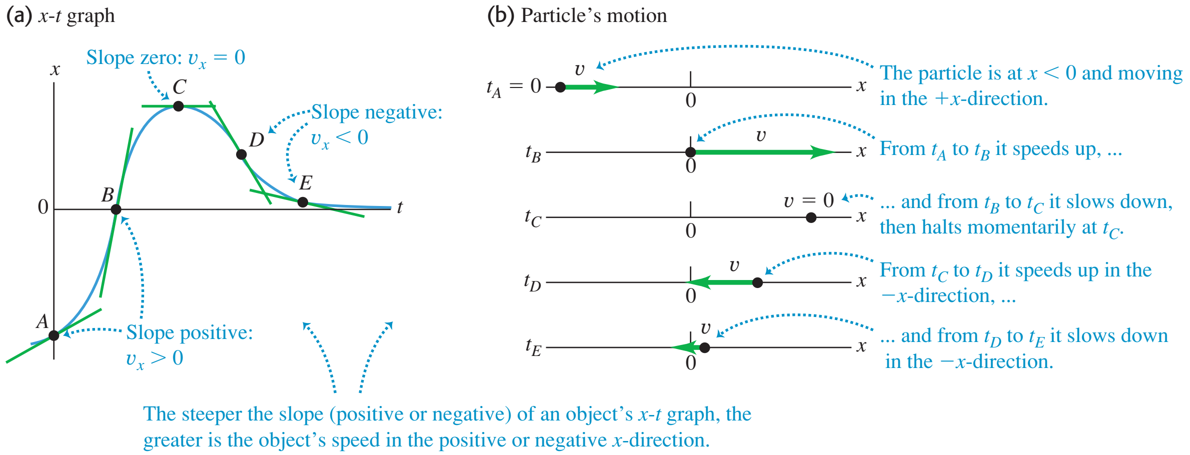
We can also find the $x$-velocity of a particle from the graph of its position as a function of time. Suppose we want to find the $x$-velocity of the dragster in Fig. 2.1 at point $P_1$. As point $P_2$ in Fig. 2.1 approaches point $P_1$, point $p_2$ in the $x$-$t$ graphs of Figs. 2.7a and 2.7b approaches point $p_1$ and the average $x$-velocity is calculated over shorter time intervals $\Delta t$. In the limit that $\Delta t \rightarrow 0$, shown in Fig. 2.7c, the slope of the line $p_1p_2$ equals the slope of the line tangent to the curve at point $p_1$. Thus, *on a graph of position as a function of time for straight-line motion, the instantaneous x-velocity at any point is equal to the slope of the tangent to the curve at that point.*

If the tangent to the $x$-$t$ curve slopes upward to the right, as in Fig. 2.7c, then its slope is positive, the $x$-velocity is positive, and the motion is in the positive $x$-direction. If the tangent slopes downward to the right, the slope of the $x$-$t$ graph

**2.7** Using an $x$-$t$ graph to go from (a), (b) average $x$-velocity to (c) instantaneous $x$-velocity $v_x$. In (c) we find the slope of the tangent to the $x$-$t$ curve by dividing any vertical interval (with distance units) along the tangent by the corresponding horizontal interval (with time units).



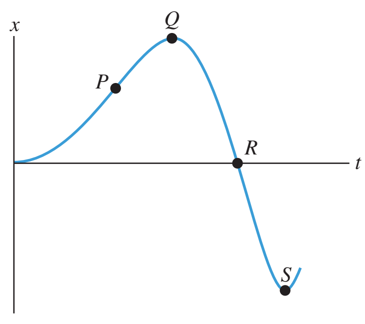

**2.8** (a) The $x$-$t$ graph of the motion of a particular particle. The slope of the tangent at any point equals the velocity at that point. (b) A motion diagram showing the position and velocity of the particle at each of the times labeled on the $x$-$t$ graph.




and the $x$-velocity are negative, and the motion is in the negative $x$-direction. When the tangent is horizontal, the slope and the $x$-velocity are zero. Figure 2.8 illustrates these three possibilities.

Figure 2.8 actually depicts the motion of a particle in two ways: as (a) an $x$-$t$ graph and (b) a **motion diagram** that shows the particle's position at various instants (like frames from a video of the particle's motion) as well as arrows to represent the particle's velocity at each instant. We will use both $x$-$t$ graphs and motion diagrams in this chapter to help you understand motion. You will find it worth your while to draw *both* an $x$-$t$ graph and a motion diagram as part of solving any problem involving motion.

**2.9** An $x$-$t$ graph for a particle.




**Test Your Understanding of Section 2.2** Figure 2.9 is an $x$-$t$ graph of the motion of a particle. (a) Rank the values of the particle's $x$-velocity $v_x$ at the points $P$, $Q$, $R$, and $S$ from most positive to most negative. (b) At which points is $v_x$ positive? (c) At which points is $v_x$ negative? (d) At which points is $v_x$ zero? (e) Rank the values of the particle's *speed* at the points $P$, $Q$, $R$, and $S$ from fastest to slowest.

## 2.3 Average and Instantaneous Acceleration

Just as velocity describes the rate of change of position with time, *acceleration* describes the rate of change of velocity with time. Like velocity, acceleration is a vector quantity. When the motion is along a straight line, its only nonzero component is along that line. As we'll see, acceleration in straight-line motion can refer to either speeding up or slowing down.

### Average Acceleration

Let's consider again a particle moving along the $x$-axis. Suppose that at time $t_1$ the particle is at point $P_1$ and has $x$-component of (instantaneous) velocity $v_{1x}$, and at a later time $t_2$ it is at point $P_2$ and has $x$-component of velocity $v_{2x}$. So the $x$-component of velocity changes by an amount $\Delta v_x = v_{2x} - v_{1x}$ during the time interval $\Delta t = t_2 - t_1$.

We define the **average acceleration** of the particle as it moves from $P_1$ to $P_2$ to be a vector quantity whose $x$-component $a_{\text{av-}x}$ (called the **average *x*-acceleration**) equals $\Delta v_x$, the change in the $x$-component of velocity, divided by the time interval $\Delta t$:

$$a_{\text{av-}x} = \frac{v_{2x} - v_{1x}}{t_2 - t_1} = \frac{\Delta v_x}{\Delta t} \quad \text{(average } x\text{-acceleration, straight-line motion)} \qquad (2.4)$$

For straight-line motion along the $x$-axis we will often call $a_{\text{av-}x}$ simply the average acceleration. (We'll encounter the other components of the average acceleration vector in Chapter 3.)

If we express velocity in meters per second and time in seconds, then average acceleration is in meters per second per second, or $(\text{m/s})/\text{s}$. This is usually written as $\text{m/s}^2$ and is read "meters per second squared."

**CAUTION** **Acceleration vs. velocity** Be very careful not to confuse acceleration with velocity! Velocity describes how a body's position changes with time; it tells us how fast and in what direction the body moves. Acceleration describes how the velocity changes with time; it tells us how the speed and direction of motion are changing. It may help to remember the phrase "acceleration is to velocity as velocity is to position." It can also help to imagine yourself riding along with the moving body. If the body accelerates forward and gains speed, you feel pushed backward in your seat; if it accelerates backward and loses speed, you feel pushed forward. If the velocity is constant and there's no acceleration, you feel neither sensation. (We'll see the reason for these sensations in Chapter 4.)

### Example 2.2 Average acceleration

An astronaut has left an orbiting spacecraft to test a new personal maneuvering unit. As she moves along a straight line, her partner on the spacecraft measures her velocity every 2.0 s, starting at time $t = 1.0$ s:

| $t$ | $v_x$ | $t$ | $v_x$ |
|---|---|---|---|
| 1.0 s | 0.8 m/s | 9.0 s | −0.4 m/s |
| 3.0 s | 1.2 m/s | 11.0 s | −1.0 m/s |
| 5.0 s | 1.6 m/s | 13.0 s | −1.6 m/s |
| 7.0 s | 1.2 m/s | 15.0 s | −0.8 m/s |

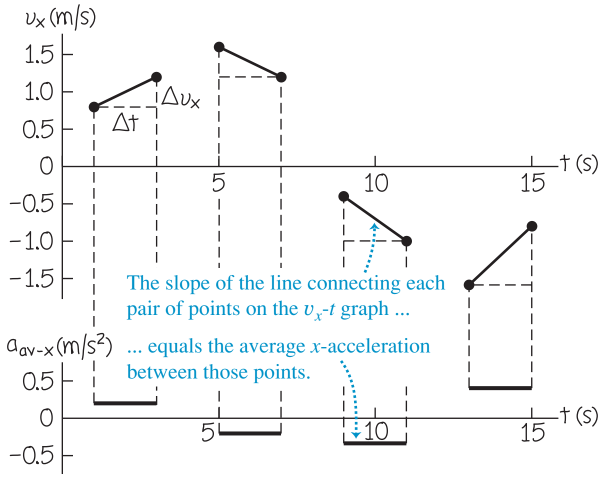
Find the average $x$-acceleration, and state whether the speed of the astronaut increases or decreases over each of these 2.0-s time intervals: (a) $t_1 = 1.0$ s to $t_2 = 3.0$ s; (b) $t_1 = 5.0$ s to $t_2 = 7.0$ s; (c) $t_1 = 9.0$ s to $t_2 = 11.0$ s; (d) $t_1 = 13.0$ s to $t_2 = 15.0$ s.

**SOLUTION**

**IDENTIFY and SET UP:** We'll use Eq. (2.4) to determine the average acceleration $a_{\text{av-}x}$ from the change in velocity over each time interval. To find the changes in speed, we'll use the idea that speed $v$ is the magnitude of the instantaneous velocity $v_x$.

The upper part of Fig. 2.10 is our graph of the $x$-velocity as a function of time. On this $v_x$-$t$ graph, the slope of the line connecting the endpoints of each interval is the average $x$-acceleration $a_{av\text{-}x} = \Delta v_x / \Delta t$ for that interval. The four slopes (and thus the *signs* of the average accelerations) are, respectively, positive, negative, negative, and positive. The third and fourth slopes (and thus the average accelerations themselves) have greater magnitude than the first and second.

**2.10** Our graphs of $x$-velocity versus time (top) and average $x$-acceleration versus time (bottom) for the astronaut.




**EXECUTE:** Using Eq. (2.4), we find:

(a) $a_{av\text{-}x} = (1.2\ \text{m/s} - 0.8\ \text{m/s})/(3.0\ \text{s} - 1.0\ \text{s}) = 0.2\ \text{m/s}^2$. The speed (magnitude of instantaneous $x$-velocity) increases from 0.8 m/s to 1.2 m/s.

(b) $a_{av\text{-}x} = (1.2\ \text{m/s} - 1.6\ \text{m/s})/(7.0\ \text{s} - 5.0\ \text{s}) = -0.2\ \text{m/s}^2$. The speed decreases from 1.6 m/s to 1.2 m/s.

(c) $a_{av\text{-}x} = [-1.0\ \text{m/s} - (-0.4\ \text{m/s})]/(11.0\ \text{s} - 9.0\ \text{s}) = -0.3\ \text{m/s}^2$. The speed increases from 0.4 m/s to 1.0 m/s.

(d) $a_{av\text{-}x} = [-0.8\ \text{m/s} - (-1.6\ \text{m/s})]/(15.0\ \text{s} - 13.0\ \text{s}) = 0.4\ \text{m/s}^2$. The speed decreases from 1.6 m/s to 0.8 m/s.

In the lower part of Fig. 2.10, we graph the values of $a_{av\text{-}x}$.

**EVALUATE:** The signs and relative magnitudes of the average accelerations agree with our qualitative predictions. For future reference, note this connection among speed, velocity, and acceleration: Our results show that when the average $x$-acceleration has the *same* direction (same algebraic sign) as the initial velocity, as in intervals (a) and (c), the astronaut goes faster. When $a_{av\text{-}x}$ has the *opposite* direction (opposite algebraic sign) from the initial velocity, as in intervals (b) and (d), she slows down. Thus positive $x$-acceleration means speeding up if the $x$-velocity is positive [interval (a)] but slowing down if the $x$-velocity is negative [interval (d)]. Similarly, negative $x$-acceleration means speeding up if the $x$-velocity is negative [interval (c)] but slowing down if the $x$-velocity is positive [interval (b)].

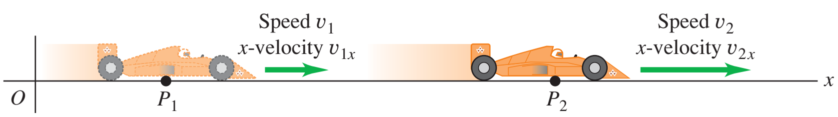
## Instantaneous Acceleration

We can now define **instantaneous acceleration** following the same procedure that we used to define instantaneous velocity. As an example, suppose a race car driver is driving along a straightaway as shown in Fig. 2.11. To define the instantaneous acceleration at point $P_1$, we take the second point $P_2$ in Fig. 2.11 to be closer and closer to $P_1$ so that the average acceleration is computed over shorter and shorter time intervals. *The instantaneous acceleration is the limit of the average acceleration as the time interval approaches zero.* In the language of calculus, *instantaneous acceleration equals the derivative of velocity with time.* Thus

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} \quad \text{(instantaneous } x\text{-acceleration, straight-line motion)} \tag{2.5}$$

Note that $a_x$ in Eq. (2.5) is really the $x$-component of the acceleration vector, or the **instantaneous $x$-acceleration;** in straight-line motion, all other components of this vector are zero. From now on, when we use the term "acceleration," we will always mean instantaneous acceleration, not average acceleration.

**2.11** A Grand Prix car at two points on the straightaway.



## Example 2.3 Average and instantaneous accelerations

Suppose the *x*-velocity $v_x$ of the car in Fig. 2.11 at any time $t$ is given by the equation

$$v_x = 60 \text{ m/s} + (0.50 \text{ m/s}^3)t^2$$

(a) Find the change in *x*-velocity of the car in the time interval $t_1 = 1.0$ s to $t_2 = 3.0$ s. (b) Find the average *x*-acceleration in this time interval. (c) Find the instantaneous *x*-acceleration at time $t_1 = 1.0$ s by taking $\Delta t$ to be first 0.1 s, then 0.01 s, then 0.001 s. (d) Derive an expression for the instantaneous *x*-acceleration as a function of time, and use it to find $a_x$ at $t = 1.0$ s and $t = 3.0$ s.

**SOLUTION**

**IDENTIFY and SET UP:** This example is analogous to Example 2.1 in Section 2.2. (Now is a good time to review that example.) In Example 2.1 we found the average *x*-velocity from the change in position over shorter and shorter time intervals, and we obtained an expression for the instantaneous *x*-velocity by differentiating the position as a function of time. In this example we have an exact parallel. Using Eq. (2.4), we'll find the average *x-acceleration* from the change in *x-velocity* over a time interval. Likewise, using Eq. (2.5), we'll obtain an expression for the instantaneous *x-acceleration* by differentiating the *x-velocity* as a function of time.

**EXECUTE:** (a) Before we can apply Eq. (2.4), we must find the *x*-velocity at each time from the given equation. At $t_1 = 1.0$ s and $t_2 = 3.0$ s, the velocities are

$$v_{1x} = 60 \text{ m/s} + (0.50 \text{ m/s}^3)(1.0 \text{ s})^2 = 60.5 \text{ m/s}$$
$$v_{2x} = 60 \text{ m/s} + (0.50 \text{ m/s}^3)(3.0 \text{ s})^2 = 64.5 \text{ m/s}$$

The change in *x*-velocity $\Delta v_x$ between $t_1 = 1.0$ s and $t_2 = 3.0$ s is

$$\Delta v_x = v_{2x} - v_{1x} = 64.5 \text{ m/s} - 60.5 \text{ m/s} = 4.0 \text{ m/s}$$

(b) The average *x*-acceleration during this time interval of duration $t_2 - t_1 = 2.0$ s is

$$a_{\text{av-}x} = \frac{v_{2x} - v_{1x}}{t_2 - t_1} = \frac{4.0 \text{ m/s}}{2.0 \text{ s}} = 2.0 \text{ m/s}^2$$

During this time interval the *x*-velocity and average *x*-acceleration have the same algebraic sign (in this case, positive), and the car speeds up.

(c) When $\Delta t = 0.1$ s, we have $t_2 = 1.1$ s. Proceeding as before, we find

$$v_{2x} = 60 \text{ m/s} + (0.50 \text{ m/s}^3)(1.1 \text{ s})^2 = 60.605 \text{ m/s}$$
$$\Delta v_x = 0.105 \text{ m/s}$$
$$a_{\text{av-}x} = \frac{\Delta v_x}{\Delta t} = \frac{0.105 \text{ m/s}}{0.1 \text{ s}} = 1.05 \text{ m/s}^2$$

You should follow this pattern to calculate $a_{\text{av-}x}$ for $\Delta t = 0.01$ s and $\Delta t = 0.001$ s; the results are $a_{\text{av-}x} = 1.005 \text{ m/s}^2$ and $a_{\text{av-}x} = 1.0005 \text{ m/s}^2$, respectively. As $\Delta t$ gets smaller, the average *x*-acceleration gets closer to $1.0 \text{ m/s}^2$, so the instantaneous *x*-acceleration at $t = 1.0$ s is $1.0 \text{ m/s}^2$.

(d) By Eq. (2.5) the instantaneous *x*-acceleration is $a_x = dv_x/dt$. The derivative of a constant is zero and the derivative of $t^2$ is $2t$, so

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}[60 \text{ m/s} + (0.50 \text{ m/s}^3)t^2]$$
$$= (0.50 \text{ m/s}^3)(2t) = (1.0 \text{ m/s}^3)t$$

When $t = 1.0$ s,

$$a_x = (1.0 \text{ m/s}^3)(1.0 \text{ s}) = 1.0 \text{ m/s}^2$$

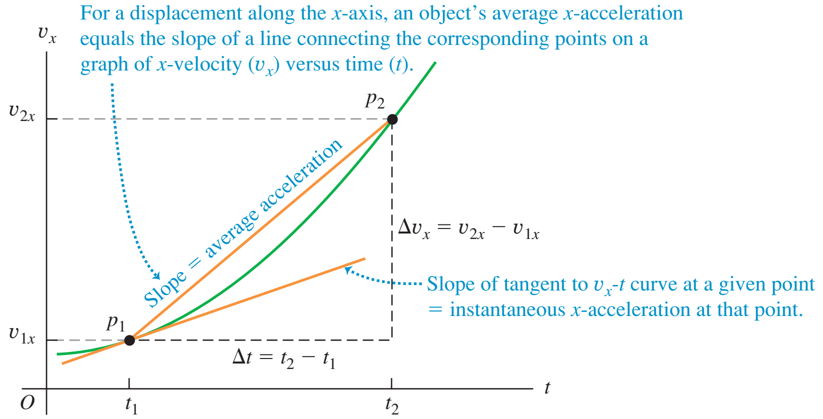
When $t = 3.0$ s,

$$a_x = (1.0 \text{ m/s}^3)(3.0 \text{ s}) = 3.0 \text{ m/s}^2$$

**EVALUATE:** Neither of the values we found in part (d) is equal to the average *x*-acceleration found in part (b). That's because the car's instantaneous *x*-acceleration varies with time. The rate of change of acceleration with time is sometimes called the "jerk."

## Finding Acceleration on a $v_x$-$t$ Graph or an $x$-$t$ Graph

In Section 2.2 we interpreted average and instantaneous *x*-velocity in terms of the slope of a graph of position versus time. In the same way, we can interpret average and instantaneous *x*-acceleration by using a graph with instantaneous velocity $v_x$ on the vertical axis and time $t$ on the horizontal axis—that is, a **$v_x$-$t$ graph** (Fig. 2.12). The points on the graph labeled $p_1$ and $p_2$ correspond to points $P_1$ and $P_2$ in Fig. 2.11. The average *x*-acceleration $a_{\text{av-}x} = \Delta v_x/\Delta t$ during this interval is the slope of the line $p_1p_2$. As point $P_2$ in Fig. 2.11 approaches point $P_1$, point $p_2$ in the $v_x$-$t$ graph of Fig. 2.12 approaches point $p_1$, and the slope of the line $p_1p_2$ approaches the slope of the line tangent to the curve at point $p_1$. Thus, *on a graph of x-velocity as a function of time, the instantaneous x-acceleration at any point is equal to the slope of the tangent to the curve at that point.* Tangents drawn at different points along the curve in Fig. 2.12 have different slopes, so the instantaneous *x*-acceleration varies with time.




**2.12** A $v_x$-$t$ graph of the motion in Fig. 2.11.

**CAUTION The signs of $x$-acceleration and $x$-velocity** By itself, the algebraic sign of the $x$-acceleration does *not* tell you whether a body is speeding up or slowing down. You must compare the signs of the $x$-velocity and the $x$-acceleration. When $v_x$ and $a_x$ have the *same* sign, the body is speeding up. If both are positive, the body is moving in the positive direction with increasing speed. If both are negative, the body is moving in the negative direction with an $x$-velocity that is becoming more and more negative, and again the speed is increasing. When $v_x$ and $a_x$ have *opposite* signs, the body is slowing down. If $v_x$ is positive and $a_x$ is negative, the body is moving in the positive direction with decreasing speed; if $v_x$ is negative and $a_x$ is positive, the body is moving in the negative direction with an $x$-velocity that is becoming less negative, and again the body is slowing down. Table 2.3 summarizes these ideas, and Fig. 2.13 illustrates some of these possibilities.

The term "deceleration" is sometimes used for a decrease in speed. Because it may mean positive or negative $a_x$, depending on the sign of $v_x$, we avoid this term.

We can also learn about the acceleration of a body from a graph of its *position* versus time. Because $a_x = dv_x/dt$ and $v_x = dx/dt$, we can write

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} \tag{2.6}$$

### Table 2.3 Rules for the Sign of $x$-Acceleration

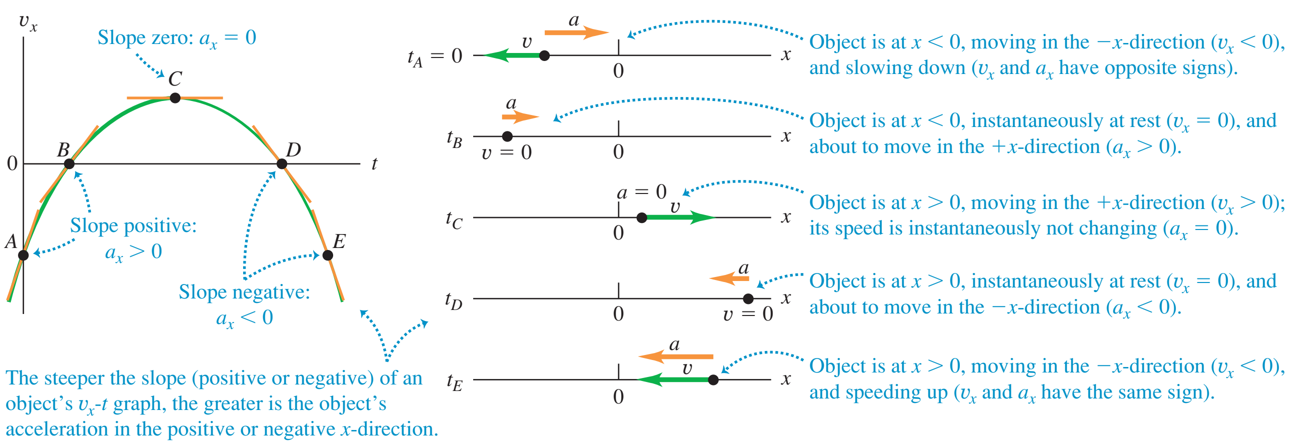
| If $x$-velocity is: | . . . $x$-acceleration is: |
|---|---|
| Positive & increasing (getting more positive) | Positive: Particle is moving in $+x$-direction & speeding up |
| Positive & decreasing (getting less positive) | Negative: Particle is moving in $+x$-direction & slowing down |
| Negative & increasing (getting less negative) | Positive: Particle is moving in $-x$-direction & slowing down |
| Negative & decreasing (getting more negative) | Negative: Particle is moving in $-x$-direction & speeding up |

*Note:* These rules apply to both the average $x$-acceleration $a_{av\text{-}x}$ and the instantaneous $x$-acceleration $a_x$.

**2.13** (a) A $v_x$-$t$ graph of the motion of a different particle from that shown in Fig. 2.8. The slope of the tangent at any point equals the $x$-acceleration at that point. (b) A motion diagram showing the position, velocity, and acceleration of the particle at each of the times labeled on the $v_x$-$t$ graph. The positions are consistent with the $v_x$-$t$ graph; for instance, from $t_A$ to $t_B$ the velocity is negative, so at $t_B$ the particle is at a more negative value of $x$ than at $t_A$.

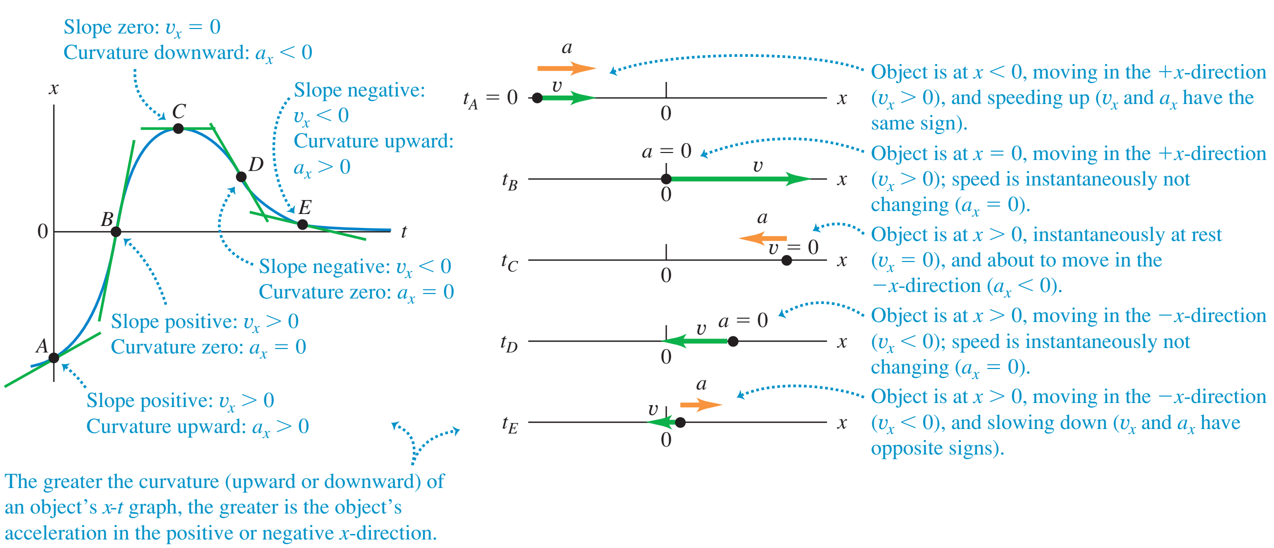
(a) $v_x$-$t$ graph for an object moving on the $x$-axis

(b) Object's position, velocity, and acceleration on the $x$-axis



**2.14** (a) The same $x$-$t$ graph as shown in Fig. 2.8a. The $x$-velocity is equal to the *slope* of the graph, and the acceleration is given by the *concavity* or *curvature* of the graph. (b) A motion diagram showing the position, velocity, and acceleration of the particle at each of the times labeled on the $x$-$t$ graph.

(a) $x$-$t$ graph (b) Object's motion




That is, $a_x$ is the second derivative of $x$ with respect to $t$. The second derivative of any function is directly related to the *concavity* or *curvature* of the graph of that function (Fig. 2.14). Where the $x$-$t$ graph is concave up (curved upward), the $x$-acceleration is positive and $v_x$ is increasing; at a point where the $x$-$t$ graph is concave down (curved downward), the $x$-acceleration is negative and $v_x$ is decreasing. At a point where the $x$-$t$ graph has no curvature, such as an inflection point, the $x$-acceleration is zero and the velocity is not changing. Figure 2.14 shows all three of these possibilities.

Examining the curvature of an $x$-$t$ graph is an easy way to decide what the *sign* of acceleration is. This technique is less helpful for determining numerical values of acceleration because the curvature of a graph is hard to measure accurately.

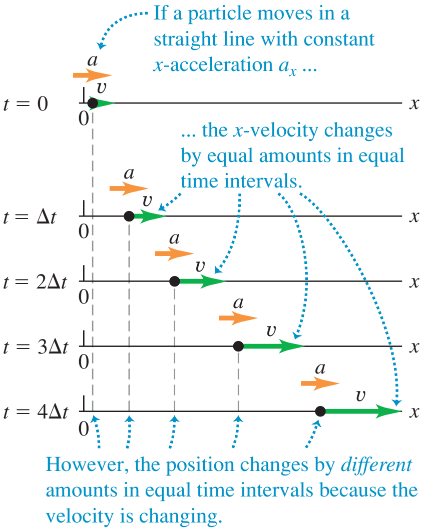
**Test Your Understanding of Section 2.3** Look again at the $x$-$t$ graph in Fig. 2.9 at the end of Section 2.2. (a) At which of the points $P$, $Q$, $R$, and $S$ is the $x$-acceleration $a_x$ positive? (b) At which points is the $x$-acceleration negative? (c) At which points does the $x$-acceleration appear to be zero? (d) At each point state whether the velocity is increasing, decreasing, or not changing.

## 2.4 Motion with Constant Acceleration

The simplest kind of accelerated motion is straight-line motion with *constant* acceleration. In this case the velocity changes at the same rate throughout the motion. As an example, a falling body has a constant acceleration if the effects of the air are not important. The same is true for a body sliding on an incline or along a rough horizontal surface, or for an airplane being catapulted from the deck of an aircraft carrier.

Figure 2.15 is a motion diagram showing the position, velocity, and acceleration for a particle moving with constant acceleration. Figures 2.16 and 2.17 depict this same motion in the form of graphs. Since the $x$-acceleration is constant, the **$a_x$-$t$ graph** (graph of $x$-acceleration versus time) in Fig. 2.16 is a horizontal line. The graph of $x$-velocity versus time, or $v_x$-$t$ graph, has a constant *slope* because the acceleration is constant, so this graph is a straight line (Fig. 2.17).

**2.15** A motion diagram for a particle moving in a straight line in the positive $x$-direction with constant positive $x$-acceleration $a_x$. The position, velocity, and acceleration are shown at five equally spaced times.



When the $x$-acceleration $a_x$ is constant, the average $x$-acceleration $a_{\text{av-}x}$ for any time interval is the same as $a_x$. This makes it easy to derive equations for the position $x$ and the $x$-velocity $v_x$ as functions of time. To find an expression for $v_x$, we first replace $a_{\text{av-}x}$ in Eq. (2.4) by $a_x$:

$$a_x = \frac{v_{2x} - v_{1x}}{t_2 - t_1} \tag{2.7}$$

Now we let $t_1 = 0$ and let $t_2$ be any later time $t$. We use the symbol $v_{0x}$ for the $x$-velocity at the initial time $t = 0$; the $x$-velocity at the later time $t$ is $v_x$. Then Eq. (2.7) becomes

$$a_x = \frac{v_x - v_{0x}}{t - 0} \qquad \text{or}$$

$$v_x = v_{0x} + a_x t \qquad \text{(constant } x\text{-acceleration only)} \tag{2.8}$$

In Eq. (2.8) the term $a_x t$ is the product of the constant rate of change of $x$-velocity, $a_x$, and the time interval $t$. Therefore it equals the *total* change in $x$-velocity from the initial time $t = 0$ to the later time $t$. The $x$-velocity $v_x$ at any time $t$ then equals the initial $x$-velocity $v_{0x}$ (at $t = 0$) plus the change in $x$-velocity $a_x t$ (Fig. 2.17).

Equation (2.8) also says that the change in $x$-velocity $v_x - v_{0x}$ of the particle between $t = 0$ and any later time $t$ equals the *area* under the $a_x$-$t$ graph between those two times. You can verify this from Fig. 2.16: Under this graph is a rectangle of vertical side $a_x$, horizontal side $t$, and area $a_x t$. From Eq. (2.8) this is indeed equal to the change in velocity $v_x - v_{0x}$. In Section 2.6 we'll show that even if the $x$-acceleration is not constant, the change in $x$-velocity during a time interval is still equal to the area under the $a_x$-$t$ curve, although in that case Eq. (2.8) does not apply.

Next we'll derive an equation for the position $x$ as a function of time when the $x$-acceleration is constant. To do this, we use two different expressions for the average $x$-velocity $v_{\text{av-}x}$ during the interval from $t = 0$ to any later time $t$. The first expression comes from the definition of $v_{\text{av-}x}$, Eq. (2.2), which is true whether or not the acceleration is constant. We call the position at time $t = 0$ the *initial position*, denoted by $x_0$. The position at the later time $t$ is simply $x$. Thus for the time interval $\Delta t = t - 0$ the displacement is $\Delta x = x - x_0$, and Eq. (2.2) gives

$$v_{\text{av-}x} = \frac{x - x_0}{t} \tag{2.9}$$

We can also get a second expression for $v_{\text{av-}x}$ that is valid only when the $x$-acceleration is constant, so that the $x$-velocity changes at a constant rate. In this case the average $x$-velocity for the time interval from 0 to $t$ is simply the average of the $x$-velocities at the beginning and end of the interval:

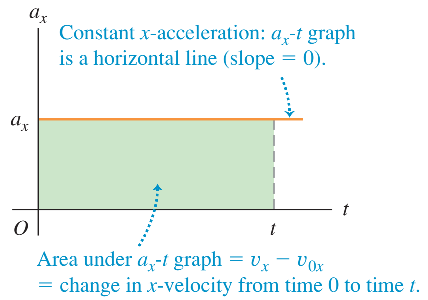
$$v_{\text{av-}x} = \frac{v_{0x} + v_x}{2} \qquad \text{(constant } x\text{-acceleration only)} \tag{2.10}$$

(This equation is *not* true if the $x$-acceleration varies during the time interval.) We also know that with constant $x$-acceleration, the $x$-velocity $v_x$ at any time $t$ is given by Eq. (2.8). Substituting that expression for $v_x$ into Eq. (2.10), we find

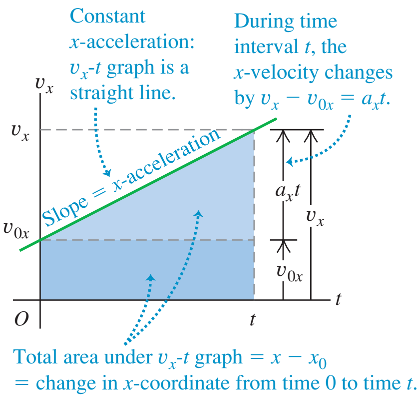
$$v_{\text{av-}x} = \tfrac{1}{2}(v_{0x} + v_{0x} + a_x t)$$

$$= v_{0x} + \tfrac{1}{2}a_x t \qquad \text{(constant } x\text{-acceleration only)} \tag{2.11}$$

**2.16** An acceleration-time ($a_x$-$t$) graph for straight-line motion with constant positive $x$-acceleration $a_x$.




**2.17** A velocity-time ($v_x$-$t$) graph for straight-line motion with constant positive $x$-acceleration $a_x$. The initial $x$-velocity $v_{0x}$ is also positive in this case.




**MasteringPHYSICS**

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**ActivPhysics 1.3:** Predicting Motion from Graphs
**ActivPhysics 1.4:** Predicting Motion from Equations
**ActivPhysics 1.5:** Problem-Solving Strategies for Kinematics
**ActivPhysics 1.6:** Skier Races Downhill

**Application Testing Humans at High Accelerations**

In experiments carried out by the U.S. Air Force in the 1940s and 1950s, humans riding a rocket sled demonstrated that they could withstand accelerations as great as 440 $m/s^2$. The first three photos in this sequence show Air Force physician John Stapp speeding up from rest to 188 m/s (678 km/h = 421 mi/h) in just 5 s. Photos 4–6 show the even greater magnitude of acceleration as the rocket sled braked to a halt.

Finally, we set Eqs. (2.9) and (2.11) equal to each other and simplify:

$$v_{0x} + \tfrac{1}{2}a_x t = \frac{x - x_0}{t} \qquad \text{or}$$

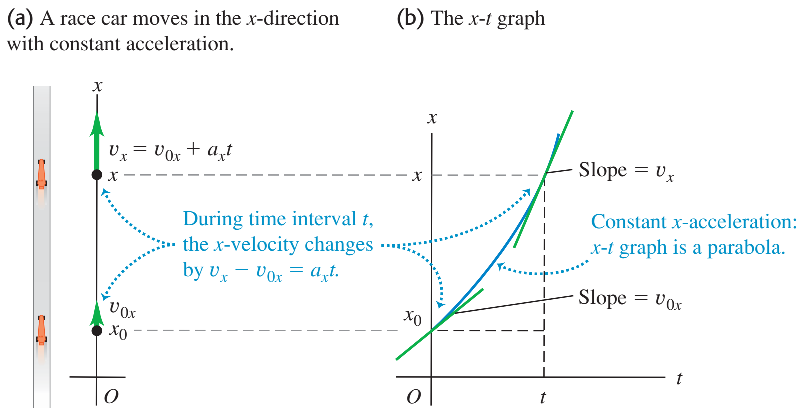
$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2 \qquad \text{(constant } x\text{-acceleration only)} \qquad (2.12)$$

Here's what Eq. (2.12) tells us: If at time $t = 0$ a particle is at position $x_0$ and has $x$-velocity $v_{0x}$, its new position $x$ at any later time $t$ is the sum of three terms—its initial position $x_0$, plus the distance $v_{0x}t$ that it would move if its $x$-velocity were constant, plus an additional distance $\frac{1}{2}a_x t^2$ caused by the change in $x$-velocity.

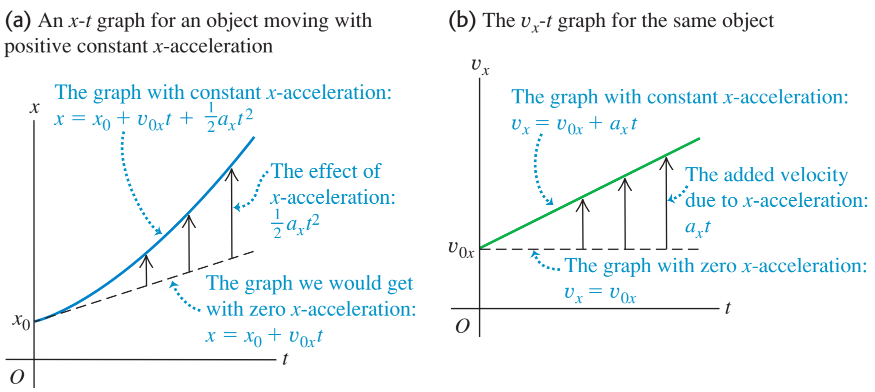
A graph of Eq. (2.12)—that is, an $x$-$t$ graph for motion with constant $x$-acceleration (Fig. 2.18a)—is always a *parabola.* Figure 2.18b shows such a graph. The curve intercepts the vertical axis ($x$-axis) at $x_0$, the position at $t = 0$. The slope of the tangent at $t = 0$ equals $v_{0x}$, the initial $x$-velocity, and the slope of the tangent at any time $t$ equals the $x$-velocity $v_x$ at that time. The slope and $x$-velocity are continuously increasing, so the $x$-acceleration $a_x$ is positive; you can also see this because the graph in Fig. 2.18b is concave up (it curves upward). If $a_x$ is negative, the $x$-$t$ graph is a parabola that is concave down (has a downward curvature).

If there is zero $x$-acceleration, the $x$-$t$ graph is a straight line; if there is a constant $x$-acceleration, the additional $\frac{1}{2}a_x t^2$ term in Eq. (2.12) for $x$ as a function of $t$ curves the graph into a parabola (Fig. 2.19a). We can analyze the $v_x$-$t$ graph in the same way. If there is zero $x$-acceleration this graph is a horizontal line (the $x$-velocity is constant); adding a constant $x$-acceleration gives a slope to the $v_x$-$t$ graph (Fig. 2.19b).

**2.18** (a) Straight-line motion with constant acceleration. (b) A position-time ($x$-$t$) graph for this motion (the same motion as is shown in Figs. 2.15, 2.16, and 2.17). For this motion the initial position $x_0$, the initial velocity $v_{0x}$, and the acceleration $a_x$ are all positive.




**2.19** (a) How a constant $x$-acceleration affects a body's (a) $x$-$t$ graph and (b) $v_x$-$t$ graph.



Just as the change in $x$-velocity of the particle equals the area under the $a_x$-$t$ graph, the displacement—that is, the change in position—equals the area under the $v_x$-$t$ graph. To be specific, the displacement $x - x_0$ of the particle between $t = 0$ and any later time $t$ equals the area under the $v_x$-$t$ graph between those two times. In Fig. 2.17 we divide the area under the graph into a dark-colored rectangle (vertical side $v_{0x}$, horizontal side $t$, and area $v_{0x}t$) and a light-colored right triangle (vertical side $a_x t$, horizontal side $t$, and area $\frac{1}{2}(a_x t)(t) = \frac{1}{2}a_x t^2$). The total area under the $v_x$-$t$ graph is

$$x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2$$

in agreement with Eq. (2.12).

The displacement during a time interval is always equal to the area under the $v_x$-$t$ curve. This is true even if the acceleration is *not* constant, although in that case Eq. (2.12) does not apply. (We'll show this in Section 2.6.)

It's often useful to have a relationship for position, $x$-velocity, and (constant) $x$-acceleration that does not involve the time. To obtain this, we first solve Eq. (2.8) for $t$ and then substitute the resulting expression into Eq. (2.12):

$$t = \frac{v_x - v_{0x}}{a_x}$$

$$x = x_0 + v_{0x}\left(\frac{v_x - v_{0x}}{a_x}\right) + \tfrac{1}{2}a_x\left(\frac{v_x - v_{0x}}{a_x}\right)^2$$

We transfer the term $x_0$ to the left side and multiply through by $2a_x$:

$$2a_x(x - x_0) = 2v_{0x}v_x - 2v_{0x}^2 + v_x^2 - 2v_{0x}v_x + v_{0x}^2$$

Finally, simplifying gives us

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0) \qquad \text{(constant } x\text{-acceleration only)} \qquad (2.13)$$

We can get one more useful relationship by equating the two expressions for $v_{\text{av-}x}$, Eqs. (2.9) and (2.10), and multiplying through by $t$. Doing this, we obtain

$$x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t \qquad \text{(constant } x\text{-acceleration only)} \qquad (2.14)$$

Note that Eq. (2.14) does not contain the $x$-acceleration $a_x$. This equation can be handy when $a_x$ is constant but its value is unknown.

Equations (2.8), (2.12), (2.13), and (2.14) are the *equations of motion with constant acceleration* (Table 2.4). By using these equations, we can solve *any* problem involving straight-line motion of a particle with constant acceleration.

For the particular case of motion with constant $x$-acceleration depicted in Fig. 2.15 and graphed in Figs. 2.16, 2.17, and 2.18, the values of $x_0$, $v_{0x}$, and $a_x$ are all positive. We invite you to redraw these figures for cases in which one, two, or all three of these quantities are negative.

MasteringPHYSICS

**PhET:** The Moving Man
**ActivPhysics 1.8:** Seat Belts Save Lives
**ActivPhysics 1.9:** Screeching to a Halt
**ActivPhysics 1.11**: Car Starts, Then Stops
**ActivPhysics 1.12:** Solving Two-Vehicle Problems
**ActivPhysics 1.13:** Car Catches Truck
**ActivPhysics 1.14:** Avoiding a Rear-End Collision

**Table 2.4 Equations of Motion with Constant Acceleration**

| Equation | | Includes Quantities | | | |
|---|---|---|---|---|---|
| $v_x = v_{0x} + a_x t$ | (2.8) | $t$ | | $v_x$ | $a_x$ |
| $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ | (2.12) | $t$ | $x$ | | $a_x$ |
| $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ | (2.13) | | $x$ | $v_x$ | $a_x$ |
| $x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t$ | (2.14) | $t$ | $x$ | $v_x$ | |

## Problem-Solving Strategy 2.1 Motion with Constant Acceleration

**IDENTIFY** *the relevant concepts:* In most straight-line motion problems, you can use the constant-acceleration equations (2.8), (2.12), (2.13), and (2.14). If you encounter a situation in which the acceleration *isn't* constant, you'll need a different approach (see Section 2.6).

**SET UP** *the problem* using the following steps:

1. Read the problem carefully. Make a motion diagram showing the location of the particle at the times of interest. Decide where to place the origin of coordinates and which axis direction is positive. It's often helpful to place the particle at the origin at time $t = 0$; then $x_0 = 0$. Remember that your choice of the positive axis direction automatically determines the positive directions for $x$-velocity and $x$-acceleration. If $x$ is positive to the right of the origin, then $v_x$ and $a_x$ are also positive toward the right.
2. Identify the physical quantities (times, positions, velocities, and accelerations) that appear in Eqs. (2.8), (2.12), (2.13), and (2.14) and assign them appropriate symbols — $x$, $x_0$, $v_x$, $v_{0x}$, and $a_x$, or symbols related to those. Translate the prose into physics: "*When* does the particle arrive at its highest point" means "What is the value of $t$ when $x$ has its maximum value?" In Example 2.4 below, "Where is the motorcyclist when his velocity is 25 m/s?" means "What is the value of $x$ when $v_x = 25$ m/s?" Be alert for implicit information. For example, "A car sits at a stop light" usually means $v_{0x} = 0$.
3. Make a list of the quantities such as $x$, $x_0$, $v_x$, $v_{0x}$, $a_x$, and $t$. Some of them will be known and some will be unknown. Write down the values of the known quantities, and decide which of the unknowns are the target variables. Make note of the *absence* of any of the quantities that appear in the four constant-acceleration equations.
4. Use Table 2.4 to identify the applicable equations. (These are often the equations that don't include any of the absent quantities that you identified in step 3.) Usually you'll find a single equation that contains only one of the target variables. Sometimes you must find two equations, each containing the same two unknowns.
5. Sketch graphs corresponding to the applicable equations. The $v_x$-$t$ graph of Eq. (2.8) is a straight line with slope $a_x$. The $x$-$t$ graph of Eq. (2.12) is a parabola that's concave up if $a_x$ is positive and concave down if $a_x$ is negative.
6. On the basis of your accumulated experience with such problems, and taking account of what your sketched graphs tell you, make any qualitative and quantitative predictions you can about the solution.

**EXECUTE** *the solution:* If a single equation applies, solve it for the target variable, *using symbols only*; then substitute the known values and calculate the value of the target variable. If you have two equations in two unknowns, solve them simultaneously for the target variables.

**EVALUATE** *your answer:* Take a hard look at your results to see whether they make sense. Are they within the general range of values that you expected?

## Example 2.4 Constant-acceleration calculations

A motorcyclist heading east through a small town accelerates at a constant 4.0 m/s$^2$ after he leaves the city limits (Fig. 2.20). At time $t = 0$ he is 5.0 m east of the city-limits signpost, moving east at 15 m/s. (a) Find his position and velocity at $t = 2.0$ s. (b) Where is he when his velocity is 25 m/s?

**SOLUTION**

**IDENTIFY and SET UP:** The $x$-acceleration is constant, so we can use the constant-acceleration equations. We take the signpost as the origin of coordinates ($x = 0$) and choose the positive $x$-axis to point east (see Fig. 2.20, which is also a motion diagram). The known variables are the initial position and velocity, $x_0 = 5.0$ m and $v_{0x} = 15$ m/s, and the acceleration, $a_x = 4.0$ m/s$^2$. The unknown target variables in part (a) are the values of the position $x$ and the $x$-velocity $v_x$ at $t = 2.0$ s; the target variable in part (b) is the value of $x$ when $v_x = 25$ m/s.

**EXECUTE:** (a) Since we know the values of $x_0$, $v_{0x}$, and $a_x$, Table 2.4 tells us that we can find the position $x$ at $t = 2.0$ s by using Eq. (2.12) and the $x$-velocity $v_x$ at this time by using Eq. (2.8):

$$\begin{aligned} x &= x_0 + v_{0x}t + \tfrac{1}{2}a_xt^2 \\ &= 5.0\text{ m} + (15\text{ m/s})(2.0\text{ s}) + \tfrac{1}{2}(4.0\text{ m/s}^2)(2.0\text{ s})^2 \\ &= 43\text{ m} \end{aligned}$$

$$\begin{aligned} v_x &= v_{0x} + a_xt \\ &= 15\text{ m/s} + (4.0\text{ m/s}^2)(2.0\text{ s}) = 23\text{ m/s} \end{aligned}$$

(b) We want to find the value of $x$ when $v_x = 25$ m/s, but we don't know the time when the motorcycle has this velocity. Table 2.4 tells us that we should use Eq. (2.13), which involves $x$, $v_x$, and $a_x$ but does *not* involve $t$:

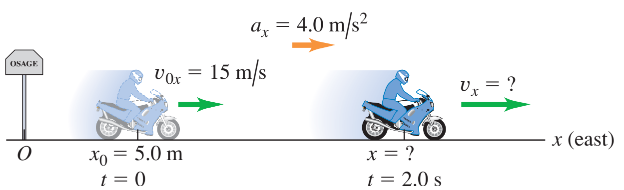
$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$$

Solving for $x$ and substituting the known values, we find

$$\begin{aligned} x &= x_0 + \frac{v_x^2 - v_{0x}^2}{2a_x} \\ &= 5.0\text{ m} + \frac{(25\text{ m/s})^2 - (15\text{ m/s})^2}{2(4.0\text{ m/s}^2)} = 55\text{ m} \end{aligned}$$

**EVALUATE:** You can check the result in part (b) by first using Eq. (2.8), $v_x = v_{0x} + a_xt$, to find the time at which $v_x = 25$ m/s, which turns out to be $t = 2.5$ s. You can then use Eq. (2.12), $x = x_0 + v_{0x}t + \frac{1}{2}a_xt^2$, to solve for $x$. You should find $x = 55$ m, the same answer as above. That's the long way to solve the problem, though. The method we used in part (b) is much more efficient.

**2.20** A motorcyclist traveling with constant acceleration.



## Example 2.5 Two bodies with different accelerations

A motorist traveling with a constant speed of 15 m/s (about 34 mi/h) passes a school-crossing corner, where the speed limit is 10 m/s (about 22 mi/h). Just as the motorist passes the school-crossing sign, a police officer on a motorcycle stopped there starts in pursuit with a constant acceleration of 3.0 m/s$^2$ (Fig. 2.21a). (a) How much time elapses before the officer passes the motorist? (b) What is the officer's speed at that time? (c) At that time, what distance has each vehicle traveled?

### SOLUTION

**IDENTIFY and SET UP:** The officer and the motorist both move with constant acceleration (equal to zero for the motorist), so we can use the constant-acceleration formulas. We take the origin at the sign, so $x_0 = 0$ for both, and we take the positive direction to the right. Let $x_P$ and $x_M$ represent the positions of the officer and the motorist at any time; their initial velocities are $v_{P0x} = 0$ and $v_{M0x} = 15$ m/s, and their accelerations are $a_{Px} = 3.0$ m/s$^2$ and $a_{Mx} = 0$. Our target variable in part (a) is the time when the officer passes the motorist—that is, when the two vehicles are at the same position $x$; Table 2.4 tells us that Eq. (2.12) is useful for this part. In part (b) we're looking for the officer's speed $v$ (the magnitude of his velocity) at the time found in part (a). We'll use Eq. (2.8) for this part. In part (c) we'll use Eq. (2.12) again to find the position of either vehicle at this same time.

Figure 2.21b shows an $x$-$t$ graph for both vehicles. The straight line represents the motorist's motion, $x_M = x_{M0} + v_{M0x}t = v_{M0x}t$. The graph for the officer's motion is the right half of a concave–up parabola:

$$x_P = x_{P0} + v_{P0x}t + \tfrac{1}{2}a_{Px}t^2 = \tfrac{1}{2}a_{Px}t^2$$

A good sketch will show that the officer and motorist are at the same position ($x_P = x_M$) at about $t = 10$ s, at which time both have traveled about 150 m from the sign.

**EXECUTE:** (a) To find the value of the time $t$ at which the motorist and police officer are at the same position, we set $x_P = x_M$ by equating the expressions above and solving that equation for $t$:

$$v_{M0x}t = \tfrac{1}{2}a_{Px}t^2$$

$$t = 0 \quad \text{or} \quad t = \frac{2v_{M0x}}{a_{Px}} = \frac{2(15\ \text{m/s})}{3.0\ \text{m/s}^2} = 10\ \text{s}$$

Both vehicles have the same $x$-coordinate at *two* times, as Fig. 2.21b indicates. At $t = 0$ the motorist passes the officer; at $t = 10$ s the officer passes the motorist.

(b) We want the magnitude of the officer's $x$-velocity $v_{Px}$ at the time $t$ found in part (a). Substituting the values of $v_{P0x}$ and $a_{Px}$ into Eq. (2.8) along with $t = 10$ s from part (a), we find

$$v_{Px} = v_{P0x} + a_{Px}t = 0 + (3.0\ \text{m/s}^2)(10\ \text{s}) = 30\ \text{m/s}$$

The officer's speed is the absolute value of this, which is also 30 m/s.

(c) In 10 s the motorist travels a distance

$$x_M = v_{M0x}t = (15\ \text{m/s})(10\ \text{s}) = 150\ \text{m}$$

and the officer travels

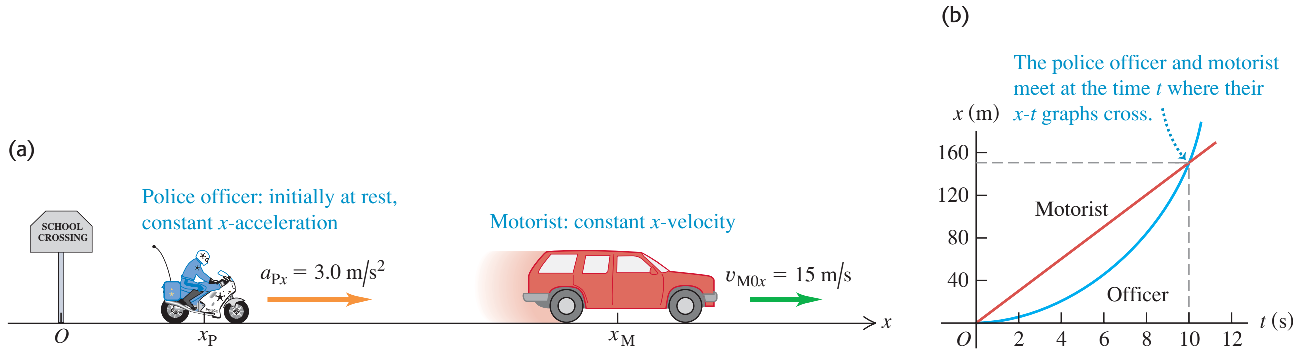
$$x_P = \tfrac{1}{2}a_{Px}t^2 = \tfrac{1}{2}(3.0\ \text{m/s}^2)(10\ \text{s})^2 = 150\ \text{m}$$

This verifies that they have gone equal distances when the officer passes the motorist.

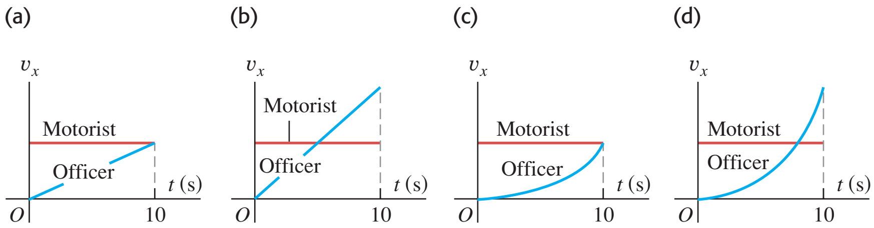
**EVALUATE:** Our results in parts (a) and (c) agree with our estimates from our sketch. Note that at the time when the officer passes the motorist, they do *not* have the same velocity. At this time the motorist is moving at 15 m/s and the officer is moving at 30 m/s. You can also see this from Fig. 2.21b. Where the two $x$-$t$ curves cross, their slopes (equal to the values of $v_x$ for the two vehicles) are different.

Is it just coincidence that when the two vehicles are at the same position, the officer is going twice the speed of the motorist? Equation (2.14), $x - x_0 = [(v_{0x} + v_x)/2]t$, gives the answer. The motorist has constant velocity, so $v_{M0x} = v_{Mx}$, and the distance $x - x_0$ that the motorist travels in time $t$ is $v_{M0x}t$. The officer has zero initial velocity, so in the same time $t$ the officer travels a distance $\tfrac{1}{2}v_{Px}t$. If the two vehicles cover the same distance in the same amount of time, the two values of $x - x_0$ must be the same. Hence when the officer passes the motorist $v_{M0x}t = \tfrac{1}{2}v_{Px}t$ and $v_{Px} = 2v_{M0x}$—that is, the officer has exactly twice the motorist's velocity. Note that this is true no matter what the value of the officer's acceleration.

**2.21** (a) Motion with constant acceleration overtaking motion with constant velocity. (b) A graph of $x$ versus $t$ for each vehicle.



**Test Your Understanding of Section 2.4** Four possible $v_x$-$t$ graphs are shown for the two vehicles in Example 2.5. Which graph is correct?




## 2.5 Freely Falling Bodies

**2.22** Multiflash photo of a freely falling ball.

MasteringPHYSICS
**PhET:** Lunar Lander
**ActivPhysics 1.7:** Balloonist Drops Lemonade
**ActivPhysics 1.10:** Pole-Vaulter Lands

The most familiar example of motion with (nearly) constant acceleration is a body falling under the influence of the earth's gravitational attraction. Such motion has held the attention of philosophers and scientists since ancient times. In the fourth century B.C., Aristotle thought (erroneously) that heavy bodies fall faster than light bodies, in proportion to their weight. Nineteen centuries later, Galileo (see Section 1.1) argued that a body should fall with a downward acceleration that is constant and independent of its weight.

Experiment shows that if the effects of the air can be neglected, Galileo is right; all bodies at a particular location fall with the same downward acceleration, regardless of their size or weight. If in addition the distance of the fall is small compared with the radius of the earth, and if we ignore small effects due to the earth's rotation, the acceleration is constant. The idealized motion that results under all of these assumptions is called **free fall,** although it includes rising as well as falling motion. (In Chapter 3 we will extend the discussion of free fall to include the motion of projectiles, which move both vertically and horizontally.)

Figure 2.22 is a photograph of a falling ball made with a stroboscopic light source that produces a series of short, intense flashes. As each flash occurs, an image of the ball at that instant is recorded on the photograph. There are equal time intervals between flashes, so the average velocity of the ball between successive flashes is proportional to the distance between corresponding images. The increasing distances between images show that the velocity is continuously changing; the ball is accelerating downward. Careful measurement shows that the velocity change is the same in each time interval, so the acceleration of the freely falling ball is constant.

The constant acceleration of a freely falling body is called the **acceleration due to gravity,** and we denote its magnitude with the letter $g$. We will frequently use the approximate value of $g$ at or near the earth's surface:

$$g = 9.8 \text{ m/s}^2 = 980 \text{ cm/s}^2 = 32 \text{ ft/s}^2 \qquad \text{(approximate value near the earth's surface)}$$

The exact value varies with location, so we will often give the value of $g$ at the earth's surface to only two significant figures. On the surface of the moon, the acceleration due to gravity is caused by the attractive force of the moon rather than the earth, and $g = 1.6 \text{ m/s}^2$. Near the surface of the sun, $g = 270 \text{ m/s}^2$.

**CAUTION** **$g$ is always a positive number** Because $g$ is the *magnitude* of a vector quantity, it is always a *positive* number. If you take the positive direction to be upward, as we do in Example 2.6 and in most situations involving free fall, the acceleration is negative (downward) and equal to $-g$. Be careful with the sign of $g$, or else you'll have no end of trouble with free-fall problems.

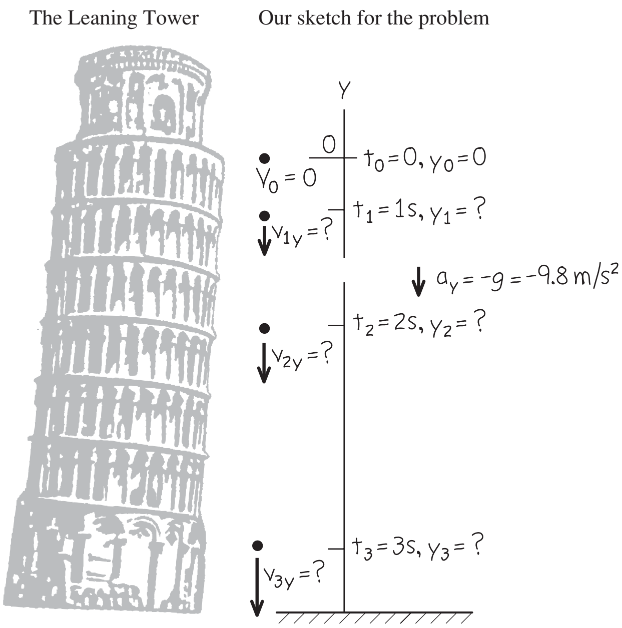
In the following examples we use the constant-acceleration equations developed in Section 2.4. You should review Problem-Solving Strategy 2.1 in that section before you study the next examples.

## Example 2.6 A freely falling coin

A one-euro coin is dropped from the Leaning Tower of Pisa and falls freely from rest. What are its position and velocity after 1.0 s, 2.0 s, and 3.0 s?

**SOLUTION**

**IDENTIFY and SET UP:** "Falls freely" means "falls with constant acceleration due to gravity," so we can use the constant-acceleration equations. The right side of Fig. 2.23 shows our motion diagram for the coin. The motion is vertical, so we use a vertical coordinate axis and call the coordinate $y$ instead of $x$. We take the origin $O$ at the starting point and the *upward* direction as positive. The initial coordinate $y_0$ and initial $y$-velocity $v_{0y}$ are both zero. The $y$-acceleration is downward (in the negative $y$-direction), so $a_y = -g = -9.8\ \text{m/s}^2$. (Remember that, by definition, $g$ itself is a positive quantity.) Our target variables are the values of $y$ and $v_y$ at the three given times. To find these, we use Eqs. (2.12) and (2.8) with $x$ replaced by $y$. Our choice of the upward direction as positive means that all positions and velocities we calculate will be negative.




**2.23** A coin freely falling from rest.

**EXECUTE:** At a time $t$ after the coin is dropped, its position and $y$-velocity are

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2 = 0 + 0 + \tfrac{1}{2}(-g)t^2 = (-4.9\ \text{m/s}^2)t^2$$

$$v_y = v_{0y} + a_y t = 0 + (-g)t = (-9.8\ \text{m/s}^2)t$$

When $t = 1.0$ s, $y = (-4.9\ \text{m/s}^2)(1.0\ \text{s})^2 = -4.9$ m and $v_y = (-9.8\ \text{m/s}^2)(1.0\ \text{s}) = -9.8\ \text{m/s}$; after 1 s, the coin is 4.9 m below the origin ($y$ is negative) and has a downward velocity ($v_y$ is negative) with magnitude 9.8 m/s.

We can find the positions and $y$-velocities at 2.0 s and 3.0 s in the same way. The results are $y = -20$ m and $v_y = -20$ m/s at $t = 2.0$ s, and $y = -44$ m and $v_y = -29$ m/s at $t = 3.0$ s.

**EVALUATE:** All our answers are negative, as we expected. If we had chosen the positive $y$-axis to point downward, the acceleration would have been $a_y = +g$ and all our answers would have been positive.

## Example 2.7 Up-and-down motion in free fall

You throw a ball vertically upward from the roof of a tall building. The ball leaves your hand at a point even with the roof railing with an upward speed of 15.0 m/s; the ball is then in free fall. On its way back down, it just misses the railing. Find (a) the ball's position and velocity 1.00 s and 4.00 s after leaving your hand; (b) the ball's velocity when it is 5.00 m above the railing; (c) the maximum height reached; (d) the ball's acceleration when it is at its maximum height.

**SOLUTION**

**IDENTIFY and SET UP:** The words "in free fall" mean that the acceleration is due to gravity, which is constant. Our target variables are position [in parts (a) and (c)], velocity [in parts (a) and (b)], and acceleration [in part (d)]. We take the origin at the point where the ball leaves your hand, and take the positive direction to be upward (Fig. 2.24). The initial position $y_0$ is zero, the initial $y$-velocity $v_{0y}$ is +15.0 m/s, and the $y$-acceleration is $a_y = -g = -9.80\ \text{m/s}^2$. In part (a), as in Example 2.6, we'll use Eqs. (2.12) and (2.8) to find the position and velocity as functions of time. In part (b) we must find the velocity at a given *position* (no time is given), so we'll use Eq. (2.13).

Figure 2.25 shows the $y$-$t$ and $v_y$-$t$ graphs for the ball. The $y$-$t$ graph is a concave-down parabola that rises and then falls, and the $v_y$-$t$ graph is a downward-sloping straight line. Note that the ball's velocity is zero when it is at its highest point.

**EXECUTE:** (a) The position and $y$-velocity at time $t$ are given by Eqs. (2.12) and (2.8) with $x$'s replaced by $y$'s:

$$\begin{aligned} y &= y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2 = y_0 + v_{0y}t + \tfrac{1}{2}(-g)t^2 \\ &= (0) + (15.0\ \text{m/s})t + \tfrac{1}{2}(-9.80\ \text{m/s}^2)t^2 \\ v_y &= v_{0y} + a_y t = v_{0y} + (-g)t \\ &= 15.0\ \text{m/s} + (-9.80\ \text{m/s}^2)t \end{aligned}$$

*Continued*

When $t = 1.00\text{ s}$, these equations give $y = +10.1\text{ m}$ and $v_y = +5.2\text{ m/s}$. That is, the ball is 10.1 m above the origin ($y$ is positive) and moving upward ($v_y$ is positive) with a speed of 5.2 m/s. This is less than the initial speed because the ball slows as it ascends. When $t = 4.00\text{ s}$, those equations give $y = -18.4\text{ m}$ and $v_y = -24.2\text{ m/s}$. The ball has passed its highest point and is 18.4 m *below* the origin ($y$ is negative). It is moving *downward* ($v_y$ is negative) with a speed of 24.2 m/s. The ball gains speed as it descends; Eq. (2.13) tells us that it is moving at the initial 15.0-m/s speed as it moves downward past the ball's launching point, and continues to gain speed as it descends further.

(b) The $y$-velocity at any position $y$ is given by Eq. (2.13) with $x$'s replaced by $y$'s:

$$\begin{aligned} v_y^2 &= v_{0y}^2 + 2a_y(y - y_0) = v_{0y}^2 + 2(-g)(y - 0) \\ &= (15.0\text{ m/s})^2 + 2(-9.80\text{ m/s}^2)y \end{aligned}$$

When the ball is 5.00 m above the origin we have $y = +5.00\text{ m}$, so

$$\begin{aligned} v_y^2 &= (15.0\text{ m/s})^2 + 2(-9.80\text{ m/s}^2)(5.00\text{ m}) = 127\text{ m}^2/\text{s}^2 \\ v_y &= \pm 11.3\text{ m/s} \end{aligned}$$

We get *two* values of $v_y$ because the ball passes through the point $y = +5.00\text{ m}$ twice, once on the way up (so $v_y$ is positive) and once on the way down (so $v_y$ is negative) (see Figs. 2.24 and 2.25a).

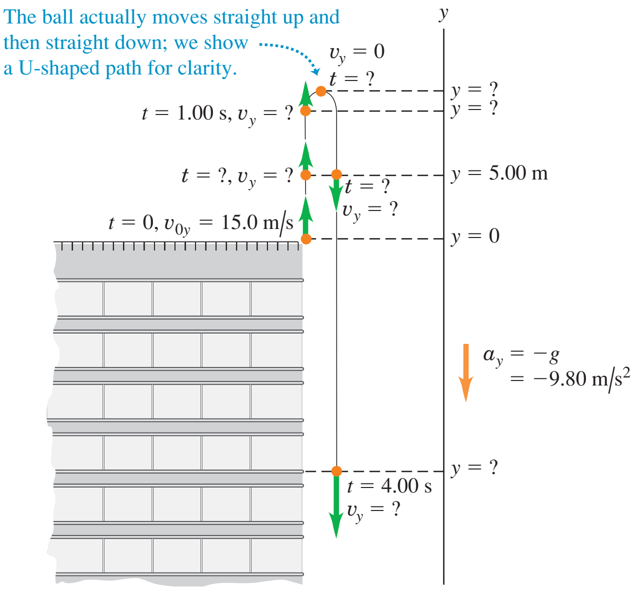
(c) At the instant at which the ball reaches its maximum height $y_1$, its $y$-velocity is momentarily zero: $v_y = 0$. We use Eq. (2.13) to find $y_1$. With $v_y = 0$, $y_0 = 0$, and $a_y = -g$, we get

$$\begin{aligned} 0 &= v_{0y}^2 + 2(-g)(y_1 - 0) \\ y_1 &= \frac{v_{0y}^2}{2g} = \frac{(15.0\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = +11.5\text{ m} \end{aligned}$$

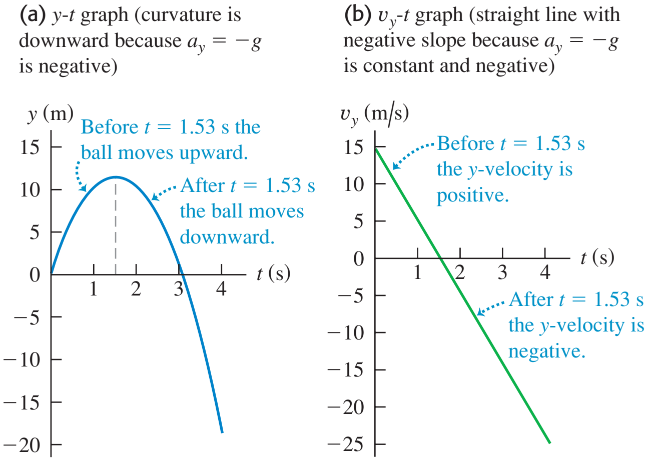
(d) **CAUTION** **A free-fall misconception** It's a common misconception that at the highest point of free-fall motion, where the velocity is zero, the acceleration is also zero. If this were so, once the ball reached the highest point it would hang there suspended in midair! Remember that acceleration is the rate of change of velocity, and the ball's velocity is continuously changing. At every point, including the highest point, and at any velocity, including zero, the acceleration in free fall is always $a_y = -g = -9.80\text{ m/s}^2$.

**EVALUATE:** A useful way to check any free-fall problem is to draw the $y$-$t$ and $v_y$-$t$ graphs as we did in Fig. 2.25. Note that these are graphs of Eqs. (2.12) and (2.8), respectively. Given the numerical values of the initial position, initial velocity, and acceleration, you can easily create these graphs using a graphing calculator or an online mathematics program.

**2.24** Position and velocity of a ball thrown vertically upward.




**2.25** (a) Position and (b) velocity as functions of time for a ball thrown upward with an initial speed of 15 m/s.




## Example 2.8 Two solutions or one?

At what time after being released has the ball in Example 2.7 fallen 5.00 m below the roof railing?

**SOLUTION**

**IDENTIFY and SET UP:** We treat this as in Example 2.7, so $y_0$, $v_{0y}$, and $a_y = -g$ have the same values as there. In this example, however, the target variable is the time at which the ball is at $y = -5.00\text{ m}$.

The best equation to use is Eq. (2.12), which gives the position $y$ as a function of time $t$:

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2 = y_0 + v_{0y}t + \tfrac{1}{2}(-g)t^2$$

This is a *quadratic* equation for $t$, which we want to solve for the value of $t$ when $y = -5.00\text{ m}$.

**EXECUTE:** We rearrange the equation so that is has the standard form of a quadratic equation for an unknown $x$, $Ax^2 + Bx + C = 0$:

$$(\tfrac{1}{2}g)t^2 + (-v_{0y})t + (y - y_0) = At^2 + Bt + C = 0$$

By comparison, we identify $A = \frac{1}{2}g$, $B = -v_{0y}$, and $C = y - y_0$. The quadratic formula (see Appendix B) tells us that this equation has *two* solutions:

$$\begin{aligned} t &= \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \\ &= \frac{-(-v_{0y}) \pm \sqrt{(-v_{0y})^2 - 4(\frac{1}{2}g)(y - y_0))}}{2(\frac{1}{2}g)} \\ &= \frac{v_{0y} \pm \sqrt{v_{0y}{}^2 - 2g(y - y_0)}}{g} \end{aligned}$$

Substituting the values $y_0 = 0$, $v_{0y} = +15.0\text{ m/s}$, $g = 9.80\text{ m/s}^2$, and $y = -5.00\text{ m}$, we find

$$t = \frac{(15.0\text{ m/s}) \pm \sqrt{(15.0\text{ m/s})^2 - 2(9.80\text{ m/s}^2)(-5.00\text{ m} - 0)}}{9.80\text{ m/s}^2}$$

You can confirm that the numerical answers are $t = +3.36\text{ s}$ and $t = -0.30\text{ s}$. The answer $t = -0.30\text{ s}$ doesn't make physical sense, since it refers to a time *before* the ball left your hand at $t = 0$. So the correct answer is $t = +3.36\text{ s}$.

**EVALUATE:** Why did we get a second, fictitious solution? The explanation is that constant-acceleration equations like Eq. (2.12) are based on the assumption that the acceleration is constant for *all* values of time, whether positive, negative, or zero. Hence the solution $t = -0.30\text{ s}$ refers to an imaginary moment when a freely falling ball was 5.00 m below the roof railing and rising to meet your hand. Since the ball didn't leave your hand and go into free fall until $t = 0$, this result is pure fiction.

You should repeat these calculations to find the times when the ball is 5.00 m *above* the origin $(y = +5.00\text{ m})$. The two answers are $t = +0.38\text{ s}$ and $t = +2.68\text{ s}$. These are both positive values of $t$, and both refer to the real motion of the ball after leaving your hand. At the earlier time the ball passes through $y = +5.00\text{ m}$ moving upward; at the later time it passes through this point moving downward. [Compare this with part (b) of Example 2.7, and again refer to Fig. 2.25a.]

You should also solve for the times when $y = +15.0\text{ m}$. In this case, both solutions involve the square root of a negative number, so there are *no* real solutions. Again Fig. 2.25a shows why; we found in part (c) of Example 2.7 that the ball's maximum height is $y = +11.5\text{ m}$, so it *never* reaches $y = +15.0\text{ m}$. While a quadratic equation such as Eq. (2.12) always has two solutions, in some situations one or both of the solutions will not be physically reasonable.

**Test Your Understanding of Section 2.5** If you toss a ball upward with a certain initial speed, it falls freely and reaches a maximum height $h$ a time $t$ after it leaves your hand. (a) If you throw the ball upward with double the initial speed, what new maximum height does the ball reach? (i) $h\sqrt{2}$; (ii) $2h$; (iii) $4h$; (iv) $8h$; (v) $16h$. (b) If you throw the ball upward with double the initial speed, how long does it take to reach its new maximum height? (i) $t/2$; (ii) $t/\sqrt{2}$; (iii) $t$; (iv) $t\sqrt{2}$; (v) $2t$.

## 2.6 Velocity and Position by Integration

This section is intended for students who have already learned a little integral calculus. In Section 2.4 we analyzed the special case of straight-line motion with constant acceleration. When $a_x$ is not constant, as is frequently the case, the equations that we derived in that section are no longer valid (Fig. 2.26). But even when $a_x$ varies with time, we can still use the relationship $v_x = dx/dt$ to find the $x$-velocity $v_x$ as a function of time if the position $x$ is a known function of time. And we can still use $a_x = dv_x/dt$ to find the $x$-acceleration $a_x$ as a function of time if the $x$-velocity $v_x$ is a known function of time.

In many situations, however, position and velocity are not known functions of time, while acceleration is (Fig. 2.27). How can we find the position and velocity in straight-line motion from the acceleration function $a_x(t)$?

We first consider a graphical approach. Figure 2.28 is a graph of $x$-acceleration versus time for a body whose acceleration is not constant. We can divide the time interval between times $t_1$ and $t_2$ into many smaller intervals, calling a typical one $\Delta t$. Let the average $x$-acceleration during $\Delta t$ be $a_{\text{av-}x}$. From Eq. (2.4) the change in $x$-velocity $\Delta v_x$ during $\Delta t$ is

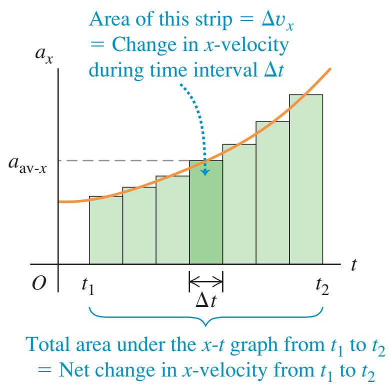
$$\Delta v_x = a_{\text{av-}x}\,\Delta t$$

Graphically, $\Delta v_x$ equals the area of the shaded strip with height $a_{\text{av-}x}$ and width $\Delta t$—that is, the area under the curve between the left and right sides of $\Delta t$. The total change in $x$-velocity during any interval (say, $t_1$ to $t_2$) is the sum of the $x$-velocity changes $\Delta v_x$ in the small subintervals. So the total $x$-velocity change is represented graphically by the *total* area under the $a_x$-$t$ curve between the vertical

**2.26** When you push your car's accelerator pedal to the floorboard, the resulting acceleration is *not* constant: The greater the car's speed, the more slowly it gains additional speed. A typical car takes twice as long to accelerate from 50 km/h to 100 km/h as it does to accelerate from 0 to 50 km/h.

**2.27** The inertial navigation system (INS) on board a long-range airliner keeps track of the airliner's acceleration. The pilots input the airliner's initial position and velocity before takeoff, and the INS uses the acceleration data to calculate the airliner's position and velocity throughout the flight.

**2.28** An $a_x$-$t$ graph for a body whose $x$-acceleration is not constant.




lines $t_1$ and $t_2$. (In Section 2.4 we showed this for the special case in which the acceleration is constant.)

In the limit that all the $\Delta t$'s become very small and their number very large, the value of $a_{\text{av-}x}$ for the interval from any time $t$ to $t + \Delta t$ approaches the instantaneous $x$-acceleration $a_x$ at time $t$. In this limit, the area under the $a_x$-$t$ curve is the *integral* of $a_x$ (which is in general a function of $t$) from $t_1$ to $t_2$. If $v_{1x}$ is the $x$-velocity of the body at time $t_1$ and $v_{2x}$ is the velocity at time $t_2$, then

$$v_{2x} - v_{1x} = \int_{v_{1x}}^{v_{2x}} dv_x = \int_{t_1}^{t_2} a_x \, dt \qquad (2.15)$$

The change in the $x$-velocity $v_x$ is the time integral of the $x$-acceleration $a_x$.

We can carry out exactly the same procedure with the curve of $x$-velocity versus time. If $x_1$ is a body's position at time $t_1$ and $x_2$ is its position at time $t_2$, from Eq. (2.2) the displacement $\Delta x$ during a small time interval $\Delta t$ is equal to $v_{\text{av-}x}\,\Delta t$, where $v_{\text{av-}x}$ is the average $x$-velocity during $\Delta t$. The total displacement $x_2 - x_1$ during the interval $t_2 - t_1$ is given by

$$x_2 - x_1 = \int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} v_x \, dt \qquad (2.16)$$

The change in position $x$—that is, the displacement—is the time integral of $x$-velocity $v_x$. Graphically, the displacement between times $t_1$ and $t_2$ is the area under the $v_x$-$t$ curve between those two times. [This is the same result that we obtained in Section 2.4 for the special case in which $v_x$ is given by Eq. (2.8).]

If $t_1 = 0$ and $t_2$ is any later time $t$, and if $x_0$ and $v_{0x}$ are the position and velocity, respectively, at time $t = 0$, then we can rewrite Eqs. (2.15) and (2.16) as follows:

$$v_x = v_{0x} + \int_0^t a_x \, dt \qquad (2.17)$$

$$x = x_0 + \int_0^t v_x \, dt \qquad (2.18)$$

Here $x$ and $v_x$ are the position and $x$-velocity at time $t$. If we know the $x$-acceleration $a_x$ as a function of time and we know the initial velocity $v_{0x}$, we can use Eq. (2.17) to find the $x$-velocity $v_x$ at any time; in other words, we can find $v_x$ as a function of time. Once we know this function, and given the initial position $x_0$, we can use Eq. (2.18) to find the position $x$ at any time.

## Example 2.9 Motion with changing acceleration

Sally is driving along a straight highway in her 1965 Mustang. At $t = 0$, when she is moving at 10 m/s in the positive $x$-direction, she passes a signpost at $x = 50$ m. Her $x$-acceleration as a function of time is

$$a_x = 2.0\ \text{m/s}^2 - (0.10\ \text{m/s}^3)t$$

(a) Find her $x$-velocity $v_x$ and position $x$ as functions of time. (b) When is her $x$-velocity greatest? (c) What is that maximum $x$-velocity? (d) Where is the car when it reaches that maximum $x$-velocity?

**SOLUTION**

**IDENTIFY and SET UP:** The $x$-acceleration is a function of time, so we *cannot* use the constant-acceleration formulas of Section 2.4. Instead, we use Eq. (2.17) to obtain an expression for $v_x$ as a function of time, and then use that result in Eq. (2.18) to find an expression for $x$ as a function of $t$. We'll then be able to answer a variety of questions about the motion.

**EXECUTE:** (a) At $t = 0$, Sally's position is $x_0 = 50$ m and her $x$-velocity is $v_{0x} = 10$ m/s. To use Eq. (2.17), we note that the integral of $t^n$ (except for $n = -1$) is $\int t^n dt = \frac{1}{n+1}t^{n+1}$. Hence we find

$$v_x = 10\text{ m/s} + \int_0^t [2.0\text{ m/s}^2 - (0.10\text{ m/s}^3)t]\,dt$$

$$= 10\text{ m/s} + (2.0\text{ m/s}^2)t - \tfrac{1}{2}(0.10\text{ m/s}^3)t^2$$

Now we use Eq. (2.18) to find $x$ as a function of $t$:

$$x = 50\text{ m} + \int_0^t [10\text{ m/s} + (2.0\text{ m/s}^2)t - \tfrac{1}{2}(0.10\text{ m/s}^3)t^2]\,dt$$

$$= 50\text{ m} + (10\text{ m/s})t + \tfrac{1}{2}(2.0\text{ m/s}^2)t^2 - \tfrac{1}{6}(0.10\text{ m/s}^3)t^3$$

Figure 2.29 shows graphs of $a_x$, $v_x$, and $x$ as functions of time as given by the equations above. Note that for any time $t$, the slope of the $v_x$-$t$ graph equals the value of $a_x$ and the slope of the $x$-$t$ graph equals the value of $v_x$.

(b) The maximum value of $v_x$ occurs when the $x$-velocity stops increasing and begins to decrease. At that instant, $dv_x/dt = a_x = 0$. So we set the expression for $a_x$ equal to zero and solve for $t$:

$$0 = 2.0\text{ m/s}^2 - (0.10\text{ m/s}^3)t$$

$$t = \frac{2.0\text{ m/s}^2}{0.10\text{ m/s}^3} = 20\text{ s}$$

(c) We find the maximum $x$-velocity by substituting $t = 20$ s, the time from part (b) when velocity is maximum, into the equation for $v_x$ from part (a):

$$v_{\text{max-}x} = 10\text{ m/s} + (2.0\text{ m/s}^2)(20\text{ s}) - \tfrac{1}{2}(0.10\text{ m/s}^3)(20\text{ s})^2$$

$$= 30\text{ m/s}$$

(d) To find the car's position at the time that we found in part (b), we substitute $t = 20$ s into the expression for $x$ from part (a):

$$x = 50\text{ m} + (10\text{ m/s})(20\text{ s}) + \tfrac{1}{2}(2.0\text{ m/s}^2)(20\text{ s})^2$$

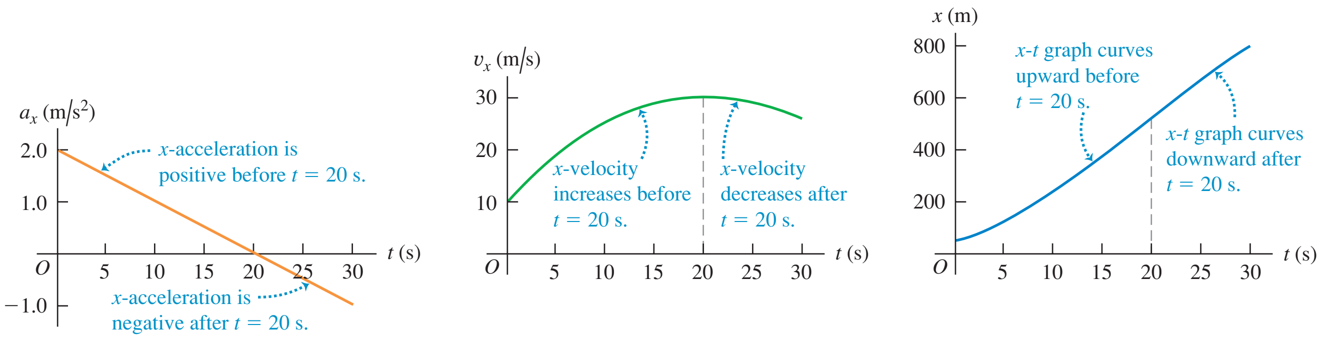
$$- \tfrac{1}{6}(0.10\text{ m/s}^3)(20\text{ s})^3$$

$$= 517\text{ m}$$

**EVALUATE:** Figure 2.29 helps us interpret our results. The top graph shows that $a_x$ is positive between $t = 0$ and $t = 20$ s and negative after that. It is zero at $t = 20$ s, the time at which $v_x$ is maximum (the high point in the middle graph). The car speeds up until $t = 20$ s (because $v_x$ and $a_x$ have the same sign) and slows down after $t = 20$ s (because $v_x$ and $a_x$ have opposite signs).

Since $v_x$ is maximum at $t = 20$ s, the $x$-$t$ graph (the bottom graph in Fig. 2.29) has its maximum positive slope at this time. Note that the $x$-$t$ graph is concave up (curved upward) from $t = 0$ to $t = 20$ s, when $a_x$ is positive. The graph is concave down (curved downward) after $t = 20$ s, when $a_x$ is negative.

**2.29** The position, velocity, and acceleration of the car in Example 2.9 as functions of time. Can you show that if this motion continues, the car will stop at $t = 44.5$ s?



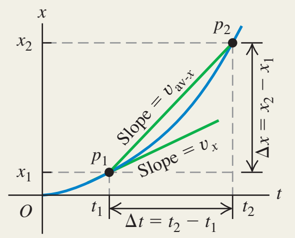

**Test Your Understanding of Section 2.6** If the $x$-acceleration $a_x$ is increasing with time, will the $v_x$-$t$ graph be (i) a straight line, (ii) concave up (i.e., with an upward curvature), or (iii) concave down (i.e., with a downward curvature)?

# CHAPTER 2 SUMMARY

**Straight-line motion, average and instantaneous *x*-velocity:** When a particle moves along a straight line, we describe its position with respect to an origin $O$ by means of a coordinate such as $x$. The particle's average $x$-velocity $v_{\text{av-}x}$ during a time interval $\Delta t = t_2 - t_1$ is equal to its displacement $\Delta x = x_2 - x_1$ divided by $\Delta t$. The instantaneous $x$-velocity $v_x$ at any time $t$ is equal to the average $x$-velocity for the time interval from $t$ to $t + \Delta t$ in the limit that $\Delta t$ goes to zero. Equivalently, $v_x$ is the derivative of the position function with respect to time. (See Example 2.1.)

$$v_{\text{av-}x} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} \quad (2.2)$$

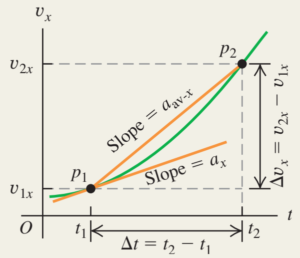
$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \quad (2.3)$$




**Average and instantaneous *x*-acceleration:** The average $x$-acceleration $a_{\text{av-}x}$ during a time interval $\Delta t$ is equal to the change in velocity $\Delta v_x = v_{2x} - v_{1x}$ during that time interval divided by $\Delta t$. The instantaneous $x$-acceleration $a_x$ is the limit of $a_{\text{av-}x}$ as $\Delta t$ goes to zero, or the derivative of $v_x$ with respect to $t$. (See Examples 2.2 and 2.3.)

$$a_{\text{av-}x} = \frac{v_{2x} - v_{1x}}{t_2 - t_1} = \frac{\Delta v_x}{\Delta t} \quad (2.4)$$

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} \quad (2.5)$$




**Straight-line motion with constant acceleration:** When the $x$-acceleration is constant, four equations relate the position $x$ and the $x$-velocity $v_x$ at any time $t$ to the initial position $x_0$, the initial $x$-velocity $v_{0x}$ (both measured at time $t = 0$), and the $x$-acceleration $a_x$. (See Examples 2.4 and 2.5.)

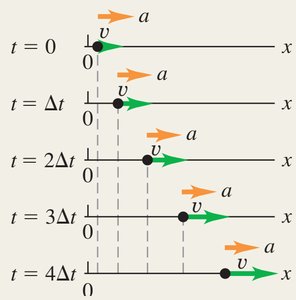
Constant $x$-acceleration only:

$$v_x = v_{0x} + a_x t \quad (2.8)$$

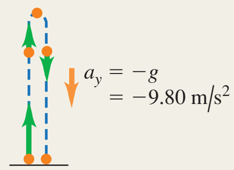
$$x = x_0 + v_{0x} t + \tfrac{1}{2} a_x t^2 \quad (2.12)$$

$$v_x^{\,2} = v_{0x}^{\,2} + 2a_x(x - x_0) \quad (2.13)$$

$$x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right) t \quad (2.14)$$



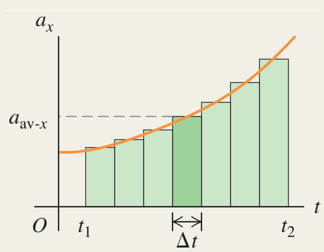

**Freely falling bodies:** Free fall is a case of motion with constant acceleration. The magnitude of the acceleration due to gravity is a positive quantity, $g$. The acceleration of a body in free fall is always downward. (See Examples 2.6–2.8.)




**Straight-line motion with varying acceleration:** When the acceleration is not constant but is a known function of time, we can find the velocity and position as functions of time by integrating the acceleration function. (See Example 2.9.)

$$v_x = v_{0x} + \int_0^t a_x\,dt \quad (2.17)$$

$$x = x_0 + \int_0^t v_x\,dt \quad (2.18)$$



## BRIDGING PROBLEM The Fall of a Superhero

The superhero Green Lantern steps from the top of a tall building. He falls freely from rest to the ground, falling half the total distance to the ground during the last 1.00 s of his fall. What is the height $h$ of the building?

### SOLUTION GUIDE

See MasteringPhysics® study area for a Video Tutor solution.

**IDENTIFY and SET UP**

1. You're told that Green Lantern falls freely from rest. What does this imply about his acceleration? About his initial velocity?
2. Choose the direction of the positive $y$-axis. It's easiest to make the same choice we used for freely falling objects in Section 2.5.
3. You can divide Green Lantern's fall into two parts: from the top of the building to the halfway point and from the halfway point to the ground. You know that the second part of the fall lasts 1.00 s. Decide what you would need to know about Green Lantern's motion at the halfway point in order to solve for the target variable $h$. Then choose two equations, one for the first part of the fall and one for the second part, that you'll use together to find an expression for $h$. (There are several pairs of equations that you could choose.)

**EXECUTE**

4. Use your two equations to solve for the height $h$. Note that heights are always positive numbers, so your answer should be positive.

**EVALUATE**

5. To check your answer for $h$, use one of the free-fall equations to find how long it takes Green Lantern to fall (i) from the top of the building to half the height and (ii) from the top of the building to the ground. If your answer for $h$ is correct, time (ii) should be 1.00 s greater than time (i). If it isn't, you'll need to go back and look for errors in how you found $h$.

# Problems

For instructor-assigned homework, go to www.masteringphysics.com

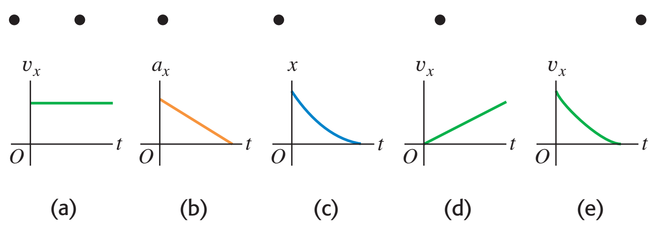
•, ••, •••: Problems of increasing difficulty. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **BIO**: Biosciences problems.

## DISCUSSION QUESTIONS

**Q2.1** Does the speedometer of a car measure speed or velocity? Explain.

**Q2.2** The top diagram in Fig. Q2.2 represents a series of high-speed photographs of an insect flying in a straight line from left to right (in the positive $x$-direction). Which of the graphs in Fig. Q2.2 most plausibly depicts this insect's motion?

Figure **Q2.2**




**Q2.3** Can an object with constant acceleration reverse its direction of travel? Can it reverse its direction *twice*? In each case, explain your reasoning.

**Q2.4** Under what conditions is average velocity equal to instantaneous velocity?

**Q2.5** Is it possible for an object (a) to be slowing down while its acceleration is increasing in magnitude; (b) to be speeding up while its acceleration is decreasing? In each case, explain your reasoning.

**Q2.6** Under what conditions does the magnitude of the average velocity equal the average speed?

**Q2.7** When a Dodge Viper is at Elwood's Car Wash, a BMW Z3 is at Elm and Main. Later, when the Dodge reaches Elm and Main, the BMW reaches Elwood's Car Wash. How are the cars' average velocities between these two times related?

**Q2.8** A driver in Massachusetts was sent to traffic court for speeding. The evidence against the driver was that a policewoman observed the driver's car alongside a second car at a certain moment, and the policewoman had already clocked the second car as going faster than the speed limit. The driver argued, "The second car was passing me. I was not speeding." The judge ruled against the driver because, in the judge's words, "If two cars were side by side, you were both speeding." If you were a lawyer representing the accused driver, how would you argue this case?

**Q2.9** Can you have a zero displacement and a nonzero average velocity? A nonzero velocity? Illustrate your answers on an $x$-$t$ graph.

**Q2.10** Can you have zero acceleration and nonzero velocity? Explain using a $v_x$-$t$ graph.

**Q2.11** Can you have zero velocity and nonzero average acceleration? Zero velocity and nonzero acceleration? Explain using a $v_x$-$t$ graph, and give an example of such motion.

**Q2.12** An automobile is traveling west. Can it have a velocity toward the west and at the same time have an acceleration toward the east? Under what circumstances?

**Q2.13** The official's truck in Fig. 2.2 is at $x_1 = 277$ m at $t_1 = 16.0$ s and is at $x_2 = 19$ m at $t_2 = 25.0$ s. (a) Sketch *two* different possible $x$-$t$ graphs for the motion of the truck. (b) Does the average velocity $v_{\text{av-}x}$ during the time interval from $t_1$ to $t_2$ have the same value for both of your graphs? Why or why not?

**Q2.14** Under constant acceleration the average velocity of a particle is half the sum of its initial and final velocities. Is this still true if the acceleration is *not* constant? Explain.

**Q2.15** You throw a baseball straight up in the air so that it rises to a maximum height much greater than your height. Is the magnitude of the acceleration greater while it is being thrown or after it leaves your hand? Explain.

**Q2.16** Prove these statements: (a) As long as you can neglect the effects of the air, if you throw anything vertically upward, it will have the same speed when it returns to the release point as when it was released. (b) The time of flight will be twice the time it takes to get to its highest point.

**Q2.17** A dripping water faucet steadily releases drops 1.0 s apart. As these drops fall, will the distance between them increase, decrease, or remain the same? Prove your answer.

**Q2.18** If the initial position and initial velocity of a vehicle are known and a record is kept of the acceleration at each instant, can you compute the vehicle's position after a certain time from these data? If so, explain how this might be done.

**Q2.19** From the top of a tall building you throw one ball straight up with speed $v_0$ and one ball straight down with speed $v_0$. (a) Which ball has the greater speed when it reaches the ground? (b) Which ball gets to the ground first? (c) Which ball has a greater displacement when it reaches the ground? (d) Which ball has traveled the greater distance when it hits the ground?

**Q2.20** A ball is dropped from rest from the top of a building of height $h$. At the same instant, a second ball is projected vertically upward from ground level, such that it has zero speed when it reaches the top of the building. When the two balls pass each other, which ball has the greater speed, or do they have the same speed? Explain. Where will the two balls be when they are alongside each other: at height $h/2$ above the ground, below this height, or above this height? Explain.

**Q2.21** An object is thrown straight up into the air and feels no air resistance. How is it possible for the object to have an acceleration when it has stopped moving at its highest point?

**Q2.22** When you drop an object from a certain height, it takes time $T$ to reach the ground with no air resistance. If you dropped it from three times that height, how long (in terms of $T$) would it take to reach the ground?

## EXERCISES

### Section 2.1 Displacement, Time, and Average Velocity

**2.1** • A car travels in the $+x$-direction on a straight and level road. For the first 4.00 s of its motion, the average velocity of the car is $v_{\text{av-}x} = 6.25$ m/s. How far does the car travel in 4.00 s?

**2.2** •• In an experiment, a shearwater (a seabird) was taken from its nest, flown 5150 km away, and released. The bird found its way back to its nest 13.5 days after release. If we place the origin in the nest and extend the $+x$-axis to the release point, what was the bird's average velocity in m/s (a) for the return flight, and (b) for the whole episode, from leaving the nest to returning?

**2.3** •• **Trip Home.** You normally drive on the freeway between San Diego and Los Angeles at an average speed of 105 km/h (65 mi/h), and the trip takes 2 h and 20 min. On a Friday afternoon, however, heavy traffic slows you down and you drive the same distance at an average speed of only 70 km/h (43 mi/h). How much longer does the trip take?

**2.4** •• **From Pillar to Post.** Starting from a pillar, you run 200 m east (the $+x$-direction) at an average speed of 5.0 m/s, and then run 280 m west at an average speed of 4.0 m/s to a post. Calculate (a) your average speed from pillar to post and (b) your average velocity from pillar to post.

**2.5** • Starting from the front door of your ranch house, you walk 60.0 m due east to your windmill, and then you turn around and slowly walk 40.0 m west to a bench where you sit and watch the sunrise. It takes you 28.0 s to walk from your house to the windmill and then 36.0 s to walk from the windmill to the bench. For the entire trip from your front door to the bench, what are (a) your average velocity and (b) your average speed?

**2.6** •• A Honda Civic travels in a straight line along a road. Its distance $x$ from a stop sign is given as a function of time $t$ by the equation $x(t) = \alpha t^2 - \beta t^3$, where $\alpha = 1.50\ \text{m/s}^2$ and $\beta = 0.0500\ \text{m/s}^3$. Calculate the average velocity of the car for each time interval: (a) $t = 0$ to $t = 2.00$ s; (b) $t = 0$ to $t = 4.00$ s; (c) $t = 2.00$ s to $t = 4.00$ s.

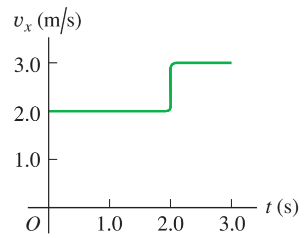
### Section 2.2 Instantaneous Velocity

**2.7** • **CALC** A car is stopped at a traffic light. It then travels along a straight road so that its distance from the light is given by $x(t) = bt^2 - ct^3$, where $b = 2.40\ \text{m/s}^2$ and $c = 0.120\ \text{m/s}^3$. (a) Calculate the average velocity of the car for the time interval $t = 0$ to $t = 10.0$ s. (b) Calculate the instantaneous velocity of the car at $t = 0$, $t = 5.0$ s, and $t = 10.0$ s. (c) How long after starting from rest is the car again at rest?

**2.8** • **CALC** A bird is flying due east. Its distance from a tall building is given by $x(t) = 28.0\ \text{m} + (12.4\ \text{m/s})t - (0.0450\ \text{m/s}^3)t^3$. What is the instantaneous velocity of the bird when $t = 8.00$ s?

**2.9** •• A ball moves in a straight line (the $x$-axis). The graph in Fig. E2.9 shows this ball's velocity as a function of time. (a) What are the ball's average speed and average velocity during the first 3.0 s? (b) Suppose that the ball moved in such a way that the graph segment after 2.0 s was $-3.0$ m/s instead of $+3.0$ m/s. Find the ball's average speed and average velocity in this case.

Figure **E2.9**




**2.10** • A physics professor leaves her house and walks along the sidewalk toward campus. After 5 min it starts to rain and she returns home. Her distance from her house as a function of time is shown in Fig. E2.10. At which of the labeled points is her velocity (a) zero? (b) constant and positive? (c) constant and negative? (d) increasing in magnitude? (e) decreasing in magnitude?

Figure **E2.10**

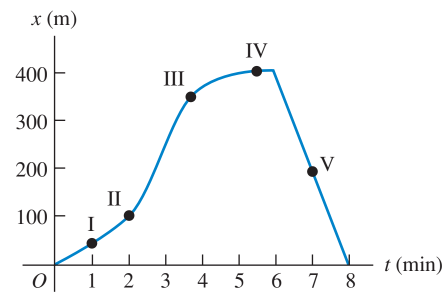
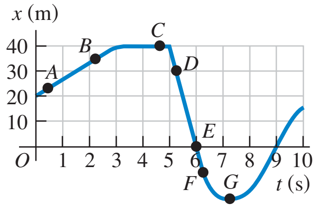


Figure **E2.11**

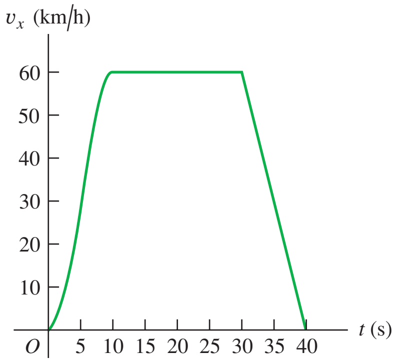



**2.11** •• A test car travels in a straight line along the $x$-axis. The graph in Fig. E2.11 shows the car's position $x$ as a function of time. Find its instantaneous velocity at points $A$ through $G$.

## Section 2.3 Average and Instantaneous Acceleration

**2.12** • Figure E2.12 shows the velocity of a solar-powered car as a function of time. The driver accelerates from a stop sign, cruises for 20 s at a constant speed of 60 km/h, and then brakes to come to a stop 40 s after leaving the stop sign. (a) Compute the average acceleration during the following time intervals: (i) $t = 0$ to $t = 10$ s; (ii) $t = 30$ s to $t = 40$ s; (iii) $t = 10$ s to $t = 30$ s; (iv) $t = 0$ to $t = 40$ s. (b) What is the instantaneous acceleration at $t = 20$ s and at $t = 35$ s?

Figure **E2.12**




**2.13** • **The Fastest (and Most Expensive) Car!** The table shows test data for the Bugatti Veyron, the fastest car made. The car is moving in a straight line (the $x$-axis).

| | | | | |
|---|---|---|---|---|
| Time (s) | 0 | 2.1 | 20.0 | 53 |
| Speed (mi/h) | 0 | 60 | 200 | 253 |

(a) Make a $v_x$-$t$ graph of this car's velocity (in mi/h) as a function of time. Is its acceleration constant? (b) Calculate the car's average acceleration (in m/s$^2$) between (i) 0 and 2.1 s; (ii) 2.1 s and 20.0 s; (iii) 20.0 s and 53 s. Are these results consistent with your graph in part (a)? (Before you decide to buy this car, it might be helpful to know that only 300 will be built, it runs out of gas in 12 minutes at top speed, and it costs $1.25 million!)

**2.14** •• **CALC** A race car starts from rest and travels east along a straight and level track. For the first 5.0 s of the car's motion, the eastward component of the car's velocity is given by $v_x(t) = (0.860\ \text{m/s}^3)t^2$. What is the acceleration of the car when $v_x = 16.0$ m/s?

**2.15** • **CALC** A turtle crawls along a straight line, which we will call the $x$-axis with the positive direction to the right. The equation for the turtle's position as a function of time is $x(t) = 50.0\ \text{cm} + (2.00\ \text{cm/s})t - (0.0625\ \text{cm/s}^2)t^2$. (a) Find the turtle's initial velocity, initial position, and initial acceleration. (b) At what time $t$ is the velocity of the turtle zero? (c) How long after starting does it take the turtle to return to its starting point? (d) At what times $t$ is the turtle a distance of 10.0 cm from its starting point? What is the velocity (magnitude and direction) of the turtle at each of these times? (e) Sketch graphs of $x$ versus $t$, $v_x$ versus $t$, and $a_x$ versus $t$, for the time interval $t = 0$ to $t = 40$ s.

**2.16** • An astronaut has left the International Space Station to test a new space scooter. Her partner measures the following velocity changes, each taking place in a 10-s interval. What are the magnitude, the algebraic sign, and the direction of the average acceleration in each interval? Assume that the positive direction is to the right. (a) At the beginning of the interval the astronaut is moving toward the right along the $x$-axis at 15.0 m/s, and at the end of the interval she is moving toward the right at 5.0 m/s. (b) At the beginning she is moving toward the left at 5.0 m/s, and at the end she is moving toward the left at 15.0 m/s. (c) At the beginning she is moving toward the right at 15.0 m/s, and at the end she is moving toward the left at 15.0 m/s.

**2.17** • **CALC** A car's velocity as a function of time is given by $v_x(t) = \alpha + \beta t^2$, where $\alpha = 3.00$ m/s and $\beta = 0.100$ m/s$^3$. (a) Calculate the average acceleration for the time interval $t = 0$ to $t = 5.00$ s. (b) Calculate the instantaneous acceleration for $t = 0$ and $t = 5.00$ s. (c) Draw $v_x$-$t$ and $a_x$-$t$ graphs for the car's motion between $t = 0$ and $t = 5.00$ s.

**2.18** •• **CALC** The position of the front bumper of a test car under microprocessor control is given by $x(t) = 2.17\ \text{m} + (4.80\ \text{m/s}^2)t^2 - (0.100\ \text{m/s}^6)t^6$. (a) Find its position and acceleration at the instants when the car has zero velocity. (b) Draw $x$-$t$, $v_x$-$t$, and $a_x$-$t$ graphs for the motion of the bumper between $t = 0$ and $t = 2.00$ s.

## Section 2.4 Motion with Constant Acceleration

**2.19** •• An antelope moving with constant acceleration covers the distance between two points 70.0 m apart in 7.00 s. Its speed as it passes the second point is 15.0 m/s. (a) What is its speed at the first point? (b) What is its acceleration?

**2.20** •• **BIO** **Blackout?** A jet fighter pilot wishes to accelerate from rest at a constant acceleration of $5g$ to reach Mach 3 (three times the speed of sound) as quickly as possible. Experimental tests reveal that he will black out if this acceleration lasts for more than 5.0 s. Use 331 m/s for the speed of sound. (a) Will the period of acceleration last long enough to cause him to black out? (b) What is the greatest speed he can reach with an acceleration of $5g$ before blacking out?

**2.21** • **A Fast Pitch.** The fastest measured pitched baseball left the pitcher's hand at a speed of 45.0 m/s. If the pitcher was in contact with the ball over a distance of 1.50 m and produced constant acceleration, (a) what acceleration did he give the ball, and (b) how much time did it take him to pitch it?

**2.22** •• **A Tennis Serve.** In the fastest measured tennis serve, the ball left the racquet at 73.14 m/s. A served tennis ball is typically in contact with the racquet for 30.0 ms and starts from rest. Assume constant acceleration. (a) What was the ball's acceleration during this serve? (b) How far did the ball travel during the serve?

**2.23** •• **BIO** **Automobile Airbags.** The human body can survive an acceleration trauma incident (sudden stop) if the magnitude of the acceleration is less than 250 m/s$^2$. If you are in an automobile accident with an initial speed of 105 km/h (65 mi/h) and you are stopped by an airbag that inflates from the dashboard, over what distance must the airbag stop you for you to survive the crash?

**2.24** • **BIO** If a pilot accelerates at more than $4g$, he begins to "gray out" but doesn't completely lose consciousness. (a) Assuming constant acceleration, what is the shortest time that a jet pilot starting from rest can take to reach Mach 4 (four times the speed of sound) without graying out? (b) How far would the plane travel during this period of acceleration? (Use 331 m/s for the speed of sound in cold air.)

**2.25** • **BIO** **Air-Bag Injuries.** During an auto accident, the vehicle's air bags deploy and slow down the passengers more gently than if they had hit the windshield or steering wheel. According to safety standards, the bags produce a maximum acceleration of $60g$ that lasts for only 36 ms (or less). How far (in meters) does a person travel in coming to a complete stop in 36 ms at a constant acceleration of $60g$?

**2.26** • **BIO** **Prevention of Hip Fractures.** Falls resulting in hip fractures are a major cause of injury and even death to the elderly. Typically, the hip's speed at impact is about 2.0 m/s. If this can be reduced to 1.3 m/s or less, the hip will usually not fracture. One way to do this is by wearing elastic hip pads. (a) If a typical pad is 5.0 cm thick and compresses by 2.0 cm during the impact of a fall, what constant acceleration (in $m/s^2$ and in $g$'s) does the hip undergo to reduce its speed from 2.0 m/s to 1.3 m/s? (b) The acceleration you found in part (a) may seem rather large, but to fully assess its effects on the hip, calculate how long it lasts.

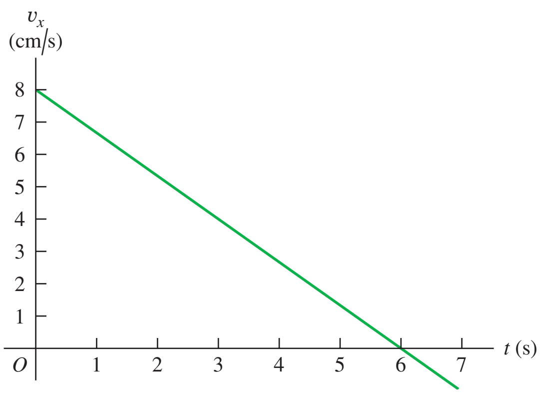
**2.27** • **BIO** **Are We Martians?** It has been suggested, and not facetiously, that life might have originated on Mars and been carried to the earth when a meteor hit Mars and blasted pieces of rock (perhaps containing primitive life) free of the surface. Astronomers know that many Martian rocks have come to the earth this way. (For information on one of these, search the Internet for "ALH 84001.") One objection to this idea is that microbes would have to undergo an enormous lethal acceleration during the impact. Let us investigate how large such an acceleration might be. To escape Mars, rock fragments would have to reach its escape velocity of 5.0 km/s, and this would most likely happen over a distance of about 4.0 m during the meteor impact. (a) What would be the acceleration (in $m/s^2$ and $g$'s) of such a rock fragment, if the acceleration is constant? (b) How long would this acceleration last? (c) In tests, scientists have found that over 40% of *Bacillius subtilis* bacteria survived after an acceleration of $450{,}000g$. In light of your answer to part (a), can we rule out the hypothesis that life might have been blasted from Mars to the earth?

**2.28** • **Entering the Freeway.** A car sits in an entrance ramp to a freeway, waiting for a break in the traffic. The driver accelerates with constant acceleration along the ramp and onto the freeway. The car starts from rest, moves in a straight line, and has a speed of 20 m/s (45 mi/h) when it reaches the end of the 120-m-long ramp. (a) What is the acceleration of the car? (b) How much time does it take the car to travel the length of the ramp? (c) The traffic on the freeway is moving at a constant speed of 20 m/s. What distance does the traffic travel while the car is moving the length of the ramp?

**2.29** •• **Launch of the Space Shuttle.** At launch the space shuttle weighs 4.5 million pounds. When it is launched from rest, it takes 8.00 s to reach 161 km/h, and at the end of the first 1.00 min its speed is 1610 km/h. (a) What is the average acceleration (in $m/s^2$) of the shuttle (i) during the first 8.00 s, and (ii) between 8.00 s and the end of the first 1.00 min? (b) Assuming the acceleration is constant during each time interval (but not necessarily the same in both intervals), what distance does the shuttle travel (i) during the first 8.00 s, and (ii) during the interval from 8.00 s to 1.00 min?

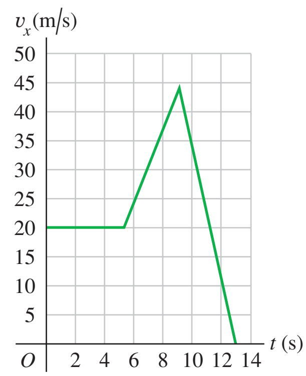
**2.30** •• A cat walks in a straight line, which we shall call the $x$-axis with the positive direction to the right. As an observant physicist, you make measurements of this cat's motion and construct a graph of the feline's velocity as a function of time (Fig. E2.30). (a) Find the cat's velocity at $t = 4.0$ s and at $t = 7.0$ s. (b) What is the cat's acceleration at $t = 3.0$ s? At $t = 6.0$ s? At $t = 7.0$ s? (c) What distance does the cat move during the first 4.5 s? From $t = 0$ to $t = 7.5$ s? (d) Sketch clear graphs of the cat's acceleration and position as functions of time, assuming that the cat started at the origin.

Figure **E2.30**



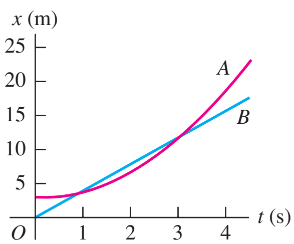

**2.31** •• The graph in Fig. E2.31 shows the velocity of a motorcycle police officer plotted as a function of time. (a) Find the instantaneous acceleration at $t = 3$ s, at $t = 7$ s, and at $t = 11$ s. (b) How far does the officer go in the first 5 s? The first 9 s? The first 13 s?

Figure **E2.31**




**2.32** • Two cars, $A$ and $B$, move along the $x$-axis. Figure E2.32 is a graph of the positions of $A$ and $B$ versus time. (a) In motion diagrams (like Figs. 2.13b and 2.14b), show the position, velocity, and acceleration of each of the two cars at $t = 0$, $t = 1$ s, and $t = 3$ s. (b) At what time(s), if any, do $A$ and $B$ have the same position? (c) Graph velocity versus time for both $A$ and $B$. (d) At what time(s), if any, do $A$ and $B$ have the same velocity? (e) At what time(s), if any, does car $A$ pass car $B$? (f) At what time(s), if any, does car $B$ pass car $A$?

Figure **E2.32**



**2.33** •• **Mars Landing.** In January 2004, NASA landed exploration vehicles on Mars. Part of the descent consisted of the following stages:

*Stage A:* Friction with the atmosphere reduced the speed from 19,300 km/h to 1600 km/h in 4.0 min.
*Stage B:* A parachute then opened to slow it down to 321 km/h in 94 s.
*Stage C:* Retro rockets then fired to reduce its speed to zero over a distance of 75 m.

Assume that each stage followed immediately after the preceding one and that the acceleration during each stage was constant. (a) Find the rocket's acceleration (in $m/s^2$) during each stage. (b) What total distance (in km) did the rocket travel during stages A, B, and C?

**2.34** • At the instant the traffic light turns green, a car that has been waiting at an intersection starts ahead with a constant acceleration of $3.20\ m/s^2$. At the same instant a truck, traveling with a constant speed of 20.0 m/s, overtakes and passes the car. (a) How far beyond its starting point does the car overtake the truck? (b) How fast is the car traveling when it overtakes the truck? (c) Sketch an $x$-$t$ graph of the motion of both vehicles. Take $x = 0$ at the intersection. (d) Sketch a $v_x$-$t$ graph of the motion of both vehicles.

## Section 2.5 Freely Falling Bodies

**2.35** •• (a) If a flea can jump straight up to a height of 0.440 m, what is its initial speed as it leaves the ground? (b) How long is it in the air?

**2.36** •• A small rock is thrown vertically upward with a speed of 18.0 m/s from the edge of the roof of a 30.0-m-tall building. The rock doesn't hit the building on its way back down and lands in the street below. Air resistance can be neglected. (a) What is the speed of the rock just before it hits the street? (b) How much time elapses from when the rock is thrown until it hits the street?

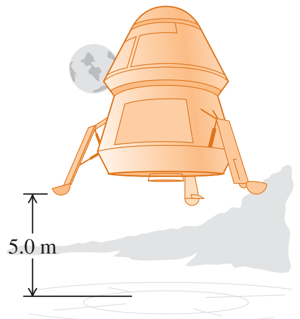
**2.37** • A juggler throws a bowling pin straight up with an initial speed of 8.20 m/s. How much time elapses until the bowling pin returns to the juggler's hand?

**2.38** •• You throw a glob of putty straight up toward the ceiling, which is 3.60 m above the point where the putty leaves your hand. The initial speed of the putty as it leaves your hand is 9.50 m/s. (a) What is the speed of the putty just before it strikes the ceiling? (b) How much time from when it leaves your hand does it take the putty to reach the ceiling?

**2.39** •• A tennis ball on Mars, where the acceleration due to gravity is $0.379g$ and air resistance is negligible, is hit directly upward and returns to the same level 8.5 s later. (a) How high above its original point did the ball go? (b) How fast was it moving just after being hit? (c) Sketch graphs of the ball's vertical position, vertical velocity, and vertical acceleration as functions of time while it's in the Martian air.

**2.40** •• **Touchdown on the Moon.** A lunar lander is making its descent to Moon Base I (Fig. E2.40). The lander descends slowly under the retro-thrust of its descent engine. The engine is cut off when the lander is 5.0 m above the surface and has a downward speed of 0.8 m/s. With the engine off, the lander is in free fall. What is the speed of the lander just before it touches the surface? The acceleration due to gravity on the moon is $1.6\ m/s^2$.

Figure **E2.40**



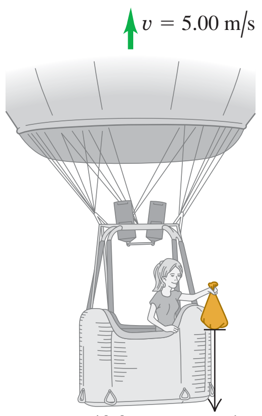

**2.41** •• **A Simple Reaction-Time Test.** A meter stick is held vertically above your hand, with the lower end between your thumb and first finger. On seeing the meter stick released, you grab it with these two fingers. You can calculate your reaction time from the distance the meter stick falls, read directly from the point where your fingers grabbed it. (a) Derive a relationship for your reaction time in terms of this measured distance, $d$. (b) If the measured distance is 17.6 cm, what is the reaction time?

**2.42** •• A brick is dropped (zero initial speed) from the roof of a building. The brick strikes the ground in 2.50 s. You may ignore air resistance, so the brick is in free fall. (a) How tall, in meters, is the building? (b) What is the magnitude of the brick's velocity just before it reaches the ground? (c) Sketch $a_y$-$t$, $v_y$-$t$, and $y$-$t$ graphs for the motion of the brick.

**2.43** •• **Launch Failure.** A 7500-kg rocket blasts off vertically from the launch pad with a constant upward acceleration of $2.25\ m/s^2$ and feels no appreciable air resistance. When it has reached a height of 525 m, its engines suddenly fail so that the only force acting on it is now gravity. (a) What is the maximum height this rocket will reach above the launch pad? (b) How much time after engine failure will elapse before the rocket comes crashing down to the launch pad, and how fast will it be moving just before it crashes? (c) Sketch $a_y$-$t$, $v_y$-$t$, and $y$-$t$ graphs of the rocket's motion from the instant of blast-off to the instant just before it strikes the launch pad.

**2.44** •• A hot-air balloonist, rising vertically with a constant velocity of magnitude 5.00 m/s, releases a sandbag at an instant when the balloon is 40.0 m above the ground (Fig. E2.44). After it is released, the sandbag is in free fall. (a) Compute the position and velocity of the sandbag at 0.250 s and 1.00 s after its release. (b) How many seconds after its release will the bag strike the ground? (c) With what magnitude of velocity does it strike the ground? (d) What is the greatest height above the ground that the sandbag reaches? (e) Sketch $a_y$-$t$, $v_y$-$t$, and $y$-$t$ graphs for the motion.

Figure **E2.44**




**2.45** • **BIO** The rocket-driven sled *Sonic Wind No. 2,* used for investigating the physiological effects of large accelerations, runs on a straight, level track 1070 m (3500 ft) long. Starting from rest, it can reach a speed of 224 m/s (500 mi/h) in 0.900 s. (a) Compute the acceleration in $m/s^2$, assuming that it is constant. (b) What is the ratio of this acceleration to that of a freely falling body ($g$)? (c) What distance is covered in 0.900 s? (d) A magazine article states that at the end of a certain run, the speed of the sled decreased from 283 m/s (632 mi/h) to zero in 1.40 s and that during this time the magnitude of the acceleration was greater than $40g$. Are these figures consistent?

**2.46** • An egg is thrown nearly vertically upward from a point near the cornice of a tall building. It just misses the cornice on the way down and passes a point 30.0 m below its starting point 5.00 s after it leaves the thrower's hand. Air resistance may be ignored.

(a) What is the initial speed of the egg? (b) How high does it rise above its starting point? (c) What is the magnitude of its velocity at the highest point? (d) What are the magnitude and direction of its acceleration at the highest point? (e) Sketch $a_y$-$t$, $v_y$-$t$, and $y$-$t$ graphs for the motion of the egg.

**2.47** •• A 15-kg rock is dropped from rest on the earth and reaches the ground in 1.75 s. When it is dropped from the same height on Saturn's satellite Enceladus, it reaches the ground in 18.6 s. What is the acceleration due to gravity on Enceladus?

**2.48** • A large boulder is ejected vertically upward from a volcano with an initial speed of 40.0 m/s. Air resistance may be ignored. (a) At what time after being ejected is the boulder moving at 20.0 m/s upward? (b) At what time is it moving at 20.0 m/s downward? (c) When is the displacement of the boulder from its initial position zero? (d) When is the velocity of the boulder zero? (e) What are the magnitude and direction of the acceleration while the boulder is (i) moving upward? (ii) Moving downward? (iii) At the highest point? (f) Sketch $a_y$-$t$, $v_y$-$t$, and $y$-$t$ graphs for the motion.

**2.49** •• Two stones are thrown vertically upward from the ground, one with three times the initial speed of the other. (a) If the faster stone takes 10 s to return to the ground, how long will it take the slower stone to return? (b) If the slower stone reaches a maximum height of $H$, how high (in terms of $H$) will the faster stone go? Assume free fall.

## Section 2.6 Velocity and Position by Integration

**2.50** • **CALC** For constant $a_x$, use Eqs. (2.17) and (2.18) to find $v_x$ and $x$ as functions of time. Compare your results to Eqs. (2.8) and (2.12).

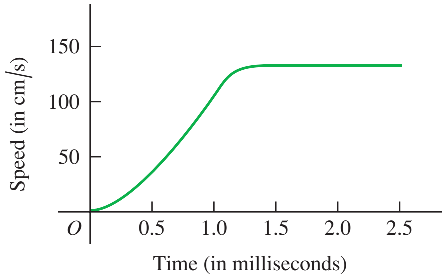
**2.51** • **CALC** A rocket starts from rest and moves upward from the surface of the earth. For the first 10.0 s of its motion, the vertical acceleration of the rocket is given by $a_y = (2.80\ \text{m/s}^3)t$, where the $+y$-direction is upward. (a) What is the height of the rocket above the surface of the earth at $t = 10.0$ s? (b) What is the speed of the rocket when it is 325 m above the surface of the earth?

**2.52** •• **CALC** The acceleration of a bus is given by $a_x(t) = \alpha t$, where $\alpha = 1.2\ \text{m/s}^3$. (a) If the bus's velocity at time $t = 1.0$ s is 5.0 m/s, what is its velocity at time $t = 2.0$ s? (b) If the bus's position at time $t = 1.0$ s is 6.0 m, what is its position at time $t = 2.0$ s? (c) Sketch $a_x$-$t$, $v_x$-$t$, and $x$-$t$ graphs for the motion.

**2.53** •• **CALC** The acceleration of a motorcycle is given by $a_x(t) = At - Bt^2$, where $A = 1.50\ \text{m/s}^3$ and $B = 0.120\ \text{m/s}^4$. The motorcycle is at rest at the origin at time $t = 0$. (a) Find its position and velocity as functions of time. (b) Calculate the maximum velocity it attains.

**2.54** •• **BIO** **Flying Leap of the Flea.** High-speed motion pictures (3500 frames/second) of a jumping, 210-$\mu$g flea yielded the data used to plot the graph given in Fig. E2.54. (See "The Flying Leap of the Flea" by M. Rothschild, Y. Schlein, K. Parker, C. Neville, and S. Sternberg in the November 1973 *Scientific American.*) This flea was about 2 mm long and jumped at a nearly vertical takeoff angle. Use the graph to answer the questions. (a) Is the acceleration of the flea ever zero? If so, when? Justify your answer. (b) Find the maximum height the flea reached in the first 2.5 ms. (c) Find the flea's acceleration at 0.5 ms, 1.0 ms, and 1.5 ms. (d) Find the flea's height at 0.5 ms, 1.0 ms, and 1.5 ms.

Figure **E2.54**




## PROBLEMS

**2.55** • **BIO** A typical male sprinter can maintain his maximum acceleration for 2.0 s and his maximum speed is 10 m/s. After reaching this maximum speed, his acceleration becomes zero and then he runs at constant speed. Assume that his acceleration is constant during the first 2.0 s of the race, that he starts from rest, and that he runs in a straight line. (a) How far has the sprinter run when he reaches his maximum speed? (b) What is the magnitude of his average velocity for a race of the following lengths: (i) 50.0 m, (ii) 100.0 m, (iii) 200.0 m?

**2.56** •• On a 20-mile bike ride, you ride the first 10 miles at an average speed of 8 mi/h. What must your average speed over the next 10 miles be to have your average speed for the total 20 miles be (a) 4 mi/h? (b) 12 mi/h? (c) Given this average speed for the first 10 miles, can you possibly attain an average speed of 16 mi/h for the total 20-mile ride? Explain.

**2.57** •• **CALC** The position of a particle between $t = 0$ and $t = 2.00$ s is given by $x(t) = (3.00\ \text{m/s}^3)t^3 - (10.0\ \text{m/s}^2)t^2 + (9.00\ \text{m/s})t$. (a) Draw the $x$-$t$, $v_x$-$t$, and $a_x$-$t$ graphs of this particle. (b) At what time(s) between $t = 0$ and $t = 2.00$ s is the particle instantaneously at rest? Does your numerical result agree with the $v_x$-$t$ graph in part (a)? (c) At each time calculated in part (b), is the acceleration of the particle positive or negative? Show that in each case the same answer is deduced from $a_x(t)$ and from the $v_x$-$t$ graph. (d) At what time(s) between $t = 0$ and $t = 2.00$ s is the velocity of the particle instantaneously not changing? Locate this point on the $v_x$-$t$ and $a_x$-$t$ graphs of part (a). (e) What is the particle's greatest distance from the origin $(x = 0)$ between $t = 0$ and $t = 2.00$ s? (f) At what time(s) between $t = 0$ and $t = 2.00$ s is the particle *speeding up* at the greatest rate? At what time(s) between $t = 0$ and $t = 2.00$ s is the particle *slowing down* at the greatest rate? Locate these points on the $v_x$-$t$ and $a_x$-$t$ graphs of part (a).

**2.58** •• **CALC** A lunar lander is descending toward the moon's surface. Until the lander reaches the surface, its height above the surface of the moon is given by $y(t) = b - ct + dt^2$, where $b = 800$ m is the initial height of the lander above the surface, $c = 60.0$ m/s, and $d = 1.05\ \text{m/s}^2$. (a) What is the initial velocity of the lander, at $t = 0$? (b) What is the velocity of the lander just before it reaches the lunar surface?

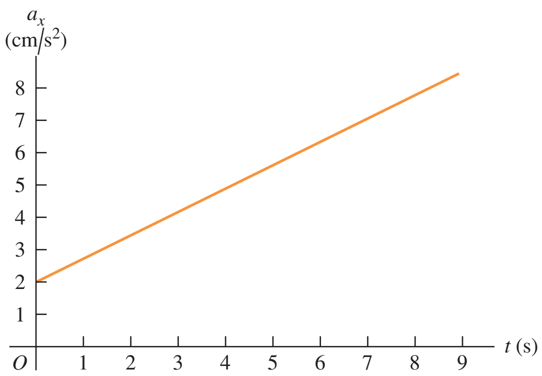
**2.59** ••• **Earthquake Analysis.** Earthquakes produce several types of shock waves. The most well known are the P-waves (P for *primary* or *pressure*) and the S-waves (S for *secondary* or *shear*). In the earth's crust, the P-waves travel at around 6.5 km/s, while the S-waves move at about 3.5 km/s. The actual speeds vary depending on the type of material they are going through. The time delay between the arrival of these two waves at a seismic recording station tells geologists how far away the earthquake occurred. If the time delay is 33 s, how far from the seismic station did the earthquake occur?

**2.60** •• **Relay Race.** In a relay race, each contestant runs 25.0 m while carrying an egg balanced on a spoon, turns around, and comes back to the starting point. Edith runs the first 25.0 m in 20.0 s. On the return trip she is more confident and takes only 15.0 s. What is the magnitude of her average velocity for (a) the first 25.0 m? (b) The return trip? (c) What is her average velocity for the entire round trip? (d) What is her average speed for the round trip?

**2.61** ••• A rocket carrying a satellite is accelerating straight up from the earth's surface. At 1.15 s after liftoff, the rocket clears the top of its launch platform, 63 m above the ground. After an additional 4.75 s, it is 1.00 km above the ground. Calculate the magnitude of the average velocity of the rocket for (a) the 4.75-s part of its flight and (b) the first 5.90 s of its flight.

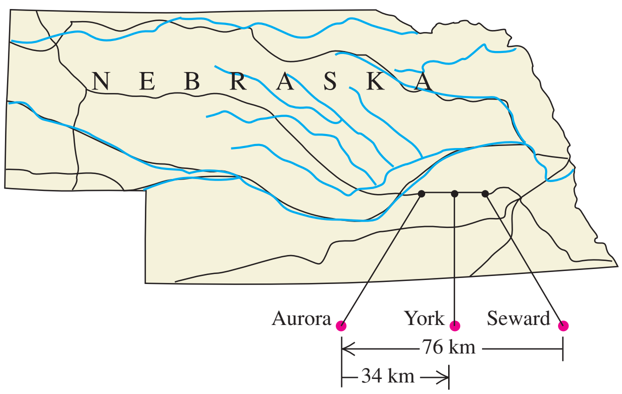
**2.62** ••• The graph in Fig. P2.62 describes the acceleration as a function of time for a stone rolling down a hill starting from rest. (a) Find the stone's velocity at $t = 2.5$ s and at $t = 7.5$ s. (b) Sketch a graph of the stone's velocity as a function of time.

Figure **P2.62**




**2.63** •• Dan gets on Interstate Highway I–80 at Seward, Nebraska, and drives due west in a straight line and at an average velocity of magnitude 88 km/h. After traveling 76 km, he reaches the Aurora exit (Fig. P2.63). Realizing he has gone too far, he turns around and drives due east 34 km back to the York exit at an average velocity of magnitude 72 km/h. For his whole trip from Seward to the York exit, what are (a) his average speed and (b) the magnitude of his average velocity?

Figure **P2.63**



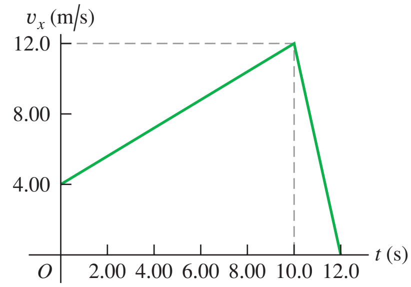

**2.64** ••• A subway train starts from rest at a station and accelerates at a rate of 1.60 m/s$^2$ for 14.0 s. It runs at constant speed for 70.0 s and slows down at a rate of 3.50 m/s$^2$ until it stops at the next station. Find the *total* distance covered.

**2.65** •• A world-class sprinter accelerates to his maximum speed in 4.0 s. He then maintains this speed for the remainder of a 100-m race, finishing with a total time of 9.1 s. (a) What is the runner's average acceleration during the first 4.0 s? (b) What is his average acceleration during the last 5.1 s? (c) What is his average acceleration for the entire race? (d) Explain why your answer to part (c) is not the average of the answers to parts (a) and (b).

**2.66** •• A sled starts from rest at the top of a hill and slides down with a constant acceleration. At some later time the sled is 14.4 m from the top, 2.00 s after that it is 25.6 m from the top, 2.00 s later 40.0 m from the top, and 2.00 s later it is 57.6 m from the top. (a) What is the magnitude of the average velocity of the sled during each of the 2.00-s intervals after passing the 14.4-m point? (b) What is the acceleration of the sled? (c) What is the speed of the sled when it passes the 14.4-m point? (d) How much time did it take to go from the top to the 14.4-m point? (e) How far did the sled go during the first second after passing the 14.4-m point?

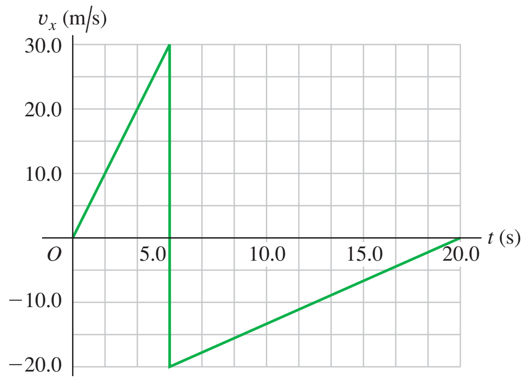
**2.67** • A gazelle is running in a straight line (the $x$-axis). The graph in Fig. P2.67 shows this animal's velocity as a function of time. During the first 12.0 s, find (a) the total distance moved and (b) the displacement of the gazelle. (c) Sketch an $a_x$-$t$ graph showing this gazelle's acceleration as a function of time for the first 12.0 s.

Figure **P2.67**




**2.68** • A rigid ball traveling in a straight line (the $x$-axis) hits a solid wall and suddenly rebounds during a brief instant. The $v_x$-$t$ graph in Fig. P2.68 shows this ball's velocity as a function of time. During the first 20.0 s of its motion, find (a) the total distance the ball moves and (b) its displacement. (c) Sketch a graph of $a_x$-$t$ for this ball's motion. (d) Is the graph shown really vertical at 5.00 s? Explain.

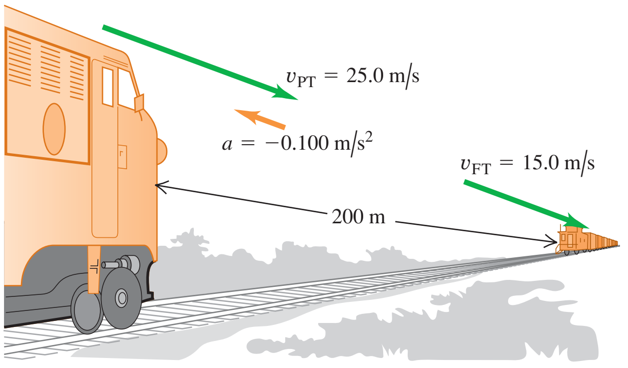
Figure **P2.68**




**2.69** ••• A ball starts from rest and rolls down a hill with uniform acceleration, traveling 150 m during the second 5.0 s of its motion. How far did it roll during the first 5.0 s of motion?

**2.70** •• **Collision.** The engineer of a passenger train traveling at 25.0 m/s sights a freight train whose caboose is 200 m ahead on

Figure **P2.70**




the same track (Fig. P2.70). The freight train is traveling at 15.0 m/s in the same direction as the passenger train. The engineer of the passenger train immediately applies the brakes, causing a constant acceleration of 0.100 $m/s^2$ in a direction opposite to the train's velocity, while the freight train continues with constant speed. Take $x = 0$ at the location of the front of the passenger train when the engineer applies the brakes. (a) Will the cows nearby witness a collision? (b) If so, where will it take place? (c) On a single graph, sketch the positions of the front of the passenger train and the back of the freight train.

**2.71** ••• Large cockroaches can run as fast as 1.50 m/s in short bursts. Suppose you turn on the light in a cheap motel and see one scurrying directly away from you at a constant 1.50 m/s. If you start 0.90 m behind the cockroach with an initial speed of 0.80 m/s toward it, what minimum constant acceleration would you need to catch up with it when it has traveled 1.20 m, just short of safety under a counter?

**2.72** •• Two cars start 200 m apart and drive toward each other at a steady 10 m/s. On the front of one of them, an energetic grasshopper jumps back and forth between the cars (he has strong legs!) with a constant horizontal velocity of 15 m/s relative to the ground. The insect jumps the instant he lands, so he spends no time resting on either car. What total distance does the grasshopper travel before the cars hit?

**2.73** • An automobile and a truck start from rest at the same instant, with the automobile initially at some distance behind the truck. The truck has a constant acceleration of 2.10 $m/s^2$, and the automobile an acceleration of 3.40 $m/s^2$. The automobile overtakes the truck after the truck has moved 40.0 m. (a) How much time does it take the automobile to overtake the truck? (b) How far was the automobile behind the truck initially? (c) What is the speed of each when they are abreast? (d) On a single graph, sketch the position of each vehicle as a function of time. Take $x = 0$ at the initial location of the truck.

**2.74** ••• Two stunt drivers drive directly toward each other. At time $t = 0$ the two cars are a distance $D$ apart, car 1 is at rest, and car 2 is moving to the left with speed $v_0$. Car 1 begins to move at $t = 0$, speeding up with a constant acceleration $a_x$. Car 2 continues to move with a constant velocity. (a) At what time do the two cars collide? (b) Find the speed of car 1 just before it collides with car 2. (c) Sketch $x$-$t$ and $v_x$-$t$ graphs for car 1 and car 2. For each of the two graphs, draw the curves for both cars on the same set of axes.

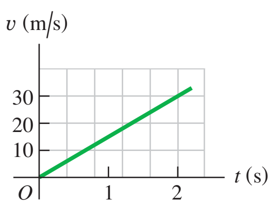
**2.75** •• A marble is released from one rim of a hemispherical bowl of diameter 50.0 cm and rolls down and up to the opposite rim in 10.0 s. Find (a) the average speed and (b) the average velocity of the marble.

**2.76** •• **CALC** An object's velocity is measured to be $v_x(t) = \alpha - \beta t^2$, where $\alpha = 4.00$ m/s and $\beta = 2.00\ m/s^3$. At $t = 0$ the object is at $x = 0$. (a) Calculate the object's position and acceleration as functions of time. (b) What is the object's maximum *positive* displacement from the origin?

**2.77** •• **Passing.** The driver of a car wishes to pass a truck that is traveling at a constant speed of 20.0 m/s (about 45 mi/h). Initially, the car is also traveling at 20.0 m/s and its front bumper is 24.0 m behind the truck's rear bumper. The car accelerates at a constant 0.600 $m/s^2$, then pulls back into the truck's lane when the rear of the car is 26.0 m ahead of the front of the truck. The car is 4.5 m long and the truck is 21.0 m long. (a) How much time is required for the car to pass the truck? (b) What distance does the car travel during this time? (c) What is the final speed of the car?

**2.78** •• On Planet X, you drop a 25-kg stone from rest and measure its speed at various times. Then you use the data you obtained to construct a graph of its speed $v$ as a function of time $t$ (Fig. P2.78). From the information in the graph, answer the following questions: (a) What is $g$ on Planet X? (b) An astronaut drops a piece of equipment from rest out of the landing module, 3.5 m above the surface of Planet X. How long will it take this equipment to reach the ground, and how fast will it be moving when it gets there? (c) How fast would an astronaut have to project an object straight upward to reach a height of 18.0 m above the release point, and how long would it take to reach that height?

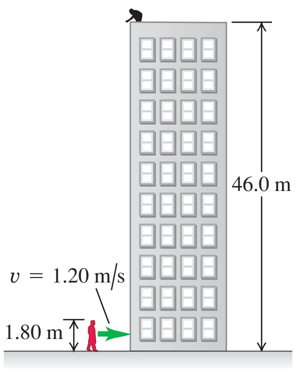
Figure **P2.78**




**2.79** ••• **CALC** The acceleration of a particle is given by $a_x(t) = -2.00\ m/s^2 + (3.00\ m/s^3)t$. (a) Find the initial velocity $v_{0x}$ such that the particle will have the same $x$-coordinate at $t = 4.00$ s as it had at $t = 0$. (b) What will be the velocity at $t = 4.00$ s?

**2.80** • **Egg Drop.** You are on the roof of the physics building, 46.0 m above the ground (Fig. P2.80). Your physics professor, who is 1.80 m tall, is walking alongside the building at a constant speed of 1.20 m/s. If you wish to drop an egg on your professor's head, where should the professor be when you release the egg? Assume that the egg is in free fall.

Figure **P2.80**




**2.81** • A certain volcano on earth can eject rocks vertically to a maximum height $H$. (a) How high (in terms of $H$) would these rocks go if a volcano on Mars ejected them with the same initial velocity? The acceleration due to gravity on Mars is 3.71 $m/s^2$, and you can neglect air resistance on both planets. (b) If the rocks are in the air for a time $T$ on earth, for how long (in terms of $T$) will they be in the air on Mars?

**2.82** •• An entertainer juggles balls while doing other activities. In one act, she throws a ball vertically upward, and while it is in the air, she runs to and from a table 5.50 m away at a constant speed of 2.50 m/s, returning just in time to catch the falling ball. (a) With what minimum initial speed must she throw the ball upward to accomplish this feat? (b) How high above its initial position is the ball just as she reaches the table?

**2.83** • Visitors at an amusement park watch divers step off a platform 21.3 m (70 ft) above a pool of water. According to the announcer, the divers enter the water at a speed of 56 mi/h (25 m/s). Air resistance may be ignored. (a) Is the announcer correct in this claim? (b) Is it possible for a diver to leap directly upward off the board so that, missing the board on the way down, she enters the water at 25.0 m/s? If so, what initial upward speed is required? Is the required initial speed physically attainable?

**2.84** ••• A flowerpot falls off a windowsill and falls past the window below. You may ignore air resistance. It takes the pot 0.420 s to pass from the top to the bottom of this window, which is 1.90 m high. How far is the top of the window below the windowsill from which the flowerpot fell?

**2.85** ••• **Look Out Below.** Sam heaves a 16-lb shot straight upward, giving it a constant upward acceleration from rest of $35.0 \text{ m/s}^2$ for 64.0 cm. He releases it 2.20 m above the ground. You may ignore air resistance. (a) What is the speed of the shot when Sam releases it? (b) How high above the ground does it go? (c) How much time does he have to get out of its way before it returns to the height of the top of his head, 1.83 m above the ground?

**2.86** ••• **A Multistage Rocket.** In the first stage of a two-stage rocket, the rocket is fired from the launch pad starting from rest but with a constant acceleration of $3.50 \text{ m/s}^2$ upward. At 25.0 s after launch, the second stage fires for 10.0 s, which boosts the rocket's velocity to 132.5 m/s upward at 35.0 s after launch. This firing uses up all the fuel, however, so after the second stage has finished firing, the only force acting on the rocket is gravity. Air resistance can be neglected. (a) Find the maximum height that the stage-two rocket reaches above the launch pad. (b) How much time after the end of the stage-two firing will it take for the rocket to fall back to the launch pad? (c) How fast will the stage-two rocket be moving just as it reaches the launch pad?

**2.87** •• **Juggling Act.** A juggler performs in a room whose ceiling is 3.0 m above the level of his hands. He throws a ball upward so that it just reaches the ceiling. (a) What is the initial velocity of the ball? (b) What is the time required for the ball to reach the ceiling? At the instant when the first ball is at the ceiling, the juggler throws a second ball upward with two-thirds the initial velocity of the first. (c) How long after the second ball is thrown do the two balls pass each other? (d) At what distance above the juggler's hand do they pass each other?

**2.88** •• A physics teacher performing an outdoor demonstration suddenly falls from rest off a high cliff and simultaneously shouts "Help." When she has fallen for 3.0 s, she hears the echo of her shout from the valley floor below. The speed of sound is 340 m/s. (a) How tall is the cliff? (b) If air resistance is neglected, how fast will she be moving just before she hits the ground? (Her actual speed will be less than this, due to air resistance.)

**2.89** ••• A helicopter carrying Dr. Evil takes off with a constant upward acceleration of $5.0 \text{ m/s}^2$. Secret agent Austin Powers jumps on just as the helicopter lifts off the ground. After the two men struggle for 10.0 s, Powers shuts off the engine and steps out of the helicopter. Assume that the helicopter is in free fall after its engine is shut off, and ignore the effects of air resistance. (a) What is the maximum height above ground reached by the helicopter? (b) Powers deploys a jet pack strapped on his back 7.0 s after leaving the helicopter, and then he has a constant downward acceleration with magnitude $2.0 \text{ m/s}^2$. How far is Powers above the ground when the helicopter crashes into the ground?

**2.90** •• **Cliff Height.** You are climbing in the High Sierra where you suddenly find yourself at the edge of a fog-shrouded cliff. To find the height of this cliff, you drop a rock from the top and 10.0 s later hear the sound of it hitting the ground at the foot of the cliff. (a) Ignoring air resistance, how high is the cliff if the speed of sound is 330 m/s? (b) Suppose you had ignored the time it takes the sound to reach you. In that case, would you have overestimated or underestimated the height of the cliff? Explain your reasoning.

**2.91** ••• **Falling Can.** A painter is standing on scaffolding that is raised at constant speed. As he travels upward, he accidentally nudges a paint can off the scaffolding and it falls 15.0 m to the ground. You are watching, and measure with your stopwatch that it takes 3.25 s for the can to reach the ground. Ignore air resistance. (a) What is the speed of the can just before it hits the ground? (b) Another painter is standing on a ledge, with his hands 4.00 m above the can when it falls off. He has lightning-fast reflexes and if the can passes in front of him, he can catch it. Does he get the chance?

**2.92** •• Determined to test the law of gravity for himself, a student walks off a skyscraper 180 m high, stopwatch in hand, and starts his free fall (zero initial velocity). Five seconds later, Superman arrives at the scene and dives off the roof to save the student. Superman leaves the roof with an initial speed $v_0$ that he produces by pushing himself downward from the edge of the roof with his legs of steel. He then falls with the same acceleration as any freely falling body. (a) What must the value of $v_0$ be so that Superman catches the student just before they reach the ground? (b) On the same graph, sketch the positions of the student and of Superman as functions of time. Take Superman's initial speed to have the value calculated in part (a). (c) If the height of the skyscraper is less than some minimum value, even Superman can't reach the student before he hits the ground. What is this minimum height?

**2.93** ••• During launches, rockets often discard unneeded parts. A certain rocket starts from rest on the launch pad and accelerates upward at a steady $3.30 \text{ m/s}^2$. When it is 235 m above the launch pad, it discards a used fuel canister by simply disconnecting it. Once it is disconnected, the only force acting on the canister is gravity (air resistance can be ignored). (a) How high is the rocket when the canister hits the launch pad, assuming that the rocket does not change its acceleration? (b) What total distance did the canister travel between its release and its crash onto the launch pad?

**2.94** •• A ball is thrown straight up from the ground with speed $v_0$. At the same instant, a second ball is dropped from rest from a height $H$, directly above the point where the first ball was thrown upward. There is no air resistance. (a) Find the time at which the two balls collide. (b) Find the value of $H$ in terms of $v_0$ and $g$ so that at the instant when the balls collide, the first ball is at the highest point of its motion.

**2.95** • **CALC** Two cars, $A$ and $B$, travel in a straight line. The distance of $A$ from the starting point is given as a function of time by $x_A(t) = \alpha t + \beta t^2$, with $\alpha = 2.60 \text{ m/s}$ and $\beta = 1.20 \text{ m/s}^2$. The distance of $B$ from the starting point is $x_B(t) = \gamma t^2 - \delta t^3$, with $\gamma = 2.80 \text{ m/s}^2$ and $\delta = 0.20 \text{ m/s}^3$. (a) Which car is ahead just after they leave the starting point? (b) At what time(s) are the cars at the same point? (c) At what time(s) is the distance from $A$ to $B$ neither increasing nor decreasing? (d) At what time(s) do $A$ and $B$ have the same acceleration?

## CHALLENGE PROBLEMS

**2.96** ••• In the vertical jump, an athlete starts from a crouch and jumps upward to reach as high as possible. Even the best athletes spend little more than 1.00 s in the air (their "hang time"). Treat the athlete as a particle and let $y_{max}$ be his maximum height above the floor. To explain why he seems to hang in the air, calculate the ratio of the time he is above $y_{max}/2$ to the time it takes him to go from the floor to that height. You may ignore air resistance.

**2.97** ••• **Catching the Bus.** A student is running at her top speed of 5.0 m/s to catch a bus, which is stopped at the bus stop. When the student is still 40.0 m from the bus, it starts to pull away, moving with a constant acceleration of $0.170 \text{ m/s}^2$. (a) For how much time and what distance does the student have to run at 5.0 m/s before she overtakes the bus? (b) When she reaches the bus, how fast is the bus traveling? (c) Sketch an *x-t* graph for both the student and the bus. Take $x = 0$ at the initial position of the student. (d) The equations you used in part (a) to find the time have a second solution, corresponding to a later time for which the student and bus are again at the same place if they continue their specified motions. Explain the significance of this second solution. How fast is the bus traveling at this point? (e) If the student's top speed is 3.5 m/s, will she catch the bus? (f) What is the *minimum* speed the student must have to just catch up with the bus? For what time and what distance does she have to run in that case?

**2.98** ••• An alert hiker sees a boulder fall from the top of a distant cliff and notes that it takes 1.30 s for the boulder to fall the last third of the way to the ground. You may ignore air resistance. (a) What is the height of the cliff in meters? (b) If in part (a) you get two solutions of a quadratic equation and you use one for your answer, what does the other solution represent?

**2.99** ••• A ball is thrown straight up from the edge of the roof of a building. A second ball is dropped from the roof 1.00 s later. You may ignore air resistance. (a) If the height of the building is 20.0 m, what must the initial speed of the first ball be if both are to hit the ground at the same time? On the same graph, sketch the position of each ball as a function of time, measured from when the first ball is thrown. Consider the same situation, but now let the initial speed $v_0$ of the first ball be given and treat the height $h$ of the building as an unknown. (b) What must the height of the building be for both balls to reach the ground at the same time (i) if $v_0$ is 6.0 m/s and (ii) if $v_0$ is 9.5 m/s? (c) If $v_0$ is greater than some value $v_{max}$, a value of $h$ does not exist that allows both balls to hit the ground at the same time. Solve for $v_{max}$. The value $v_{max}$ has a simple physical interpretation. What is it? (d) If $v_0$ is less than some value $v_{min}$, a value of $h$ does not exist that allows both balls to hit the ground at the same time. Solve for $v_{min}$. The value $v_{min}$ also has a simple physical interpretation. What is it?

# Answers

## Chapter Opening Question ?

Yes. Acceleration refers to *any* change in velocity, including both speeding up and slowing down.

## Test Your Understanding Questions

**2.1 Answer to (a): (iv), (i) and (iii) (tie), (v), (ii); answer to (b): (i) and (iii); answer to (c): (v)** In (a) the average *x*-velocity is $v_{av\text{-}x} = \Delta x/\Delta t$. For all five trips, $\Delta t = 1$ h. For the individual trips, we have (i) $\Delta x = +50$ km, $v_{av\text{-}x} = +50$ km/h; (ii) $\Delta x = -50$ km, $v_{av\text{-}x} = -50$ km/h; (iii) $\Delta x = 60 \text{ km} - 10 \text{ km} = +50$ km, $v_{av\text{-}x} = +50$ km/h; (iv) $\Delta x = +70$ km, $v_{av\text{-}x} = +70$ km/h; (v) $\Delta x = -20 \text{ km} + 20 \text{ km} = 0$, $v_{av\text{-}x} = 0$. In (b) both have $v_{av\text{-}x} = +50$ km/h.

**2.2 Answers: (a) *P*, *Q* and *S* (tie), *R*** The *x*-velocity is **(b)** positive when the slope of the *x-t* graph is positive (***P***), **(c)** negative when the slope is negative (***R***), and **(d)** zero when the slope is zero (***Q* and *S***). **(e) *R*, *P*, *Q* and *S* (tie)** The speed is greatest when the slope of the *x-t* graph is steepest (either positive or negative) and zero when the slope is zero.

**2.3 Answers: (a) *S*,** where the *x-t* graph is curved upward (concave up). **(b) *Q*,** where the *x-t* graph is curved downward (concave down). **(c) *P* and *R*,** where the *x-t* graph is not curved either up or down. **(d)** At *P*, $a_x = 0$ (velocity is **not changing**); at *Q*, $a_x < 0$ (velocity is **decreasing,** i.e., changing from positive to zero to negative); at *R*, $a_x = 0$ (velocity is **not changing**); and at *S*, $a_x > 0$ (velocity is **increasing,** i.e., changing from negative to zero to positive).

**2.4 Answer: (b)** The officer's *x*-acceleration is constant, so her $v_x$-*t* graph is a straight line, and the officer's motorcycle is moving faster than the motorist's car when the two vehicles meet at $t = 10$ s.

**2.5 Answers: (a) (iii)** Use Eq. (2.13) with $x$ replaced by $y$ and $a_y = g$; $v_y^2 = v_{0y}^2 - 2g(y - y_0)$. The starting height is $y_0 = 0$ and the *y*-velocity at the maximum height $y = h$ is $v_y = 0$, so $0 = v_{0y}^2 - 2gh$ and $h = v_{0y}^2/2g$. If the initial *y*-velocity is increased by a factor of 2, the maximum height increases by a factor of $2^2 = 4$ and the ball goes to height $4h$. **(b) (v)** Use Eq. (2.8) with $x$ replaced by $y$ and $a_y = g$; $v_y = v_{0y} - gt$. The *y*-velocity at the maximum height is $v_y = 0$, so $0 = v_{0y} - gt$ and $t = v_{0y}/g$. If the initial *y*-velocity is increased by a factor of 2, the time to reach the maximum height increases by a factor of 2 and becomes $2t$.

**2.6 Answer: (ii)** The acceleration $a_x$ is equal to the slope of the $v_x$-*t* graph. If $a_x$ is increasing, the slope of the $v_x$-*t* graph is also increasing and the graph is concave up.

## Bridging Problem

**Answer:** $h = 57.1$ m